flow rate to gas flow rate, and extremely small change of compositions in both phases, because the accuracy of $y^*$ is not satisfactory. In addition, this method usually requires a tedious procedure. Therefore, so long as the equilibrium relation is approximately linear, the modified Zivi-Brand method and the modified Nusselt solution, which do not require tedious calculations, are sufficiently available for the evaluation of the concentration of solute in both phases. On the other hand, when the equilibrium curves cannot be approximated by a linear function with respect to compositions, the graphical method is applicable to the practical absorption problem of cross-flow towers except for some special cases.

Nomenclature

\begin{align*}
T & = \text{local liquid temperature} \quad [\degree C] \\
V & = \text{effective tower volume} \quad [m^3] \\
W_G & = \text{average gas flow rate} \quad [\text{kg-mole/hr}] \\
W_L & = \text{average liquid flow rate} \quad [\text{kg-mole/hr}] \\
X & = \text{horizontal coordinate (gas flow direction)} \quad [m] \\
X_0 & = \text{dimensionless variable, defined by Eq.(30)} \quad [-] \\
\tilde{\alpha} & = \text{local concentration of solute gas in liquid phase (mole fraction)} \quad [-] \\
Y & = \text{vertical coordinate (liquid flow direction)} \quad [m] \\
Y_L & = \text{dimensionless variable, defined by Eq.(31)} \quad [-] \\
\tilde{\gamma} & = \text{local concentration of } y \text{ equilibrium with } \tilde{\alpha} \quad \text{(mole fraction)} \quad [-] \\
Z_0 & = \text{effective tower width} \quad [m] \\
\alpha & = L/m_H K_G a PV, \text{defined by Eq.(21)} \quad [-] \\
\beta & = G/K_G a PV, \text{defined by Eq.(21)} \quad [-] \\
\zeta & = \text{concentration ratio, defined by Eq.(19)} \quad [-] \\
\xi & = \text{dimensionless length, defined by Eq.(20)} \quad [-] \\
\text{(Subscripts)} \\
i & = \text{inlet mean} \\
o & = \text{outlet mean} \\
l, 2, 3, \ldots, 16 & = \text{position number, reference to Figs. 2 and 3}
\end{align*}

Literature Cited

3) Izumi, R.: Memoirs of the Faculty of Technology, Yamagata University, No. 2, 412 (1951)

THE ELUTRIATION RATE FROM A PACKED FLUIDIZED BED*

KUNIO KATO AND UTARO ITO
Department of Chemical Engineering, Gunma University, Kiryu, Japan

The particle elutriation rate from a packed fluidized bed consisting of particles of almost uniform size is measured for various reduced bed heights, $L_{mf}/L_T$, various reduced gas velocities $U/U_T$, and various packing sizes. An experimental equation for that rate is obtained.

The elutriation rate of fine particles from a packed fluidized bed comprising particles of two sizes is also measured under various experimental conditions. That rate is proportional to the concentration of fine particles in the bed. An experimental equation for the elutriation rate constant is obtained.

Finally, the mechanism of elutriation from the packed fluidized bed is discussed.

Introduction

In designing a particle-fluid reactor, such as a fluidized bed or a packed fluidized bed, it is quite important to estimate the particle elutriation rate from the column in order to determine the operating range of gas flow rate and to design a suitable particle collector. The elutriation from the fluidized bed has been studied by Leva1), Wen et al.4), Osberg et al.5), Zenz et al.6), Yagi et al.7) and Tanaka et al.8) But the elutriation from the packed fluidized bed has been little studied.

In this study, the elutriation rates from a packed fluidized bed consisting of particles of almost uniform size or particles of two sizes are measured under various operating conditions. Experimental equations

* Received on June 17, 1971
Presented at the 36th Annual Meeting of the Soc. of Chem. Engrs., Japan, April 4, 1971

144 (46) JOURNAL OF CHEMICAL ENGINEERING OF JAPAN
1. The Experimental Apparatus and Procedure

A schematic diagram of the experimental apparatus is shown in Fig. 1. The column, made of acrylic resin, is 80 mm I.D. and 1500 mm long. A porous brass plate of 2 mm thickness and 10 μm pore size is used as a gas distributor. Fluidized particles, packed materials and the systems of two-size particles used in this experiment are shown in Table 1.

A column is perfectly packed with packings and fluidized particles are put into the column until bed height at the minimum fluidization gas velocity is about 20~50 cm, and then are well fluidized with a suitable gas velocity. Suddenly, the gas velocity is quickly increased to a certain value. Particles elutriated from the column are separated from gas by a cyclone and are collected into a bag below the cyclone.

The weight of particles elutriated from the bed per unit time is measured by using a particle-path changer installed below the cyclone during a suitable time interval. The total amount of particles elutriated from the bed at any time after the elutriation is started is obtained from the weight of particles elutriated.

Bed height corresponding to the minimum fluidization gas velocity in the case of a uniform-size particle bed, and concentration of fine particles in the bed in the case of a two-size system are obtained from the total weight of particles elutriated.

To prevent static charging of the particles, the fluidized gas is saturated with water and packed materials are connected to an earthed line.

2. Experimental Results

2.1. The elutriation rate from a uniform particle size bed

The elutriation rate may be affected by the reduced gas velocity $U/U_T$, the reduced bed height $L_{mf}/L_T$, packing size, and the effective cross-sectional area of the bed. Fig. 2 shows the relation between the elutriation rate, $q$, and $L_{mf}/L_T$ in the case of PN-3
From Fig. 2, the elutriation rate is proportional to \((Lmf/\ell T)^m\). **Fig. 3** shows the relation between the elutriation rate and the reduced gas velocity, \(U/\ell T\), in the case of PN-3 packing and \(-48+65\) mesh glass beads. From Fig. 3, the elutriation rate is proportional to \(\exp(k_sU/\ell T)\).

In the packed fluidized bed, the growth of gas bubbles is restricted by the packing, and the fluidization behaviour changes with neither bed diameter nor bed height. Therefore, the elutriation rate from the column is proportional to the effective cross-sectional area of the bed, \(S_e\), and the density of fluidized particles. The elutriation rate from a column with a uniform-size particle bed may be expressed as follows;

\[
- \frac{q}{\rho_sS_e} = A_o(Lmf/\ell T)^m \cdot \exp(k_sU/\ell T)
\]

Constants \(m\) and \(k_s\) in Eq.(1) are affected by the packing size. The following empirical equations are obtained from this experiment.

\[
m = 5.5D_p^{-1.14}
\]

\[
k_s = 17.0D_p^{-1.8}
\]

In Eqs.(2) and (3), \(D_p\) is expressed in centimeters. Relation between \(q/\rho_sS_e\) and \((Lmf/\ell T)^m\) \(\exp(k_sU/\ell T)\) is shown in **Fig. 4**. The elutriation rate from a packed fluidized bed consisting of uniform size particles is

\[
- \frac{q}{\rho_sS_e} = 2.8 \times 10^{-4}(Lmf/\ell T)^m \cdot \exp(k_sU/\ell T)
\]

In Eq.(4), the value of \(m\) and \(k_s\) are obtained from Eqs.(2) and (3), respectively.

### 2.2. The elutriation rate from a packed fluidized bed consisting of two-size particles

If the fluidized particles in the bed are mixed perfectly, the following equation is obtained from the material balance of small-size particles in the bed.

\[
-(V \frac{dC}{d\theta} + C \frac{dV}{d\theta}) = \tau_eV
\]

where \(V\) is volume of fluidized bed corresponding to the minimum fluidization gas velocity, \(\ell T\) is the elutriation rate of small-size particles per unit time and unit volume.

The decrease of bed volume accompanied by the particle elutriation in \(d\theta\) is \(dV = -(\tau_eV/\rho_{sb})d\theta\). If the elutriation rate is proportional to the concentration of small-size particles in the bed, Eq.(5) becomes

\[
\frac{dC}{d\theta} = \frac{k}{\rho_{sb}} (C_t - \rho_{sb}C)
\]

where \(k\) is the elutriation rate constant [1/min].

Eq.(6) can be solved with the initial condition \(\theta = 0, C = C_0\).

\[
\ln \left(\frac{\rho_{sb}C_0 - C}{\rho_{sb}C_t - C_0}\right) = k\theta
\]

If the initial concentration of fine particles is very small, that is, \((\rho_{sb}C_0 - C)/(\rho_{sb}C_t - C_0) \approx 1\), Eq.(7) becomes

\[
-k(C_0 - C_t) = k\theta
\]

If the concentration of small-size particles in the bed is measured in an arbitrary time \(\theta\) after the experiment is started, the elutriation rate constant is obtained from Eq.(7) or (8). **Fig.(5-a)** and **Fig.(5-b)** show the relation between the concentration of small-size particles and \(\theta\) in the case of PN-5 and PN-1 packings, respectively, as some examples of this experimental data. From Fig.(5-a) and Fig.(5-b), the elutriation rate of small-size particles from a packed fluidized bed is also proportional to the concentration of small-size particles in the bed.

The elutriation rate constant in a packed fluidized bed may be considered as a function of the ratio of operation gas velocity to terminal velocity of small-size particles, \(U/\ell T\), the packing size, the length of freeboard, that is, \(Lmf/\ell T\), and the initial concentration of small-size particles.

From this experiment, the elutriation rate constant,
$k$, is proportional to $(L_{mf}/L_\tau)^{0.68}$ and $D_p^{-0.65}$, respectively, and the value of $k$ is affected by neither the initial concentration of small-size particles nor the size of the coarse particles in the bed. The elutriation of particles from the surface of the bed may be proportional to the number of bubble eruption in that surface. The elutriation rate, therefore, is proportional to the effective cross-sectional area of the bed. Fig. 6 shows the relation between $k/(D_p^{-0.65}(L_{mf}/L_\tau)^{0.68}Se)$ and $U/U_\tau$. From Fig. 6, the following experimental equation concerning the elutriation rate constant for a packed fluidized bed in the case of two-size particles is obtained.

$$k/Se = 1.27 \times 10^{-4} (L_{mf}/L_\tau)^{0.68} D_p^{-0.65} \exp \left(5.47 C_\mu / \pi^2 \right)$$  \hspace{1cm} (9)

In this experiment, the coarse-size particles are not elutriated from the column.

3. Discussion

If particles in the bed are fluidized with larger superficial gas velocity than the terminal velocity of these particles, the bed expansion is quite large. Many bubbles rise in the bed with high speed. These bubbles erupt at the bed surface and thrust masses of solids into the space above the bed. A part of these masses is elutriated from the column.

From these observations, the following mechanism concerning elutriation from a packed fluidized bed may be considered.

There are dense and dilute phases in the packed fluidized bed. The average bubble velocity in the bed is

$$U_b = \frac{RU-U_{em}}{R-1}$$  \hspace{1cm} (10)

In Eq.(10), $R$ is the bed expansion ratio, that is, $L/L_{mf}/U_{em}$ is an average gas velocity of dense phase and cloud. $U_{em}$ is of almost the same order of magnitude as $U_{em}$. The relation between bed expansion ratio and volume fraction of bubbles in the bed is expressed as

$$\frac{L-L_{mf}}{L} = \frac{R-1}{R} = \varepsilon_b$$  \hspace{1cm} (11)

The value of $\varepsilon_b$ increases as superficial gas velocity increases. From the geometrical arrangement of bubbles in the bed, the maximum value of $\varepsilon_b$ is about 0.6–0.65. The total volume flow rate of gas rising in the bed as gas bubbles is $S(L-L_{mf})U_b = S(R-1)/U_\tau$.

The average bubble diameter in the bed has not been observed yet, but it may be almost the same as the packing size in high superficial gas velocity. The number of bubbles broken at the bed surface is

$$n = \frac{S(R-1)/U_b}{\pi D_p^2/6}$$  \hspace{1cm} (12)

The rising velocity of particles in the cloud depends...
on the distance from the bubble. When bubbles erupt at the bed surface, these particles are thrown out into the space above the bed with different speeds. The total weight of these particles per unit time is

$$q_e = nC_f^* \frac{\rho}{\rho_0} (D_1 - D_2^f)$$

Where $C_f^*$ is the concentration of small-size particles in the cloud. If the bed consists of uniform-size particles, $C_f^*$ becomes the bulk density of particles.

If the particles in the freeboard are exposed to higher gas velocity than the terminal velocity of those particles, they rise up in the freeboard. Some of those particles are elutriated from the column.

If the superficial gas flow rate is increased, the bed expansion ratio is increased and the bubble velocity is also increased. Therefore the number of bubbles broken at the bed surface increases, the thickness of the bubble clouds, $T_h$, also increases, and then $q_e$ is increased extremely. The fraction of particles in freeboard which are exposed to higher gas velocity than the terminal velocity of those particles increases as the superficial velocity of gas is increased. Therefore, elutriation from a packed fluidized bed is quite sensitive to the superficial velocity of gas, as shown in Eqs. (4) and (9).

The effect of packing size upon elutriation rate is explained as follows. If the packing size is small, the average bubble size in the packed fluidized bed becomes small. The number of bubbles broken at the bed surface is, therefore, increased and the bed expansion ratio also becomes large. The elutriation rate from a packed fluidized bed increases as the packing size is decreased, as shown in Eqs. (4) and (9).

The effect of $L_{m_f}/L_T$ upon the elutriation rate is explained as follows. If $L_{m_f}/L_T$ is increased, the freeboard becomes small and even if the value of $q_e$ is not affected by that of $L_{m_f}/L_T$, the fraction of particles elutriated from the column becomes large. The elutriation rate from a packed fluidized bed increases as the value of $L_{m_f}/L_T$ is increased, as shown in Eqs. (4) and (9).

To compare the elutriation rate of fine particles from a packed fluidized bed with that from a fluidized bed, Fig. 7 shows the same correlation of elutriation rate constant for the packed fluidized bed and fluidized bed as that of Wen et al.4. In Fig. 7, $k'$ is

$$k' = \frac{kW/S}{60}.$$  

From Fig. 7, the elutriation rate from the packed fluidized bed is much larger than that from the ordinary fluidized bed. This fact is due to the following reasons. (1) As the bubble size is restricted by the packing, the number of bubbles broken at the bed surface is quite large. The value of $q_e$ is increased. Therefore, the elutriation rate from the packed fluidized bed is much larger than that from the ordinary fluidized bed. This tendency can be understood from Eqs. (4) and (9). (II) The bed expansion ratio of the packed fluidized bed is much larger than that of the ordinary fluidized bed under such a large superficial gas velocity. There is no clear transport disengaging height under this experimental condition ($L_{m_f}/L_T = 0.025 - 0.18$). When the packing size becomes small, the effect of $L_{m_f}/L_T$ upon the elutriation rate becomes sensitive, as shown in Eqs. (4) and (9). (III) The radial gas velocity distribution in the freeboard becomes uniform by the packing. The particles in the freeboard are, therefore, effectively elutriated from the column.

If a packed fluidized bed is used as a reactor and dense-phase fluidization is expected, it is not necessary to pack the packings in the whole column. But it is better to use uniform-size fluidized particles.

Conclusion

A correlation for the evaluation of the elutriation rate of particles from a packed fluidized bed consisting of uniform-size particles is proposed. The elutriation rate of fine particles from a packed fluidized bed consisting of a two-size system is propotional to the concentration of fine particles in the bed. A correlation for the evaluation of the elutriation rate constant is obtained.

The variables affecting the elutriation rate or the elutriation rate constant are as follows.

(1) The ratio of the superficial gas velocity to the terminal velocity of particles is the major factor influencing these values.

(2) The size of packing affects these values. If the size of packing is increased, the elutriation rate decreases.

148
Freeboard has an influence on the elutriation rate.

In a two-size system, the elutriation rate constant of fine particles is affected by neither the initial concentration of fine particles in the bed nor the size of the coarse particles.

Nomenclature

- \( A_0 \) = constant in Eq.(1) [cm/sec]
- \( C \) = concentration of fine particles [g/cc]
- \( C_0 \) = initial concentration of fine particles [g/cc]
- \( D_c \) = equivalent diameter of cloud [cm]
- \( D_p \) = volumetric equivalent diameter of packed materials defined as \( V/2d^2\pi \) [cm]
- \( d_p \) = diameter of fluidized particles [cm]
- \( d \) = diameter of cylindrical wire net [cm]
- \( g \) = gravitational acceleration [cm/sec²]
- \( h \) = length of cylindrical wire net [cm]
- \( k \) = elutriation rate constant [1/min]
- \( k' \) = elutriation rate constant [g/cm²-sec]
- \( k_s \) = constant in Eq.(4) [-]
- \( L \) = expanded bed height [cm]
- \( L_{mf} \) = bed height at minimum fluidization gas velocity [cm]
- \( L_T \) = distance between the gas distributor and the top of the column [cm]
- \( m \) = constant in Eq.(4) [cm]
- \( n \) = number of bubbles broken at the bed surface per unit time [1/min]
- \( q \) = elutriation rate [g/sec]
- \( q_E \) = amount of fluidized particles thrown out from the bed accompanied by eruption of gas bubbles per unit time [g/sec]
- \( R \) = bed expansion ratio [-]
- \( r_E \) = elutriation rate of fine particles [g/cm² min]
- \( S \) = cross-sectional area of the bed [cm²]
- \( T_h \) = thickness of cloud [cm]
- \( U \) = superficial gas velocity [cm/sec]
- \( U_b \) = gas velocity of bubbles [cm/sec]
- \( U_{em} \) = average gas velocity of emulsion phase and cloud [cm/sec]
- \( U_{mf} \) = minimum fluidization gas velocity [cm/sec]
- \( U_T \) = terminal velocity of particles [cm/sec]
- \( V \) = volume of bed corresponding to minimum fluidization gas velocity [cm³]
- \( \epsilon \) = packing voidage [-]
- \( \varepsilon_b \) = volume fraction of gas bubbles in bed [-]
- \( \rho \) = density of gas [g/cc]
- \( \rho_b \) = density of fluidized particles [g/cc]
- \( \rho_{eb} \) = bulk density of fluidized particles at minimum fluidized gas velocity [g/cc]
- \( \theta \) = time elapsed [min]
- \( \mu \) = gas viscosity [g/cm sec]

Literature Cited


DESIGN OF MULTISTAGE GAS-LIQUID REACTOR

MIKIO KAWAGOE, KATSUMI NAKAO AND TSUTAO OTAKE
Department of Chemical Engineering, Osaka University, Toyonaka, Japan

In order to take the dissolved gas in the bulk liquid into consideration, the absorption rate with chemical reaction and the overall reaction rate are analyzed separately on the basis of film theory. Using these rates, general design equations are derived for the multistage gas-liquid reactor.

Since stage-to-stage calculations with trial-and-error are required to solve the design equations, simplification of the solution is discussed for practical purposes.

As a result, a method of graphical analysis is presented for solving the design equations when dissolved gas in the liquid bulk may be neglected.

Furthermore, the relation between absorption and overall reaction rates is represented graphically to eliminate much of the tedious trial-and-error calculation required.

Introduction

Much experimental and theoretical work has been done in the field of gas absorption with chemical reaction and the overall reaction rate, and to both gas and liquid components, the latter being for situations where the rate controlling process is diffusion or chemical reaction. However, the design procedure for gas-liquid reactors has been developed only for reactions of first order with respect to the gas component alone and to both gas and liquid components, the latter being for situations where the rate controlling process is diffusion or chemical reaction.

Received on July 26, 1971
Presented at the 4th Autumn Meeting of the Soc. of Chem. Engrs., Japan, at Hiroshima, October 1970