we obtain

$$\left[w - \frac{2}{PeB} \frac{dw}{dz} \right]_{z=0} = \left[v + \frac{2}{PeB} \frac{dv}{dz} \right]_{z=1} e^{-\frac{PeB}{2}} \quad (132)$$

Therefore, it is obvious that condition (98) holds if and only if condition (104) holds, which completes the proof of Lemma 1.

Nomenclature

B	$= L_r/D_r$	[—]
C	$=$ concentration	[mol/l]
C_{in}	$=$ concentration at the reactor inlet	[mol/l]
C_p	$=$ heat capacity	[cal/g °C]
D_r	$=$ diameter of a reactor	[cm]
$e(z)$	$=$ function defined by Eq.(35)	
E	$=$ diffusion coefficient	[cm ² /sec]
$f(z)$	$=$ function defined by Eq.(36)	
$g(z)$	$= f(z) - e(z) - PeB/4$	
$H(\lambda)$	$=$ function of λ defined by Eq.(62)	
I	$=$ identity matrix	
K	$=$ matrix operator defined by Eq.(42)	
k_H	$=$ heat conductivity	[cal/cm sec °C]
L_r	$=$ length of a reactor	[cm]
p	$=$ eigenfunction vector of Eqs.(48), (49) and (50)	
P	$=$ matrix operator defined by Eq.(41)	
Pe_H	$=$ Péclet number for heat transfer	
Pe_M	$=$ Péclet number for mass transfer	
Q_c	$=$ total heat removal per unit time	
Q_H	$=$ total heat generation per unit time	
$R(C, T)$	$=$ rate of reaction	[mol/l sec]
t	$=$ time	[sec]
T	$=$ temperature	[°K]
T_{in}	$=$ temperature at the reactor inlet	[°K]
u	$=$ linear velocity	[cm/sec]
$u_{ij}(\lambda, z)$	$=$ entry of matrix $U(\lambda, z)$	
$U(\lambda, z)$	$=$ matrix defined by Eqs.(55), (56) and (57)	
$v(z)$	$=$ function defined by Eqs.(94), (95) and (96)	

$w(z)$	$=$ function defined by Eqs.(100), (101) and (102)	
x	$=$ distance from the reactor inlet	[cm]
z	$=$ x/L_r	
$r(\phi^*, \psi^*)$	$= (L_r/uC_{in})R(C, T)$	
$-\Delta H$	$=$ heat of reaction	[cal/mol]
ϵ_a	$=$ dimensionless activation energy	
η	$=$ function vector defined by Eqs.(51) (52) and (53)	
θ	$=$ ut/L_r	
κ	$=$ dimensionless frequency factor	
$\xi(z)$	$=$ function defined by Eqs.(106), (107) and (108)	
$\xi(z)$	$=$ function vector defined by Eqs.(70), (71) and (72)	
ρ	$=$ density	[g/l]
ϕ	$=$ dimensionless concentration deviation from the equilibrium state	
ϕ^*	$=$ dimensionless concentration	
Φ	$=$ dimensionless concentration at the equilibrium state	
Φ_{out}	$=$ assumed value of Φ at the reactor outlet	
$\chi(z)$	$=$ function defined by Eqs.(118), (119) and (120)	
ψ	$=$ dimensionless temperature deviation from the equilibrium state	
ψ^*	$=$ dimensionless temperature	
Ψ	$=$ dimensionless temperature at the equilibrium state	
Ψ_{in}	$=$ dimensionless temperature at the reactor inlet	
Ψ_{out}	$=$ assumed value of Ψ at the reactor outlet	

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SHORT COMMUNICATIONS

ON THE EXTENT OF THERMAL CONDUCTION REGIME IN ENCLOSED VERTICAL PLANE FLUID LAYERS*

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For fluid enclosed between two vertical plates at different temperatures, transport equations were integrated numerically by Poots' method, and a correlation Eq.(18) derived for estimation of Nu number. Moreover, a criterion of the conduction region were derived, using Eq.(18). Especially in the region of $L/d \gtrsim 20$, the criterion may be expressed by Eq.(20). This criterion agreed with experiments of Eckert et al and Elder, and is applied as a better criterion than Batchelor's.

When a thermal diffusion column is used to separate liquid solutions and mixed gases, it is known from the old experiments of Clusius-Dickel¹⁾ that

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natural convection in the fluid layer enclosed between two vertical plates at different temperatures plays an important part in increasing the separation. Since then the Clusius-Dickel type of column has been used to separate mixtures. On the other hand, Jones-Furry²⁾ developed a column theory to predict

Fig. 1 The coordinate system of enclosure

the separation degree. To develop the theory, they assumed that conduction heat transfer dominated in the column. Then it is necessary to answer the question of whether or not the heat transfer is dominated by thermal conduction under any condition. As investigations of the above, there are an approximate theory by Batchelor³⁾, experiments in gases by Eckert *et al.*⁴⁾ and experiments in liquids of high Prandtl number by Elder⁵⁾, and experiments and numerical solutions in gases for a system with small ratio of length to thickness by Hirata *et al.*⁶⁾

In this report, the transport equations for the enclosure between two vertical plates at different temperatures were integrated numerically by the method of Poots⁷⁾. Then a correlation was derived that the conduction region satisfied, and it was discussed and compared with Batchelor's equation and the experiments of Eckert *et al.* and Elder. As a result, it was found that the correlation could be applied sufficiently to predict the conduction region in thermal diffusion columns.

Basic Equations and Their Numerical Solutions

Consider a fluid enclosed between two vertical plates at different temperatures, as shown in **Fig. 1**. Let T_h and T_c be the absolute temperatures of the hot and cold walls, respectively. Assuming that (i) temperature difference $T_h - T_c$ is small compared with T_c , (ii) physical properties are constant except for the effect of density variation in producing buoyancy, (iii) fluid is incompressible and viscous heat dissipation may be neglected, (iv) motion is steady two-dimensional, (v) the boundary condition on the border strips is assumed to be either linear or of the form $T = T_c + (T_h - T_c)y/d$, then subject to the above restrictions the transport equations in the enclosure are expressed as follows:

continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

motion equations

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial P}{\partial x} - g\beta(T - T_c) + \nu \nabla^2 u \quad (2)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial P}{\partial y} + \nu \nabla^2 v \quad (3)$$

energy equation

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \nabla^2 T \quad (4)$$

The boundary conditions are

$$\begin{aligned} u=v=0, \quad T=T_c, \quad \text{at } y'=0 \\ u=v=0, \quad T=T_h, \quad \text{at } y'=d \\ u=v=0, \quad T=T_c + (T_h - T_c)y'/d \quad \text{at } x'=0, L \end{aligned} \quad (5)$$

Defining dimensionless variables and stream function by

$$\begin{aligned} x = x'/d, \quad y = y'/d, \quad \theta = (T - T_c)/(T_h - T_c) \\ u = (\alpha/d) \frac{\partial \phi}{\partial y}, \quad v = -(\alpha/d) \frac{\partial \phi}{\partial x} \end{aligned} \quad (6)$$

respectively, then on eliminating the pressure P the transport equations become

$$\begin{aligned} \nabla^2 \theta &= \frac{\partial \theta}{\partial x} \frac{\partial \phi}{\partial y} - \frac{\partial \theta}{\partial y} \frac{\partial \phi}{\partial x} \\ \nabla^4 \phi &= Ra \frac{\partial \theta}{\partial y} + \frac{1}{Pr} \left\{ \frac{\partial}{\partial x} (\nabla^2 \phi) \frac{\partial \phi}{\partial y} - \frac{\partial}{\partial y} (\nabla^2 \phi) \frac{\partial \phi}{\partial x} \right\} \end{aligned} \quad (7)$$

For convenience convert the region of integration from a rectangle to a square region of unit dimension by the substitutions

$$x = k\xi, \quad y = \eta \quad (8)$$

also assume that θ is expressed as the sum of the term from the thermal heat conduction and the term from the convection heat transfer

$$\theta(\xi, \eta) = \eta + \Theta(\xi, \eta) \quad (9)$$

Then the equations become

$$\begin{aligned} \nabla_k^2 \Theta &= \frac{1}{k^2} \frac{\partial^2 \Theta}{\partial \xi^2} + \frac{\partial^2 \Theta}{\partial \eta^2} = \frac{1}{k} \left(\frac{\partial \Theta}{\partial \xi} \frac{\partial \phi}{\partial \eta} - \frac{\partial \Theta}{\partial \eta} \frac{\partial \phi}{\partial \xi} - \frac{\partial \phi}{\partial \xi} \right) \\ \nabla_k^4 \phi &= Ra \left(1 + \frac{\partial \Theta}{\partial \eta} \right) + \frac{1}{kPr} \left\{ \frac{\partial}{\partial \xi} (\nabla_k^2 \phi) \frac{\partial \phi}{\partial \eta} - \frac{\partial}{\partial \eta} (\nabla_k^2 \phi) \frac{\partial \phi}{\partial \xi} \right\} \end{aligned} \quad (10)$$

The boundary conditions are

$$\begin{aligned} \phi = \frac{\partial \phi}{\partial \eta} = 0, \quad \Theta = 0, \quad \text{at } \eta = 0, 1 \\ \phi = \frac{\partial \phi}{\partial \xi} = 0, \quad \Theta = 0, \quad \text{at } \xi = 0, 1 \end{aligned} \quad (11)$$

To evaluate Θ and ϕ , let us assume that Θ and ϕ may be represented by a complete double series of orthogonal functions

$$\Theta(\xi, \eta) = \sum_{p,q=1}^{\infty} A_{p,q} \sin(p\pi\xi) \cdot \sin(q\pi\eta) \quad (12)$$

and

$$\phi(\xi, \eta) = \sum_{p,q=1}^{\infty} B_{p,q} X_p(\xi) X_q(\eta) \quad (13)$$

The double Fourier series expansion (12) satisfies boundary conditions (11), and expression (13) satisfies boundary conditions (11), provided the functions $X_p(\xi)$ are chosen to satisfy the fourth order Sturm-Liouville equation

$$\frac{d^4 X_p}{d\xi^4} = \mu_p^4 X_p \quad (14)$$

with $X_p(0) = X_p(1) = X_p'(0) = X_p'(1) = 0$ where a prime denotes differentiation with respect to ξ .

$A_{p,q}$ and $B_{p,q}$ are to be evaluated so that Eqs.(12)

Fig. 2 Mean Nusselt number vs. Pr , taking Ra and L/d as parameters

and (13) satisfy Eq.(10). By the substitution of Eqs.(12) and (13) in Eq.(10), an infinite set of algebraic equations in $A_{p,q}$ and $B_{p,q}$ are obtained as follows, using the orthogonality of the expansion functions:

$$\begin{aligned} \left(\frac{1}{k^2}m^2 + n^2\right)A_{m,n} &= -\frac{1}{k\pi^2} \sum_{r,s=1}^{\infty} B_{r,s}(a_m^s b_n^r) + \gamma_{m,n} \quad (15) \\ \left(\frac{1}{k^4}\mu_m^4 + \frac{2}{k^2}D_{mm}D_{nn} + \mu_n^4\right)B_{m,n} &= RaC_mC_n \\ -\frac{Ra}{4} \sum_{p,q=1}^{\infty} A_{p,q}(a_p^m b_q^n) - \frac{1}{k^2} \sum_{p,q=1}^{\infty} B_{p,q}D_{pm}D_{qn} + \lambda_{mn} \end{aligned} \quad (16)$$

By solving a set of simultaneous algebraic equations of $A_{p,q}$ and $B_{p,q}$, the temperature and velocity profile in the enclosure are evaluated. In evaluating $A_{p,q}$ and $B_{p,q}$, the method can be considered as approximate since the infinite double expansions have been taken as finite, although sufficient coefficients have been computed in each case to ensure that there is no change in the leading terms. From the temperatures profile evaluated, Nu number is given by

$$\begin{aligned} Nu &= \frac{hd}{\lambda} = \frac{d}{\lambda} \cdot \frac{-\int_0^l (\lambda \cdot \partial T / \partial y')_{y'=0} dx'}{T_h - T_c} \\ &= 1 + \sum_{p,q=1}^{\infty} A_{2p+1,2q} \frac{4q}{2p+1} \end{aligned} \quad (17)$$

In the right-hand side of the above equation, the first term indicates the effect of conduction heat transfer and the second term the effect of convection heat transfer. Therefore, the conduction region is considered as the region where the second term is effectively smaller than unity.

Fig. 3 Mean Nusselt number vs. Ra , taking L/d as a parameter

Discussion Results

From Eqs.(15) and (16), coefficients $A_{2p+1,2q}$ are considered as a function of three parameters of (L/d), Pr and Ra . To estimate the effect, **Figs. 2, 3** and **4** indicate the numerical results of Eq.(17).

From Fig. 2, it is found that $Nu-1$ is not dependent on Pr at constant values of L/d and Ra . In Eqs.(15) and (16), Pr number occurs in the term λ_{mn} . This term results from the inertia terms of motion equations. As natural convection is small in the enclosure, the inertia terms are negligible. So it may be considered that the effect of Pr is negligible in the conduction region.

Fig. 3 indicates a correlation between $Nu-1$ and Ra at constant values of L/d and Pr . $Nu-1$ is proportional to $Ra^{2.0}$ in the region of $Ra \lesssim 5 \times 10^3$.

Fig. 4 indicates the correlation of $(Nu-1)/Ra^{2.0}$ and L/d . A maximum value of $Nu-1$ exists near $L/d \approx 2$. This is observed experimentally⁶.

In the region of

$$0.1 \leq Pr \leq 10, \quad Ra \lesssim 5 \times 10^3, \quad 1 \leq L/d \leq 100$$

the following correlation may be obtained.

$$\begin{aligned} Nu-1 &= Ra^{2.0} \times 10^{-6} \left[\frac{0.8}{(L/d)^{2.0} + 21 \cdot (L/d)^{-3.0}} \right. \\ &\quad \left. + \frac{10}{(L/d)^{2.0} + 2^{10} \cdot (L/d)^{-1.0}} \right] \end{aligned} \quad (18)$$

Fig. 4 $(Nu-1)/Ra^{2.0}$ vs. L/d

If the condition of the conduction regime is estimated from the condition that the relation $Nu-1$ is valid within 10%, the criterion becomes

$$Ra^{2.0} \times 10^{-6} \left[\frac{0.8}{(L/d)^{2.0} + 21 \cdot (L/d)^{-3.0}} + \frac{10}{(L/d)^{2.0} + 21 \cdot (L/d)^{-1.0}} \right] \leq 0.1 \quad (19)$$

The thermal diffusion column is used to be designed at large values of L/d . From this point, in the region of $L/d \geq 20$, Eq.(19) becomes

$$Ra(L/d)^{-1} \leq 100 \quad (20)$$

On the other hand, in the region of $L/d \leq 1$, Eq.(19) becomes

$$Ra[2.4(L/d)^3 + 0.625(L/d)]^{1/2} \leq 800\sqrt{10} \quad (21)$$

Eq.(20) may be considered as the operating criterion of the thermal diffusion column. In case an operating criterion does not satisfy Eq.(20), the temperature profile in the column deviates from linearity, and has a large gradient near walls and a small gradient in the central region of the column. In this condition, the separation degree of mixtures seems to decrease in the thermal diffusion column.

Fig. 5 indicates the extent of various regimes obtained from experimental results. The curve, labeled $Nu-1 \leq 0.1$, shows Eq.(19). An experimental result at $L/d=20$ that seems to be in the conduction regime is out of the criterion of Eq.(19). If a criterion of $Nu-1 \leq 0.15$ is used to determine the extent of conduction regime, all experimental results may be explained sufficiently. Eckert *et al.* obtained an experimental correlation as shown in Fig. 5. Batchelor presented the following equation for the extent of the conduction region.

$$Ra(L/d)^{-1} \leq 500$$

Fig. 5 Extent of conduction regime

This criterion is not valid to predict the critical condition between the conduction region and the transition region, because experiments of Eckert *et al.* indicate that Batchelor's criterion exists in the transition region as shown in Fig. 5. If the criterion were derived from Eq.(18) in the region of $L/d \geq 20$, the following relation must be satisfied.

$$Nu-1 \leq 2.5$$

The above cannot be applied to determine the criterion.

From Fig. 5, it may be found that Eq.(20) can be applied as a better criterion.

Nomenclature

$A_{p,q}, B_{p,q}$ = expansion coefficients defined by Eqs.(15) and (16).

$$a_m^s = 2 \int_0^1 x_s(\xi) \sin(m\pi\xi) d\xi$$

$$b_m^r = 2 \int_0^1 X_r'(\eta) \sin(m\pi\eta) d\eta$$

$$C_m = \int_0^1 X_m d\xi$$

$$D_{mn} = \int_0^1 X_m'' X_n d\xi$$

d = thickness of fluid layer

g = gravitational acceleration

$$H(q, s, n) = \int_0^1 X_q'' X_s' X_n d\xi$$

$$K(p, r, m) = \int_0^1 X_p''' X_r X_m d\xi$$

$$L(s, q, n) = \int_0^1 X_q' X_s X_n d\xi$$

L = height of fluid layer

P = pressure

Pr = Prandtl number

Ra = Rayleigh number

$$R(r, p, m) = \int_0^1 X_r(\xi) \cos(p\pi\xi) \sin(m\pi\xi) d\xi$$

$$T(s, q, n) = \int_0^1 X_s(\eta) \sin(q\pi\eta) \sin(n\pi\eta) d\eta$$

T = temperature

T_c, T_h = temperature of cold and hot wall

u, v = velocity in the x and y direction

x = distance from corner

y = distance from wall
 α = thermal diffusivity
 β = thermal expansion coefficient
 $\gamma_{mn} = -(1/4k) \sum_{p,q=1}^{\infty} A_{p,q} \left[B_{r,s} \left\{ pR(r,p,m)T(s,q,n) - qR(s,q,n)T(r,p,m) \right\} \right]$
 ν = kinematic viscosity
 ρ = density of fluid
 $\theta = (T - T_c)/(T_h - T_c)$
 $\Theta = \theta - \eta$
 ψ = stream function defined by Eq.(18)
 $\lambda_{mn} = (1/kPr) \sum_{p,q=1}^{\infty} B_{p,q} \left[\sum_{r,s=1}^{\infty} B_{r,s} \left\{ (1/k^2)K(p,r,m)L(s,q,n) + H(q,s,n)L(p,r,m) - (1/k^2)H(p,r,m)L(q,s,n) - K(q,s,n)L(r,p,m) \right\} \right]$

$\xi = x/(L/d)$
 $\eta = y$ } non-dimensional variables defined by Eq.(10)

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AGITATION EFFECT ON THE SEPARATION OF BINARY MIXTURE IN HIGH VACUUM DISTILLATION*

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An experiment on distillation of E.H.P.-mT.C.P. system was performed by use of pot still in a range of residual air pressure from 1.0×10^{-3} mmHg to 3.5×10^{-1} mmHg. The effects of residual air pressure and liquid surface temperature on relative volatility have become clear. The agitation effect on relative volatility was also investigated.

An experiment on distillation of E.H.P.(di-2-ethylhexyl-phthalate)-mT.C.P.(m-tricresyl-phosphate) mixture was performed by use of a pot still in a range of residual air pressure from 1.0×10^{-3} mmHg to 3.5×10^{-1} mmHg. The effects of residual air pressure and liquid surface temperature on relative volatility were investigated. The liquid phase in the pot still was agitated by a magnetic stirrer, and the agitation effect on relative volatility was investigated. In this work, the authors could not discover a torpid liquid surface^{1,4)}, and we considered that there is no effect of torpidity.

Experiments and Discussion

The experiments were performed by use of the pot still described in the previous paper³⁾. At six residual air pressures (1.0×10^{-3} mmHg, 3.5×10^{-3} mmHg, 1.0×10^{-2} mmHg, 3.5×10^{-2} mmHg, 1.0×10^{-1} mmHg and 3.5×10^{-1} mmHg), the distillation of E.H.P.-mT.C.P. system was performed in the composition range from 30 mole% to 35 mole%. The relative volatility (α) was calculated by Eq.(1).

$$\alpha = y(1-x)/(1-y)x \quad (1)$$

The effects of liquid surface temperature and residual air pressure on relative volatility

The variation of α with liquid surface temperature is shown in Fig. 1. At constant liquid surface temperature, α increases as residual air pressure is raised. On the other hand, α increases as the liquid surface temperature is lowered at constant residual air pressure.

The agitation effect on relative volatility

The liquid phase in the pot still was agitated by a magnetic stirrer. The rotational speeds were 180 r.p.m., 240 r.p.m., 300 r.p.m., and 360 r.p.m.. The agitation effects on α are shown in Fig. 2, which shows that the most effective revolution speed is 300 r.p.m., independent of residual air pressure. The agitation effect on α increases as liquid surface temperature is lowered. At high liquid surface temperature, there is no agitation effect on α .

Up to a rotational speed of 300 r.p.m., the value of α increases with rotational speed. But the value of α measured at a rotational speed of 360 r.p.m. was smaller than that measured at 300 r.p.m.. The authors consider that these facts may be explained by the following reasons. Sufficient disorder in liquid phase is not yet obtained at a rotational speed

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