EVALUATION OF PERFORMANCE OF MIXING APPARATUS FOR HIGH VISCOSITY FLUIDS

YASUHIRO MURAKAMI, KATSUMASA FUJIMOTO, TAKAFUMI SHIMADA**, AKIO YAMADA AND KENJI ASANO
Department of Chemical Engineering, Kyushu University, Fukuoka, Japan

There exists no universally accepted technique for evaluating the functional status of equipment for high-viscosity mixing based upon generalized basic information such as flow patterns and shearing deformation characteristics. This paper deals with the measurement of flow patterns. After obtaining the overall flow patterns in a mixing vessel, circulation capacity and fluid deformations which cover information about energy dissipation were correlated with the homogenizing time. The physical meanings of $C_1 = n T_M$ and $C_2 = T_M / \mu$, are interpreted by using a concept of striation thickness.

A measure of an averaged dimensionless shear rate number $\frac{\mu}{P_g \mu / n}$ is plotted against the dimensionless nearest distance between moving part and fixed wall in the mixing vessel. This diagram could give an useful comparison as to averaged shear rate characteristics for any device for high-viscosity mixing regardless of geometrical configuration. A novel technique is also proposed for quick evaluation of the shear distribution in a given apparatus.

Introduction

There have been a few studies of devices for high-viscosity mixing. However, there exists no universally accepted technique for evaluating their functional status based upon generalized basic information such as flow patterns and shearing and tensile deformation characteristics in the mixing equipment. This paper deals with the measurement of flow patterns and power consumption in several types of mixers used in the high-viscosity range, including anchor, paddle and double-helical ribbon mixers and an apparatus with two agitator axes having multidisks.

After obtaining the overall flow patterns in a mixing vessel, circulating capacity and fluid deformations which cover information as to the energy dissipation in an agitated field are correlated with the homogenizing time.

As a first conclusion, the authors give a physical meaning to the mixing number, $C_2 = n T_M$, where $n$ is rotational speed and $T_M$ is mixing time. The relation between power consumption and mixing time is also considered to evaluate the shearing functions of several widely used high-viscosity mixing devices from a practical point of view. As a second conclusion, the authors give a physical meaning to the constant, $C_2 = T_M / \mu$, by using a simplified form of Mohr's striation thickness theory.

As will be shown later, a measure of an averaged dimensionless shear rate number, $\frac{\mu}{P_g \mu / n}$, is plotted against the dimensionless nearest distance between moving part and fixed wall in the mixing vessel. This diagram could give an useful comparison as to averaged shear rate characteristics for any high-viscosity mixing device regardless of geometrical configuration.

A novel technique is also proposed for quick evaluation of the shear distribution in a given apparatus. Part of the experimental results were obtained by using deformation characteristics of fine copper wires.

1. Flow Patterns

1.1 Experimental apparatus and procedures

Impellers and vessels used for the measurement of flow patterns were made of acrylic resin, with geometrical configuration as shown in Fig. 1. The cylindrical vessel was enclosed in a square-sided acrylic vessel filled with the same fluid to reduce optical distortion.

The velocity was determined by taking photographs of tracer polystyrene spheres which were illuminated with a slit ray of a mercury lamp. Fig. 2 shows the camera angle and the direction of slit ray, in which the slit widths were 3 mm and 5 mm for horizontal sections and vertical sections, respectively. For horizontal sections, the camera and the impeller
were synchronously rotated to observe the stream lines simultaneously. The slit widths were determined by considering fluid velocity through the slit width and the shutter speed of the camera.

The average diameter and density of tracer polystyrene spheres used were 0.047 cm and 1.04 g/cm³, respectively. The terminal velocity of tracers was about 0.00022 cm/sec in this experiment, which was about 0.015% of blade-tip velocity and about 0.30% of average axial velocity $v_z$. Aqueous solutions of corn syrup, with a viscosity of about 200 poises, were used as highly viscous materials. Velocity data were obtained by measuring tracer loci on doubly enlarged photographs with slide calipers.

### 1.2 Resulting flow patterns

Figs. 3 and 4 show examples of tracer loci. Figs. 3, 4, 5 and 6 show representative flow patterns for four types of mixers.

**Anchor:** Tangential flow is dominant and becomes smaller with distance away from the impeller. Axial flow does not exist except in the bottom region at a low Reynolds number. Radial flow is recognizable near the impeller, but it should be noted that the essential exchanges in a radial direction hardly exist because of creeping flow, as shown in Fig. 3.

**Large paddle:** Axial flow, as in the case of the anchor mixer, does not exist. On the whole, shear action is strong but the flow is a circulating flow. Therefore the fluid elements are considerably sheared, but there is no exchange of fluid elements in either hemicylinder on either side of the paddle.

**Helical ribbon:** Tangential flow is almost the same as for the anchor. Axial flow is strongly recognizable,
forming an assumed complex flow tube which has an elliptical section along the blade. The fluid elements are circulated in this flow tube, where the shear action is poor inside the tube (Region II) and strong on the outside (Region I). They are given a high shear action and there exists a strong exchange of fluid elements. Radial flow is not so large.

Mixing apparatus with two agitator axes having multidisks: This apparatus has a wide free surface, and the region of free surface rotates almost like a solid body. Shear action suffers between disks. In this region of free surface, although quantitative measurement could not be made, axial flow is recognized from visual observation of tip tracer. Therefore, this region is good enough for axial circulation and delivers to the shear region between disks. However, this axial circulation is not so large.

1.3 Distributions of dissipation energy

From the flow pattern, the distribution of dissipation energy was obtained by the following formula.

\[ P_{\text{g,e}/\mu} = 2 \left( \frac{\partial v_r}{\partial r} \right)^2 + \left( \frac{1}{r} \left( \frac{\partial v_r}{\partial \theta} + v_r \right) \right)^2 + \left( \frac{\partial v_z}{\partial z} \right)^2 \]

\[ + \left( \frac{\partial v_r}{\partial \theta} \right)^2 + \left( \frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z} \right)^2 \]

\[ + \frac{1}{r^2} \frac{\partial v_r}{\partial r} + \frac{\partial v_r}{\partial r} \left( \frac{\partial v_r}{\partial r} \right) \]

\[ A, P, H \]

Fig. 7 shows the distributions in a given horizontal section. It becomes clear that the high-shear region is maldistributed near the blade. Consequently, for high-viscosity apparatus it is a problem how the fluid moves into and out of the high-shear region. In Eq.(1) the underlined terms A, P and H are dominant for an anchor, a paddle and a helical ribbon, respectively.

2. An Evaluation of Circulation Capacity of High-Viscosity Mixing Devices

The flow pattern is a very important factor in moving fluid readily into every part of a vessel. The relation \( nT_M = C_1 \) might be connected with axial circulation flow rate. The relations between the circulation capacity given by the flow pattern and a constant \( C_1 \) are summarized in Table 1. They are well correlated as \( nT_M = 3N_e \) by considering axial flow rate only. An axial circulation capacity may also be considered as an efficiency which shows whether the supplied power is used effectively in mixing processes.

3. A Process of Homogenizing by Shear Action in High-Viscosity Mixing Devices

In this section, the shear rate obtained from the flow pattern is correlated with homogenizing time in high-viscosity mixers in comparison with a uniform shear mixer. The following discussion is based on a concept of striation thickness by W. D. Mohr5,8). The physical meaning of the criterion of final state of homogenization by conventional methods does not seem to be evident.

3.1 Simplification of complicated mixing system

Applying the theory of Mohr’s striation thickness to a uniform shear field, the following formula is derived.

\[ \frac{L_0}{L_M} = M = \frac{T_M}{T_M} \quad (M \gg 1) \]

Where \( L_0 \) and \( L_M \) are the scales of fluid element at the original and the final state of mixing, respectively. In an industrial operation, the original scale, \( L_0 \), is considered to be the diameter of inlet pipe or of mixing vessel and the final scale, \( L_M \), is determined as desired for each mixing system.

A tangential annular flow between two coaxial cylinders is taken as a uniform shear field model. If the ratio \( \kappa \) of inner to outer diameter is assumed as \( \kappa = 1 \), the shear rate becomes \( M = 2\pi \kappa n/(1-\kappa) \). Therefore Eq.(2) gives the following relation for mixing time.

\[ nT_M = \frac{L_0}{L_M} \left( \frac{1}{1-\kappa} \right) \]

On the other hand, the power consumption is shown as

\[ \sqrt{P_{\text{g,e}/\mu}} = 4\pi nL_M/(1-\kappa) \]

From Eqs.(3) and (4),

\[ T_{\text{ny/P_{g,e}/\mu}} = [2/(1+\kappa)]L_0/L_M \]

The above relation seems to explain the physical meaning of \( T_{\text{ny/P_{g,e}/\mu}} \) is const., that is, the term \( 2/(1+\kappa) \) may be a constant depending on the geometrical configuration of the apparatus, and the term \( L_0/L_M \) may be a constant depending on the essential scale of fluid elements at the final state of mixing. The former term contains complicated factors for generally used mixers.

It is for the case of \( \kappa = 1 \) that the stage of homogenizing process could be considered ideal.

The ideal limit of \( T_{\text{ny/P_{g,e}/\mu}} \) may be equal to

<table>
<thead>
<tr>
<th>Impeller</th>
<th>( N_p )</th>
<th>( N_e )</th>
<th>( N_r )</th>
<th>( 3N_e )</th>
<th>( nT_M )</th>
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<tr>
<td>Anchor</td>
<td>0.37</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td>Ribbon</td>
<td>0.37</td>
<td>0.081</td>
<td>11</td>
<td>33</td>
<td>33</td>
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<tr>
<td>Screw</td>
<td>0.42</td>
<td>14</td>
<td>42</td>
<td>45</td>
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</table>

Table 1 Mixing time and circulation capacity

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Table 2 Mixing characteristics

<table>
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<tr>
<th>Investigator</th>
<th>$nT_M$</th>
<th>$T_MI/P_{edg}[/]$</th>
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<tr>
<td>Helical ribbon</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nagata et al.</td>
<td>33</td>
<td>600</td>
</tr>
<tr>
<td>Gray</td>
<td>25.3</td>
<td>500</td>
</tr>
<tr>
<td>Helical screw</td>
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<td></td>
</tr>
<tr>
<td>Nagata et al.</td>
<td>45</td>
<td>400</td>
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<tr>
<td>Gray</td>
<td>220</td>
<td>900</td>
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<tr>
<td>Apparatus with two agitator axes having multidisks</td>
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<td></td>
</tr>
<tr>
<td>$l/De$</td>
<td>$nT_M$</td>
<td>$T_MI/P_{edg}[/]$</td>
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<tr>
<td>0.2</td>
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<td>721</td>
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<td>0.6</td>
<td>44.2</td>
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<tr>
<td>0.8</td>
<td>45.8</td>
<td>554</td>
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Table 3 Mixing time and striation thickness for helical ribbon mixer

<table>
<thead>
<tr>
<th>$L_0$ [cm]</th>
<th>$T_M$ [sec]</th>
<th>$L_{M,p}$ [cm]</th>
<th>$L_{M,f}$ [cm]</th>
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<tr>
<td>0.70</td>
<td>230</td>
<td>$1.18 \times 10^{-2}$</td>
<td>$2.02 \times 10^{-2}$</td>
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<tr>
<td>1.00</td>
<td>236</td>
<td>1.65</td>
<td>2.81</td>
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<td>1.00</td>
<td>200</td>
<td>1.96</td>
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<td>1.23</td>
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<td>1.23</td>
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<td>1.70</td>
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</tr>
<tr>
<td>2.00</td>
<td>317</td>
<td>2.45</td>
<td>4.19</td>
</tr>
<tr>
<td>av.</td>
<td></td>
<td>$2.00 \times 10^{-2}$</td>
<td>$3.42 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

In widely used high-viscosity mixing devices which have good circulation, the following relations have been generally concluded to exist in laminar flow system, as shown in Table 2.

$$nT_M = C_1 \text{ (const.)} \quad (6)$$

$$T_MI/P_{edg} = C_2 \text{ (const.)} \quad (7)$$

The scaling-up under the condition of Eq.(6) is considered to be that under a constant scale ratio. The observed scale $L_W$ is determined dependently on the mixing-time criterion system. Therefore, it should be emphasized that the relation of Eq.(6) or (7) does not provide the condition of the scaling-up measure based on the absolute scale of fluid elements, but on the scale ratio.

In the case of extending the above discussion to generally used mixers, it is required for the content of the term $\sqrt{P_{edg}}[/]$ that shear action is dominant in the mixing device. The authors considered a helical ribbon mixer as a generally used mixer, in which shear action seems to be fairly dominant (80%) from the analysis of Eq.(1). Two types of amount of shear in a helical ribbon mixer are derived. One is the product $M_p$ of $T_M$ by $\sqrt{P_{edg}}[/]$ from power consumption, and the other is the product $M_f$ of $T_M$ by $\bar{M}$ from the flow pattern as follows.

$$M_p = 19.3 nT_M \quad (8)$$

$$M_f = N_c(M_1 + N_4 t_1) \quad (9)$$

$$T_M = N_c(t_1 + t_4) \quad (10)$$

The latter is the mean amount of shear, considering the flow rate as follows.

$$\bar{M} = \int_{y_1}^{y_f} \frac{M_1 ds}{v d s} \quad (11)$$

where $t_1 = t_4$, $M_1 = 18.6 n$, $M_f = 4.0 n$

$$M_f = 11.3 nT_M \quad (11)$$

The measurement of mixing time is carried out by a method of reaction of I$_2$-Na$_2$S$_2$O$_3$ (6), in which a minor component is I$_2$-corn syrup solution, 15.5 g I$_2$/kg corn syrup solution, and a major component is Na$_2$S$_2$O$_3$-corn syrup solution, 15.4 g Na$_2$S$_2$O$_3$-5H$_2$O/kg corn syrup solution. The shear rate in a cone rotor mixer and the averaged shear rate in a helical ribbon mixer derived from power consumption were made nearly identical, 2.4 sec$^{-1}$ and 2.57 sec$^{-1}$ respectively, because a chemical reaction is used as criterion of the final state of mixing.

3.3 Results and discussion

The scale $L_W$ at the adjudged final state of mixing for a cone rotor mixer is determined as $L_W = 0.0018$ cm, although the criterion is fairly delicate.

The experimental results for a helical ribbon mixer are summarized in Fig. 8 and Table 3. Table 3 provides the value of two kinds of final scale, $L_{M,f}$ and $L_{M,p}$, calculated by Eqs.(8) and (11) as to different values of original scale, $L_0$, through the mixing time, $T_M$, determined by the above-mentioned method. From the above-mentioned considerations...
and experimental results, the relation of Eq. (5) seems to be available for mixing equipment dominated by shear rate action. Two types of value of scale $L_M$ are calculated by using the two types of shear rate stated previously. $L_{Mf}=0.0020$ cm by Eq. (8) and $L_{Mr}=0.0034$ cm by Eq. (11). On comparing the two values, the scale $L_{Mr}$ from the power consumption agree fairly well with the scale $L_M$ of a cone rotor mixer.

4. Shear Characteristics of High-Viscosity Mixing Devices

The following method seems to give a useful measure of evaluating the shear characteristics in the mixer. The object of this method is an evaluation of some high-viscosity mixers and some flow fields regardless of geometrical configurations of the mixing device.

This method was suggested by Mr. H. Kajimoto of Mitsubishi Heavy Industries Ltd., and the authors developed the method to evaluate the shearing characteristics of various kinds of apparatus. The authors take $\sqrt{P_g/\mu \mu/n}$ or $D/P_g/\mu \mu/a$ as a measure of dimensionless shear rate, which is obtained from the equation of correlation to power consumption. It has another variable indicating the close clearance between rotator or moving part and fixed wall.

This is the ratio $\kappa$ of impeller diameter to that of the vessel for generally used agitator systems, or inner to outer diameter for cylindrical annulus, the ratio $C/D$ of representative clearance to vessel diameter for an apparatus having any clearance, or the ratio $l/D_d$ of the distance between disks to the disk diameter for an apparatus with two agitator axes having multidisks. These ratios can be considered to be one criterion of shear field.

Fig. 9 shows these relations for some typical mixing devices and flow system. This figure gives some practical techniques for evaluating the state of averaged shear characteristics for the apparatus.

5. Distributions of Shear Rate in High-Viscosity Mixing Devices

The authors propose a novel technique of quick evaluation of shear rate distribution in a mixing device. This method makes use of the plastic deformation of fine copper wires. By assuming a simplified flow field, as shown in Fig. 10, the distribution of deformation angle of the wires is converted to that of shear rate as follows.

$$q = \frac{F_l}{\mu} \left( \frac{m}{m+2} \right)^{m+2/m} \left( \frac{m}{m+2} \right)^{m+2/m}$$

$$q = 14.3 \theta^{m/3} l^{m/3}$$

where $m=3/5$ and $F_l=2.98$ [G-cm$^{m/5}$], which were determined experimentally for the copper wires used. For the drag coefficient for a circular cylinder,

$$C_d = 8\pi [5 - \log(Re/8)]$$

$$= 1.6/Re \quad \text{for} \ 10^{-2} < Re < 10^{-5}$$

$$q = 0.8\mu (v_1 - v_2)/g_c = 0.8\mu l_w \dot{N}/g_c$$
The above situation for flow field is not strictly valid for actual flow, but this method is a clue to understanding flow field characteristics.

Annealed straight copper wires used in this experiment are of \( l_w = 2.0 \ \text{cm} \) and \( d_w = 0.02 \ \text{cm} \). They are deformed moderately under the condition of fluid viscosity of about 500 poises. The wires are intermixed slowly and uniformly in 1.43 \( l \) of working fluid, and the effect on the flow pattern due to suspended wires is supposed to be negligible. Fig. 10 shows example of deformed wires.

Fig. 11 shows the distribution of shear characteristics in four types of mixers. A herical ribbon mixer, which is considered the most available at present for high viscosity fluids, shows a sharp distribution in a short time. An anchor shows a region in spite of elapsed time, which is considered as a dead space or poorly mixed zone. As to a large paddle, it is interesting that the distribution becomes fairly sharp with passage of time. The distribution in the mixing apparatus with two agitator axes having multidisks is fairly sharp in a short time and is very similar to that of the helical ribbon mixer. The similarity of the helical ribbon mixer to the mixing apparatus with two agitator axes having multidisks is confirmed in the characteristics of the constant \( C_2 \) of Eq.(7), as shown in Table 2, and in the distribution of shearing characteristics in Fig. 9.

Conclusions

1. The authors present some flow patterns for typical devices for high-viscosity mixing. From the flow pattern the distribution of dissipation energy is presented. It is maldistributed near the blade.

2. The process of homogenizing in the mixers is discussed and the physical meaning of the \( nT_M = C_1 \) or \( T_nP_{g1}/\mu = C_2 \) is interpreted. It is confirmed that a constant \( C_1 \) can be determined by axial circulation capacity only. A constant \( C_2 \) is discussed as a ratio of scale at original to adjudged final state of mixing, not the absolute scale of fluid elements.

3. A method of evaluating some flow system is presented as shown in Fig. 9 from the viewpoint of shear characteristics.

4. A technique for quick evaluation of the shear distribution in a given apparatus, using deformation of metal wires, is proposed.

5. The similarity of the helical ribbon mixer to the mixing apparatus with two agitator axes having multidisks is confirmed from the following practical points of view, the averaged shear characteristics as shown in Fig. 9, the homogenizing time characteristics as shown in Table 2 and the distribution curve of shear rate in these mixers as shown in Fig. 11.

Acknowledgement

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Nomenclature

- \( C \) = clearance between moving part and fixed wall [cm]
- \( C_1 \) = a constant in Eq.(6) [-]
- \( C_2 \) = a constant in Eq.(7) [-]
- \( D \) = vessel diameter [cm]
- \( D_d \) = disk diameter [cm]
- \( d \) = impeller diameter [cm]
- \( F \) = plastic modulus in Eq.(13) [G/cm²]
- \( g_c \) = gravitational conversion factor [g-cm/G-sec²]
- \( I_m \) = integral term due to non-linearity in Eq.(13) [cm²]
- \( L_q \) = scale of fluid element at original state [cm]
- \( L_M \) = that at final state of mixing [cm]
- \( L_{M_f} \) = that calculated by Eq.(11) [cm]
- \( L_{M_P} \) = that calculated by Eq.(8) [cm]
- \( l_w \) = distance between disks [cm]
- \( l_e \) = length of fine wire [cm]
- \( M \) = net amount of shear supplied [-]
- \( M_f \) = that obtained from flow pattern [-]
- \( M_p \) = that obtained from power consumption [-]
- \( M_r \) = shear rate [sec⁻¹]
- \( M_t \) = average shear rate in outer region of flow tube
**LIMITING SHERWOOD NUMBER FOR DILUTE SPHERE-PACKED BEDS**

TERUKATSU MIYAUCHI  
Department of Chemical Engineering, University of Tokyo, Tokyo, Japan

The limiting (or lowest) Sherwood number for dilute sphere-packed beds is determined from the mass transfer data taken at zero flow rate by the diffusion current method utilizing potassium ferro- and ferri-cyanides system. These mass transfer data are enhanced by the influence of natural convection. The experimental Sherwood numbers at zero flow rate are plotted against \((\text{particle size})^{3/4}\), and the limiting Sherwood number \(S_{h_{po}}\) is determined from linear extrapolation of the plot to zero particle size. \(S_{h_{po}}\) thus obtained agrees fairly well with the theoretical value of 2.0 (\(\varepsilon_f/1.5\)), where \(\varepsilon_f\) is the void fraction of the beds.

The limiting (or lowest) Sherwood number\(^7,^9\) is defined for the film coefficient of mass transfer between an active particle and fluid when the fluid is stagnant and mass transfer takes place only by pure molecular diffusion of a transferring component through the fluid medium. This Sherwood number is given\(^7,^9\) theoretically by the following equation for sufficiently dilute bed, where \(D_{eff}\) is approximated by \((\varepsilon_f/1.5)D_g\):  

\[
S_{h_{po}}=1.5(\varepsilon_f)S_{h_{po}}=2.0
\]  

For dilute sphere-packed beds direct measurements to determine \(S_{h_{po}}\) have not been successful due to the influence of natural convection taking place at zero flow rate\(^1,^8\). The purpose of this note is to determine \(S_{h_{po}}\) from the Sherwood number measured at zero flow rate. Fig. 1 shows \(S_{h_{po}}=(k_f d_p/D_{eff})\) for lower flow rate regime measured by three kinds of experimental method, as follows:

1. Sublimation of camphor sphere by downward flowing air stream: Measurement of the Sherwood number by upflowing or downflowing fluid stream was discussed in detail by Gaffney and Drew\(^2\) regarding the influence of natural convection. From the discussion the data by Kitaura et al.\(^5\) are considered to have given appropriate results. In fact, their data approach the limiting Sherwood number reasonably well with decreasing fluid velocity (refer to curve AB in Fig. 1).

2. Measurement by Na-isotope exchange: The data by Kawazoe et al.\(^4\) were measured by utilizing the rate of \(^{22}\text{Na}^+\) and \(^{23}\text{Na}^+\) exchange between a flow-