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AN ANALYSIS OF THE LIGAMENT-TYPE DISINTEGRATION OF THIN LIQUID FILM AT THE EDGE OF A ROTATING DISK*

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An analysis of ligament-type disintegration was made by equating the rate of dissipation of kinetic energy, surface energy and viscous deformation energy of disturbance of the liquid film to the motive power by centrifugal force at the edge of a rotating disk.

The number of ligaments generated in disintegration of liquid film is given as a function of the Weber number and the stability number.

This analysis gives an interpretation on the basis of the energy balance for the experimental results by Hinze and his co-worker, who used a rotating cup and represented the same expression as the present paper.

Introduction

Theoretical studies of the disintegration of thin liquid film and liquid column in a gas-liquid system have been made by Squire⁸⁾, Hagerty³⁾, Dombrowski^{1,2)}, and Weber¹¹⁾ on the basis of the wave theory.

When the liquid is fed at a moderate rate through a nozzle to the center of a rotating disk, the liquid spreads out on the disk in a thin circular film which breaks up into ligaments at the edge of the disk.

This is a form of disintegration of liquid film different from that studied by Squire and Dombrowski. The stream of ligaments further breaks up into drops. In the event that the final step of breakup into drops is due to the disintegration of liquid column, the primary disintegration of liquid from film to ligaments follows an important rule in the sense that it forms ligamentary bridges to the final step of breakup into drops.

Experimental results on the number of ligaments generated in disintegration of liquid film at the edge of a rotating disk have been reported^{4,5)}, but no analytical solution of how many ligaments should be generated at the edge of a disk has been published.

In the present study, the motion of liquid was treated two-dimensionally, and analytical results on the number of ligaments were obtained and compared with the experimental ones. To test the analytical results, the author further expanded them to the estimation of the growth of ligaments at the time of disintegration,

and compared the results with those reported by Weber¹¹⁾ in the case of liquid column disintegration.

1. Theory

The liquid flowing down at the center of a rotating disk spreads out on the surface in the form of circular film and reaches the edge of the disk.

Surface tension works on the film edge under the state of a gas-liquid-gas system. The liquid film becomes unstable departing from the edge, and changes into several ligamentary streams due to the growth of disturbance. The stream of ligaments actually observed is close to the involute of a circle for the reason that the radial velocity of the film is negligibly small in comparison with the tangential velocity of the disk. (cf. Appendix)

This property, based on experimental knowledge, gives an advantage in analysis in that only the growth rate of disturbance may be considered in the radial direction.

Here, the disturbance potential is taken into account at the edge of the liquid film and an analysis is then made with the aid of the small-disturbance method^{3,7)}, on the assumption that the number of ligaments is decided by the wave number which gives the maximum growth rate of disturbance. Assuming that the motion along the z -axis is uniform in the circular cylindrical coordinates (r, θ, z) , the analysis is treated as a two-dimensional problem.

The disturbance potential can readily be derived from the Laplace equation as in Eq.(1).

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Fig. 1 Disturbance at the edge of disk

Here, the wave velocity $(K\sigma/\rho a)^{1/2}$ is assumed to be negligible in comparison with the peripheral speed of a disk ωa .

Let the displacement be given by Eq.(2) (cf. Fig. 1), where α is a function of time.

$$\phi = A \left(\frac{r}{a} \right)^K \cos K(\theta - \omega t) \quad (1)$$

$$\eta = r - a = \alpha \cos K(\theta - \omega t) \quad (2)$$

The initial condition is $\alpha = \alpha_0$ at $t=0$ at the circumferential edge of the disk, and the kinetic boundary condition of the film at its edge is expressed by Eq.(3).

$$\frac{\partial \eta}{\partial t} + \omega a \frac{\partial \eta}{\partial \theta} = - \frac{\partial \phi}{\partial r} \Big|_{r=a} \quad (3)$$

Substituting Eqs.(1) and (2) into Eq.(3), the constant A is given as

$$A = -\dot{\alpha} a / K \quad (4)$$

The dot (.) here signifies the differential with respect to time. The value of A in Eq.(4) is substituted into Eq.(1) to yield the disturbance potential.

$$\phi = -\frac{\dot{\alpha} a}{K} \left(\frac{r}{a} \right)^K \cos K(\theta - \omega t) \quad (5)$$

In the following, the analysis is made by equating the rate of dissipation of kinetic energy, surface energy, and viscous deformation energy of disturbance of the liquid film to the motive power by centrifugal force at the edge of a rotating disk.

Kinetic energy

Since motion along the z -axis is uniform, the liquid film can be considered to be of unit thickness.

The kinetic energy per wavelength is given by Eq.(6).

$$\frac{\rho}{2} \int \phi \frac{\partial \phi}{\partial r} a d\theta = \frac{\rho \dot{\alpha}^2 a^2}{2K} \int_{-\pi/K + \omega t}^{\pi/K + \omega t} \cos^2 K(\theta - \omega t) d\theta \quad (6)$$

The average kinetic energy of disturbance per unit area is obtained by dividing the numerical value of Eq.(6) by $2\pi a/K$.

$$\text{Average energy per unit area} = \rho \dot{\alpha}^2 / 4K \quad (7)$$

Surface energy

The surface energy due to stretching of the film edge is determined. In the light of the circumstance that disturbance grows on the outside of the edge of the disk, surface tension is considered to be at work on both sides of the film, and the surface energy per unit

area is obtained directly in the same manner as in Eq.(7).

$$\begin{aligned} & \frac{K\sigma}{\pi a} \left[\int \sqrt{1 + \left(\frac{\partial \eta}{\partial \theta} \right)^2} a d\theta - \int a d\theta \right] \\ &= \frac{K\sigma}{2\pi a} \int_{-\pi/K + \omega t}^{\pi/K + \omega t} \frac{\alpha^2 K^2}{a^2} \sin^2 K(\theta - \omega t) a d\theta = \frac{\alpha^2 K^2 \sigma}{2a^2} \quad (8) \end{aligned}$$

Energy of centrifugal force

The average energy acting on unit area is obtained by multiplying the centrifugal force working on the edge of the disk by displacement η and is expressed as in Eq.(9). In this case, α_0 is treated as sufficiently small in comparison with the radius of the disk.

$$\begin{aligned} \frac{\rho \omega^2 K}{2\pi} \int \eta^2 a d\theta &= \frac{\rho \omega^2 a K \alpha^2}{2\pi} \int_{-\pi/K + \omega t}^{\pi/K + \omega t} \cos^2 K(\theta - \omega t) d\theta \\ &= \frac{1}{2} \rho \omega^2 a \alpha^2 \quad (9) \end{aligned}$$

Energy lost in unit time by viscous force

The energy lost with resistance to deformation due to the viscous force is obtained by determining the radial velocity of disturbance u and the tangential velocity of disturbance v from disturbance potential, multiplying u by the stress component in the radial direction p_{rr} and v by the stress component in the tangential direction $p_{r\theta}$, and summing these products.

$$\begin{aligned} & \frac{K}{2\pi a} \int_{-\pi/K + \omega t}^{\pi/K + \omega t} (P_{rr} u + P_{r\theta} v) a d\theta \\ &= \frac{2\mu(K-1)}{a} \dot{\alpha}^2 + \frac{\mu}{2a} \dot{\alpha}^2 = \frac{\mu}{2a} (4K-3) \dot{\alpha}^2 \quad (10) \end{aligned}$$

$$\text{where } P_{rr} = -P + 2\mu \frac{\partial u}{\partial r}$$

$$= -P + 2\mu \frac{(K-1)}{a} \dot{\alpha} \cos K(\theta - \omega t) \quad (11)$$

$$\begin{aligned} P_{r\theta} &= \mu \left(\frac{\partial v}{\partial r} + \frac{\partial u}{a \partial \theta} - \frac{v}{a} \right) \\ &= -(2K-1) \mu \frac{\dot{\alpha}}{a} \sin K(\theta - \omega t) \quad (12) \end{aligned}$$

$$\begin{aligned} P_{rr} u + P_{r\theta} v &= -P \dot{\alpha} \cos K(\theta - \omega t) \\ &+ \mu \frac{\dot{\alpha}^2}{a} \sin^2 K(\theta - \omega t) + \frac{2\mu(K-1)}{a} \dot{\alpha}^2 \quad (13) \end{aligned}$$

When the values of Eqs.(7), (8), (9), and (10) are rearranged by equating to the motive power of centrifugal force as mentioned at the beginning, Eq.(14) is obtained.

$$\begin{aligned} \frac{d}{dt} \left[-\frac{\rho}{2} \omega^2 a \alpha^2 \right] &= \frac{d}{dt} \left[\frac{\rho a}{4K} \dot{\alpha}^2 + \frac{K^2 \sigma}{2a^2} \alpha^2 \right] \\ &+ \frac{\mu}{2a} (4K-3) \dot{\alpha}^2 \quad (14) \end{aligned}$$

From this, the following linear equation is derived.

$$\ddot{\alpha} + K(4K-3) \frac{\mu}{\rho a^2} \dot{\alpha} + 2 \left(\frac{K^3 \sigma}{\rho a^3} - K \omega^2 \right) \alpha = 0 \quad (15)$$

The solution of Eq.(15) which fits the phenomenon of disintegration of liquid film is given by Eq.(16).

$$\alpha = \alpha_0 \exp \left\{ \frac{1}{2} K(4K-3) \frac{\mu}{\rho a^2} [\sqrt{\varphi} - 1] t \right\} \quad (16)$$

From Eq.(16), the growth rate of disturbance β is

given as

$$\beta = \frac{1}{t} \ln \frac{\alpha}{\alpha_0} = \frac{1}{2} K(4K-3) \frac{\mu}{\rho a^2} [\sqrt{\varphi} - 1] \quad (17)$$

Here, φ is defined as shown in Eq.(18).

$$\left. \begin{aligned} \varphi &= 1 + \frac{8St^{-1}}{K(4K-3)^2} (We - K^2) \\ We &= \rho \omega^2 a^3 / \sigma, \quad St = \mu^2 / \rho \sigma a \end{aligned} \right\} \quad (18)$$

Designating the breakup time t^* , it is possible to obtain the total growth of disturbance $\ln(\alpha/\alpha_0)$ by putting $t=t^*$ in Eq.(16). As is described later, t^* was considered for two cases, ligament-type disintegration and drop-wise disintegration.

1) Ligament-type disintegration

Since the radial velocity of the liquid film is small compared with the tangential velocity, it is assumed that ligaments grow in the form of the involute of a circle from the circumferential edge of the disk (cf. Appendix). Although this assumption seems fairly bold, it is not contradictory to the form of a stream of liquid actually observed by a stroboscope synchronized to the rotational speed of the disk.

The time required for disintegration is given by Eq.(19), where l_c is the continuous length of ligament measured along the liquid stream. Substituting Eq.(19) into Eq.(16), the total growth of disturbance in ligament-type disintegration is given by the following equation.

$$t^* = \frac{1}{\omega} \sqrt{\frac{2l_c}{a}} \quad (19)$$

$$\ln(\alpha/\alpha_0) = \frac{1}{\sqrt{2}} Re^{-1} K(4K-3) (\sqrt{\varphi} - 1) \sqrt{\frac{l_c}{a}} \quad (20)$$

where $Re = \rho \omega a^2 / \mu$.

The substitution of Eq.(19) into Eq.(16) may be considered unreasonable in mathematical treatment, because the solution based on the small-disturbance method is valid only when the magnitude of the displacement is small. But Eq.(20) is based on the same linearizing assumption as that in Weber's analysis, and therefore the value obtained from Eq.(20) may be compared with the total growth of disturbance obtained by Weber.

2) Drop-wise disintegration

On the assumption that a drop is released from the edge of the disk with a volume equal to that of the maximum drop size, t^* is given as

$$t^* = \frac{\pi}{6} \frac{d_{max}^3}{\delta(Q/2\pi a)} \quad (21)$$

Tanasawa's equation⁹⁾ is quoted for the experimental value of maximum drop size.

$$d_{max} = 3.2 \left(\frac{\sigma}{\rho \omega^2 a} \right)^{1/2} \quad (22)$$

As for the pitch of drops δ , the experimental results of this author and co-worker obtained for the water-ethanol system are quoted⁵⁾.

$$\delta = 10.5 \left(\frac{\sigma}{\rho \omega^2 a} \right)^{1/2} \quad (23)$$

Substituting Eqs.(22) and (23) into Eq.(21), the value of t^* can be estimated as

$$t^* = 10.3 \left(\frac{\sigma}{\rho \omega^2 Q} \right) \quad (24)$$

Therefore, the growth of disturbance in dropwise disintegration is expressed by the following equation.

$$\ln(\alpha/\alpha_0) = 5.15 Re^{-1} \left(\frac{\sigma}{\rho \omega Q} \right) K(4K-3) (\sqrt{\varphi} - 1) \quad (25)$$

The wave number K_{opt} which gives the maximum growth rate of disturbance β_{max} or maximum total growth of disturbance $\ln(\alpha/\alpha_0)_{max}$ is obtained by differentiating Eq.(17) with wave number under the condition of $d\beta/dK=0$. The value of K which satisfies this condition is shown by Eq.(26).

$$(8K-3)(\sqrt{\varphi} - 1) = \frac{8St^{-1}K}{(4K-3)\sqrt{\varphi}} + \frac{12St^{-1}(4K-1)(We-K^2)}{K(4K-3)^2\sqrt{\varphi}} \quad (26)$$

Since φ is a function of K alone when the Weber number and the stability number are given, Eq.(26) can be solved with respect to the Weber number as follows, although it can also be solved by the trial method to obtain K_{opt} ;

$$We = K_{opt}^2 \left[3 + (8K_{opt}-3)St \left\{ 1 + \sqrt{1 + \frac{1}{K_{opt}St}} \right\} \right] \quad (27)$$

Eq.(27) yields a Weber number corresponding to a given wave number K_{opt} . The sign for the radical sign was chosen positive, which agrees with the experimental results.

2. Comparison of Experimental Results and Analytical Results

2-1 Number of ligaments

Eq.(27) describes the case of non-viscous fluids when the stability number is zero, or $St=0$.

In this case, the value of K_{opt} is given by Eq.(28) and agrees well with the results as expressed by Eq.(29) which this author and co-worker⁵⁾ obtained in the case of drop-wise disintegration of the water-ethanol system on a rotating disk.

$$K_{opt} = 0.58 We^{1/2} \quad (28)$$

$$K_{exp} = 0.6 We^{1/2} \quad (29)$$

The results of calculation of Eq.(27) are shown in **Fig. 2**. The stability number is chosen as a parameter here, and the slope of the straight line indicating the relationship between the Weber number and K_{opt} decreases slightly as the stability number increases.

It is observed that $K_{opt} \propto We^{1/2}$ at $St=0$ while $K_{opt} \propto We^{0.37}$ at $St=10^{-2}$. The relationship between K_{opt} and the stability number is shown in **Fig. 3**. Let us find the value of coefficient C and the exponents m and n , by assuming that the number of ligaments K is represented by Eq.(30).

$$K = C \cdot We^m \cdot St^n \quad (30)$$

Regarding the curve in **Fig. 3** at $St=10^{-2}$ to be approximately a straight line, n comes to be -0.2 from the determination of the slope for $We=10^4$. Then, $m=0.37$ from **Fig. 2**.

Since $K_{opt}=30$ for $We=10^4$, the coefficient C in Eq.(30) is given numerically as

Fig. 2 Solution of Eq.(27) by graphical representation

Fig. 3 Relation between K_{opt} and stability number

Fig. 4 Comparison of theoretical and experimental results

Fig. 5 Comparison of theoretical and experimental results

Fig. 6 Comparison of theoretical and experimental results

Fig. 7 Curve of dimensionless growth rate for wave number

$$C = 30 \times (10^{-2})^{0.2} / (10^4)^{0.37} \div 0.4$$

In short, K_{opt} in the case of $St = 10^{-2}$ is given by Eq.(31), and the experimental results of dimensional analysis by Hinze and his co-worker⁴⁾ who used this form of representation are represented by Eq.(32).

Eq.(32) shows the number of ligaments obtained using a rotating cup.

$$K_{opt} = 0.4 We^{0.37} \cdot St^{-0.2} \quad (31)$$

$$K_{exp} = 0.574 We^{5/12} \cdot St^{-1/6} \quad (32)$$

The calculated results of Eq.(27) and the number of ligaments K_{exp} obtained by this author and co-work-

er on an aqueous solution of millet jelly using a rotating disk (flat disk) are compared in **Figs. 4, 5, and 6** for different values of the stability number with various combinations of disk diameter—6, 8, and 10 cm—and viscosity—5.05 and 1.28 poises.

The analytical results show values 20–30% lower than experimental ones, but the trend of both is the same.

In **Fig. 7** are shown numerical values of the dimensionless growth rate $(2 \rho a^2 / \mu) \beta$ calculated by Eq.(17) in the case where $St = 10^{-2}$ and $We = 10^5$ for the wave

Table 1 Continuous length of ligaments and total growth of ligament-type disintegration

$a=3$ cm	Run No.	ρ [g/cc]	μ [g/cm.s]	σ [dyn/cm]	Q [cc/s]	N [rpm]	l_c [—]	$We \times 10^{-4}$ [—]	K_{opt} [—]	$\ln(\alpha/\alpha_0)$ [cm]
$St 4.10 \times 10^{-3}$	1	1.31	1.18	87.1	3.2	2620	3.12	3.06	57	11.6
	2	1.31	1.18	87.1	3.2	5620	1.93	12.96	102	11.8
	3	1.31	1.18	87.1	3.2	3930	2.23	6.88	80	11.5
	4	1.31	1.18	87.1	4.5	2620	3.58	3.06	57	12.4
	5	1.31	1.18	87.1	0.9	2620	2.34	3.06	57	10.0
$St 1.75 \times 10^{-2}$	65	0.88	1.26	34.3	0.8	2620	2.88	5.23	51	10.0
	66	0.88	1.26	34.3	0.8	4060	2.13	12.56	70	10.2
	67	0.88	1.26	34.3	0.8	5640	1.56	24.2	89	9.7
	68	0.88	1.26	34.3	0.5	2590	1.50	5.11	50	7.3
	69	0.88	1.26	34.3	1.1	2590	4.87	5.11	50	13.2
	70	0.88	1.26	34.3	1.4	2590	5.37	5.11	50	13.7

Table 2 Total growth of drop-wise disintegration

Run No.	Q [cc/s]	N [rpm]	a [cm]	K_{exp}	$\ln(\alpha/\alpha_0)$		
506	0.9	3340	3	232	15.9	$\sigma=32.6$ dyne/cm 43.26 dyne/cm	$\rho=0.927$ g/cc 0.92 g/cc
341	1.1	2570	3	112	19.9		
447	2.1	3890	4	288	10.1		
382	3.1	2580	5	288	9.8	57.8 dyne/cm 43.26 dyne/cm	0.99 g/cc 0.92 g/cc
624	1.8	6000	4	357	6.9		
442	2.1	2900	4	252	11.0		
431	3.3	2550	4	184	9.2		

number K . In this case, the wave number ($K=74$) which gives the maximum value of growth rate certainly agrees with the value of K_{opt} under the same condition in Fig. 2.

2-2 Examination of total growth of disturbance $\ln(\alpha/\alpha_0)$

Weber¹¹⁾ reported that the total growth of disturbance $\ln(\alpha/\alpha_0)$ had a constant value (=12) in his study of liquid-column disintegration.

The calculated value of $\ln(\alpha/\alpha_0)$ in the present study is also expected to be 12 because disturbance on the liquid film grows into ligaments before disintegration.

For ligament-type disintegration, the total growth can be obtained from Eq. (20). For this calculation, it is necessary to know the value of the continuous length of ligament from the edge of the disk l_c . The continuous length of ligament⁶⁾ experimentally determined is shown in Table 1 together with the experimental conditions. The image instantaneously photographed was enlarged by a projector, the length of ligament from the edge of the disk to the point of disintegration was measured, and an average length of 40 to 50 ligaments was taken.

The total growth determined from the assumption that the number of ligaments is given by K_{opt} under these experimental conditions is shown in Table 1.

The total growth in the case of drop-wise disintegration is obtained by substituting the experimental number of ligaments K_{exp} into Eq. (25), and is shown in Table 2 together with the experimental conditions.

The growth shows large values when the flow rate is particularly small; it is reported that such cases do not obey the Rayleigh-type disintegration¹⁰⁾ due to the rapid increase of periodicity at the time when

liquid drops are released.

In spite of the fact that the motion of liquid film generated on a rotating disk is dealt with, good agreement of the trend between experiment and analysis with respect to the number of ligaments without consideration of skin friction between the disk and the liquid film has been obtained. This is probably due to the circumstance that the disintegration of liquid film takes place at the outside of the circumferential edge of the disk. Apart from the present analysis, this author and co-worker obtained another experimental result⁵⁾, i.e., the number of ligaments is related to dimensionless terms $\rho Q^2/\sigma a^3$ and $\rho Q/\mu a$ in the range of large change in flow rate, and it becomes the same as the number of ligaments obtained in the case of non-viscous fluids independent of viscosity for the case of particularly small flow rate. The reason for this seems to be that, when the flow rate is exceptionally small, the rate of deformation of disturbance is small, and hence the viscous resistance to deformation becomes small.

Furthermore, it is considered that the periodicity of release of liquid drops is not taken into consideration in this method of analysis, and that many more ligaments should be generated than the calculated value, if periodicity is involved in this problem and the liquid fed on the disk cannot be discharged.

In this paper, experimental results for the cases where the values of $\rho Q^2/\sigma a^3$ are on the order of 10^{-3} were compared with analytical results.

Conclusion

The analysis is made by equating the rate of dissipation of kinetic energy, surface energy and viscous

Fig. A-1

deformation energy of disturbance of the liquid film to the motive power by centrifugal force at the edge of rotating disk. From the assumption that the number of ligaments is decided when liquid film disintegrates at the wave number giving the maximum growth rate of disturbance, the number of ligaments was expressed as a function of the Weber number and the stability number.

This analysis showed values 20–30% lower than the experimental ones, but it gives an interpretation on an energy-balance basis for the experimental results by Hinze and his co-worker, who used a rotating cup and presented the same expression as does this paper.

Appendix

Let us suppose that liquid present at the point $A(\xi, \zeta)$ at $t=0$ reaches the point $C(x, y)$ after t seconds. If the liquid moves from point A in the tangential direction of the disk with the same velocity as the tangential velocity, then $AB=AC$ in Fig. A-1. The values of x and y are given by Eq. (A-2).

$$\xi = a \cos \omega t, \quad \zeta = a \sin \omega t \quad (\text{A-1})$$

$$\left. \begin{aligned} x &= \xi + a\omega t \sin \omega t \\ &= a \cos \omega t + a\omega t \sin \omega t \\ y &= \zeta - a\omega t \cos \omega t \\ &= a \sin \omega t - a\omega t \cos \omega t \end{aligned} \right\} \quad (\text{A-2})$$

The length of ligament from the circumferential edge of the disk to point $C(x, y)$ is given by Eq. (A-3).

$$l = \int \sqrt{dx^2 + dy^2} = a\omega^2 \int_0^t t dt \quad (\text{A-3})$$

Designating the continuous length of ligament to the point of disintegration as l_c , the breakup time t^* can be estimated by Eq. (A-4). This results from ignoring the radial velocity of film in comparison with the tangential velocity.

$$t^* = \frac{1}{\omega} \sqrt{\frac{2l_c}{a}} \quad (\text{A-4})$$

Nomenclature

A	= constant in Eq.(1)	[cm ² /sec]
a	= radius of disk	[cm]
d_{max}	= maximum diameter of drop	[cm]
K	= number of ligaments or wave number of disturbance	[—]
K_{exp}	= number of ligaments obtained by experiment	[—]
K_{opt}	= wave number of maximum growth rate	[—]
l_c	= continuous length of ligament	[cm]
$\ln(\alpha/\alpha_0)$	= total growth of disturbance	[—]
P	= static pressure	[g/cm·sec ²]
P_{rr}	= stress component in r -direction	[g/cm·sec ²]
$P_{r\theta}$	= stress component in θ -direction	[g/cm·sec ²]
Q	= volumetric flow rate	[cm ³ /sec]
Re	= Reynolds number = $\rho\omega a^2/\mu$	[—]
r	= distance in radial direction from center of disk	[cm]
St	= stability number = $\mu^2/\rho\sigma a$	[—]
t	= time	[sec]
t^*	= breakup time	[sec]
u	= radial velocity of disturbance	[cm/sec]
v	= tangential velocity of disturbance	[cm/sec]
α	= amplitude of disturbance	[cm]
α_0	= initial amplitude of disturbance	[cm]
β	= growth rate of disturbance	[sec ⁻¹]
β_{max}	= maximum growth rate of disturbance	[sec ⁻¹]
δ	= pitch of drops in drop-wise disintegration	[cm]
η	= displacement from circumferential edge of disk	[cm]
θ	= angle	[—]
μ	= viscosity of liquid	[g/cm·sec]
ρ	= density of liquid	[g/cm ³]
σ	= surface tension	[dyn/cm]
ϕ	= disturbance potential	[cm ² /sec]
ω	= angular velocity of disk	[sec ⁻¹]

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