COMPARISON OF MOMENT METHOD AND FOURIER ANALYSIS IN CHROMATOGRAPH*

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Analytical solution was obtained for response to delta tracer signal which is injected into a packed bed of non-adsorbing porous particles. Moment method and Fourier analysis were compared in estimating parameters from delta response with truncation error in tail.

Since Kubin⁸⁾ and Kucera⁹⁾ proposed a moment method for evaluating parameters of packed-bed reactors, much attention has been focused on chromatographic parameter estimation techniques.

In experiments Smith *et al.*^{4,11,12)} have applied the moment method to several cases. Some other methods have also been proposed: weighted moment method¹⁰⁾, transfer function fitting method^{7,10)} and Fourier transform fitting method⁵⁾.

Anderssen and White²⁾ compared these estimation methods using experimental signals for a packed bed of non-porous particles and have concluded that the best method was time domain fitting, and that the Fourier transform fitting and the weighted moment methods also gave quite accurate values. A similar conclusion has been reached by Clements⁵⁾ and Böxkes and Hofmann³⁾.

Gangwall, Hudgins, Bryson and Silveston⁶⁾ have demonstrated that Fourier analysis was sufficiently usable.

The present paper concerns a packed bed of non-adsorbing porous particles. The purposes of this investigation are, first, to obtain an analytical solution for delta response and, second, to compare the moment method and Fourier analysis in illustrating the influence of truncation errors in tailing portion on the estimated parameter values.

Analytical Solution

Let us consider a case where a non-adsorbable tracer pulse signal is imposed on a steadily flowing fluid in a packed bed of porous spherical particles of uniform size. The differential equation for the bulk concentration is

$$\frac{\partial C}{\partial t} = D_{ax} \frac{\partial^2 C}{\partial X^2} - U \frac{\partial C}{\partial X} - \frac{3D_e}{R} \frac{1-\varepsilon_b}{\varepsilon_b} \left(\frac{\partial c_i}{\partial r} \right)_{r=R} \quad (1)$$

The concentration in the particle is

$$\frac{\partial c_i}{\partial t} = \frac{D_e}{\varepsilon_p} \left(\frac{\partial^2 c_i}{\partial r^2} + \frac{2}{r} \frac{\partial c_i}{\partial r} \right) \quad (2)$$

with

$$C=0, \quad \text{at } t=0 \quad (3)$$

$$c_i=0, \quad \text{at } t=0 \quad (4)$$

$$D_e \frac{\partial c_i}{\partial r} = k_f (C - c_i), \quad \text{at } r=R \quad (5)$$

$$C = \delta(t=0), \quad \text{at } X=0 \quad (6)$$

$$\frac{\partial C}{\partial X} = 0, \quad \text{at } X=L \quad (7)$$

The system of Eqs.(1) to (7) may be made dimensionless:

$$\frac{\partial C}{\partial \tau} = \beta \frac{\partial^2 C}{\partial x^2} - \frac{\partial C}{\partial x} - \frac{3\eta}{\kappa} \left(\frac{\partial c_i}{\partial \rho} \right)_{\rho=1} \quad (8)$$

$$\kappa \frac{\partial c_i}{\partial \tau} = \frac{\partial^2 c_i}{\partial \rho^2} + \frac{2}{\rho} \frac{\partial c_i}{\partial \rho} \quad (9)$$

$$C(x, 0) = 0 \quad (10)$$

$$c_i(\rho, x, 0) = 0 \quad (11)$$

$$\gamma \left(\frac{\partial c_i}{\partial \rho} \right)_{\rho=1} = C - (c_i)_{\rho=1} \quad (12)$$

$$C(0, \tau) = \delta(\tau=0) \quad (13)$$

$$\left(\frac{\partial C}{\partial x} \right)_{x=1} = 0 \quad (14)$$

The tracer concentration in the effluent fluid is (Appendix A)

$$C_e = 2\beta \sum_{n=1}^{\infty} \sum_{j=1}^{\infty} \frac{\alpha_n \sin \alpha_n}{1 - \frac{\sin 2\alpha_n}{2\alpha_n}}$$

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$$\times \frac{(1-\gamma)\sin z_j + \gamma z_j \cos z_j}{A_{n,j}\sin z_j + B_{n,j}\cos z_j} e^{-(2\beta z_j^2 \tau - \kappa)/(2\alpha\beta)} \quad (15)$$

where

$$A_{n,j} = 1 - \gamma + \frac{3\gamma}{2} + \frac{\kappa\gamma}{2} \left(\frac{1}{4\beta} + \beta\alpha_n^2 - \frac{z_j^2}{\kappa} \right) \quad (16)$$

$$B_{n,j} = \gamma z_j - \frac{\kappa}{2z_j} \left(\frac{1}{4\beta} + \beta\alpha_n^2 - \frac{z_j^2}{\kappa} \right) \quad (17)$$

α_n is a root of

$$\tan \alpha = -2\beta\alpha \quad (18)$$

and z_j is a root of

$$\tan z = -z \frac{\gamma \left(\frac{1}{4\beta} + \beta\alpha_n^2 - \frac{z^2}{\kappa} \right) + \frac{3\gamma}{\kappa}}{(1-\gamma) \left(\frac{1}{4\beta} + \beta\alpha_n^2 - \frac{z^2}{\kappa} \right) - \frac{3\gamma}{\kappa}} \quad (19)$$

Eq.(15) is a function of $\beta, \gamma, \eta, \kappa$ and τ or $\varepsilon_p, \varepsilon_b, L/D_p, D_e/D_v, N_{Sc}N_{Re}, N_{Pe}, N_{Sh}$ and Ut/L . Assuming the following data

$$\varepsilon_p = 0.5, \quad \varepsilon_b = 0.4, \\ L/D_p = 10, \quad N_{Sc}N_{Re} = 10,$$

Eq.(15) is illustrated for several combinations of $D_e/D_v, N_{Pe}$ and N_{Sh} values in **Fig. 1**.

Moments

The Laplace transform, $\bar{C}_e(s)$, of the response, C_e ,

$$\bar{C}_e(s) = \int_0^\infty C_e e^{-s\tau} d\tau \quad (20)$$

may be obtained by applying the transform to the system of Eqs.(8) to (14) and solving them:

$$\bar{C}_e(s) = \frac{2\phi e^{1/(2\beta)}}{\left(\frac{1}{2\beta} + \phi \right) e^\phi - \left(\frac{1}{2\beta} - \phi \right) e^{-\phi}} \quad (21)$$

where

$$\phi = \frac{1}{2\beta} \sqrt{1 + 4\beta \left[s + \frac{3\gamma}{\kappa\gamma} \left(1 - \frac{1}{1 - \gamma + \gamma\sqrt{\kappa s} \coth \sqrt{\kappa s}} \right) \right]} \quad (22)$$

Substituting Eq.(21) into the following theorem (note that $\int_0^\infty C_e d\tau = 1$):

$$\left(\begin{array}{l} \text{nth absolute} \\ \text{moment of} \\ \text{output} \\ \text{signal} \end{array} \right) = \mu_n = \frac{\int_0^\infty C_e \tau^n d\tau}{\int_0^\infty C_e d\tau} = (-1)^n \left(\frac{d^n \bar{C}_e}{ds^n} \right)_{s=0} \quad (23)$$

one obtains

$$\mu_1 = (1 + \eta)f \quad (24)$$

$$\mu_2 = \frac{2}{3} \kappa\gamma \left(\gamma + \frac{1}{5} \right) f + [2(1 + 3\beta)f - 3](1 + \eta)^2 + 2\mu_1^2 \quad (25)$$

where

Fig. 1 Delta responses calculated by Eq.(15), with assumption of $\varepsilon_p=0.5, \varepsilon_b=0.4, N_{Sc}N_{Re}=10$ and $L/D_p=10$

$$f = 1 - \beta + \beta e^{-1/\beta} \quad (26)$$

Note that when L/D_p is so large that β is sufficiently small, Eqs.(24) and (25), respectively, reduce to

$$\mu_1 = (1 + \eta)(1 - \beta) \quad (24')$$

$$\doteq (1 + \eta) \quad (24'')$$

$$\mu_2 = \frac{2}{3} \kappa\gamma \left(\gamma + \frac{1}{5} \right) (1 - \beta) + (1 - 4\beta^2)(1 + \eta)^2 \quad (25')$$

$$\doteq \frac{2}{3} \kappa\gamma \left(\gamma + \frac{1}{5} \right) + (1 + \eta)^2 \quad (25'')$$

second central moment $= \mu_2 - \mu_1^2$

$$= \frac{2}{3} \kappa\gamma \left(\gamma + \frac{1}{5} \right) (1 - \beta) + 2\beta \left(1 - \frac{5}{2}\beta \right) (1 + \eta)^2 \quad (25A')$$

$$\doteq \frac{2}{3} \kappa\gamma \left(\gamma + \frac{1}{5} \right) + 2\beta(1 + \eta)^2 \quad (25A'')$$

where Eqs.(24''), (25'') and (25A'') are identical with those derived earlier by Kubin⁸ and Kucera⁹ under the assumption of infinite bed length.

The error of approximating Eqs.(24'), (25') and (25A'), respectively, to Eqs.(24''), (25'') and (25A'') depends on the value of β .

When Eqs.(24'') and (25A'') are used for parameter estimation, as was done by Smith *et al.*^{4,11,12}, the maximum error caused is considered to be the approximation of $1 - 2.5\beta \doteq 1$. If this error is expected to be within 5 per cent, it should be that $\beta \leq 0.02$. For beds of fine particles Smith *et al.* have obtained low N_{Pe} values, but their L/D_p is sufficiently large so that the approximation of infinite bed length is sufficiently satisfied. For beds with $N_{Pe}=2$, as is expected for beds

Fig. 2 Comparison of original outputs (Curves A' and A'') and the predicted outputs resulting from moment method and Fourier analysis; $\varepsilon_p=0.5$, $\varepsilon_b=0.4$, $N_{Sc}N_{Re}=10$, $L/D_p=10$ and $D_e/D_v=0.1$ are assumed.

of relatively large particles in the turbulent flow region, the condition of $\beta \leq 0.02$ is satisfied when $L/D_p \geq 25$.

Delta Response in Fourier Series

Substitution of $s=i\omega$ into Eq.(21) gives

$$\bar{C}_e(i\omega) = Re(\bar{C}_e) + iIm(\bar{C}_e) \quad (27)$$

where $Re(\bar{C}_e)$ and $Im(\bar{C}_e)$ are shown in Appendix B.

In the following Fourier series of C_e in the range of $0 < \tau < T$

$$C_e = 2b_0 + \sum_{n=1}^{\infty} \left(a_n \sin \frac{n\pi\tau}{T} + b_n \cos \frac{n\pi\tau}{T} \right) \quad (28)$$

the Fourier coefficients are related to $\bar{C}_e(in\pi/T)$ as follows⁶⁾:

$$a_n = \frac{1}{T} \int_0^T C_e \sin \frac{n\pi\tau}{T} d\tau \quad (29)$$

$$= -\frac{1}{T} Im \left(\bar{C}_e \left(i \frac{n\pi}{T} \right) \right) \quad (30)$$

$$b_n = \frac{1}{T} \int_0^T C_e \cos \frac{n\pi\tau}{T} d\tau \quad (31)$$

$$= \frac{1}{T} Re \left(\bar{C}_e \left(i \frac{n\pi}{T} \right) \right) \quad (32)$$

Comparison of Moment Method and Fourier Analysis

Curve A on **Fig. 2** is a theoretical peak with the following values:

$$\varepsilon_p=0.5, \quad \varepsilon_b=0.4, \quad L/D_p=10, \quad N_{Sc}N_{Re}=10, \\ D_e/D_v=0.1, \quad N_{Pe}=2, \quad N_{Sh}=1$$

Consider the hypothetical output signals, Curve A' and A'', which are identical with Curve A except for

Table 1 Data estimated by moment method and Fourier analysis

		Curve A'		Curve A''	
		N_{Pe}	N_{Sh}	N_{Pe}	N_{Sh}
Moment method (a)		1.43	4.98	1.09	-2.54
Fourier analysis (b)	$n=1$	2.48	0.14	(c)	(c)
	2	1.86	0.47	1.55	0.36
	3	1.87	0.78	1.44	0.80
	4	1.95	0.96	1.45	1.28
	5	2.15	0.96	1.50	1.66
	6	2.80	0.79	1.65	1.65
mean		2.19	0.68	1.52	1.15

- (a) $\mu_1=1.63$ and $\mu_2=3.35$, for Curve A'
 $\mu_1=1.59$ and $\mu_2=3.13$, for Curve A''
 (b) It is assumed that $T=20$ so that $T/(\text{the time at which tail vanishes})=20/5=4$ for Curve A',
 and $=20/4.5=4.4$ for Curve A''.
 (c) No solution is found.

the tailing portion. The only difference is that Curve A' (zero concentration at $\tau > 5$) and Curve A'' (zero concentration at $\tau > 4.5$) have a short tail, while Curve A has a long tail. Note that the tail of Curve A is

$$\tau=4.5 \quad 5 \\ C_e=0.018 \quad 0.010$$

Since the tail is not accurately measurable, such small misrecording and/or experimental error will be quite frequently incurred:

$$\frac{(C_e)_{\tau=4.5}}{\text{peak value of } C_e} = 2.8\%$$

$$\frac{(C_e)_{\tau=5}}{\text{peak value of } C_e} = 1.5\%$$

Our interest is what parameter values would be determined from these hypothetical outputs.

Let us assume that all the parameter values except N_{Pe} and N_{Sh} are known and that the N_{Pe} and N_{Sh} are to be determined by the moment method (N_{Pe} from Eq.(24) for μ_1 and N_{Sh} from Eq.(25) for μ_2) and Fourier analysis (solving simultaneously each set of a_n and b_n for N_{Pe} and N_{Sh}).

The computed data are shown in **Table 1**. It should be noted that the originally assumed data are $N_{Pe}=2$ and $N_{Sh}=1$. In the case of Curve A', the Fourier analysis looks better than the moment method as far as the estimated data are concerned, although the predicted output resulting from the Fourier analysis (Curve C, Fig. 2) deviates from Curve A' as much as that from the moment method (Curve B).

From Curve A'', which has the shortest tail, the moment method gives negative N_{Sh} value. With regard to the Fourier analysis, from a_1 and b_1 no solution (for N_{Pe} and N_{Sh}) is found to exist, at least in the range of $N_{Sh}=0.1-10$, but the mean N_{Pe} and N_{Sh} values for $n=2$ to 6 may be regarded as reasonable.

The moment method is much simpler than the Fourier analysis, but the former has a disadvantage in that the higher moments are seriously affected by error

in the tail. The Fourier analysis is not so sensitive to error in the tail, but it also has a disadvantage, in that the estimated data depend on n and the period T .

Finally, it is concluded that Fourier analysis may be more safely applied than the moment method to signals of which the tail is not accurately measured.

Appendix A

Similarly to Amundson¹⁾, the following change of variables converts the system of Eqs. (8) to (14) to a form in which a finite Fourier transform can be applied:

$$W = C e^{-\frac{x}{2\beta}}, w = c_i e^{-\frac{x}{2\beta}}$$

Then Eqs.(8) to (14) become

$$\frac{\partial W}{\partial \tau} = \beta \frac{\partial^2 W}{\partial x^2} - \frac{1}{4\beta} W - \frac{3\eta}{\kappa} \left(\frac{\partial w}{\partial \rho} \right)_{\rho=1} \quad (A1)$$

$$\kappa \frac{\partial w}{\partial \tau} = \frac{\partial^2 w}{\partial \rho^2} + \frac{2}{\rho} \frac{\partial w}{\partial \rho} \quad (A2)$$

$$W(x, 0) = 0 \quad (A3)$$

$$w(\rho, x, 0) = 0 \quad (A4)$$

$$\gamma \left(\frac{\partial w}{\partial \rho} \right)_{\rho=1} = W - (w)_{\rho=1} \quad (A5)$$

$$W(0, \tau) = \delta(\tau=0) \quad (A6)$$

$$\left(\frac{\partial W}{\partial x} \right)_{x=1} + \frac{1}{2\beta} (W)_{x=1} = 0 \quad (A7)$$

When the following Fourier transform is applied:

$$G_n = \int_0^1 W \sin \alpha_n x \, dx = G \quad (A8)$$

the inverse transform is given by

$$W = 2 \sum_{n=1}^{\infty} \frac{\sin \alpha_n x}{1 - \frac{\sin 2\alpha_n}{2\alpha_n}} G \quad (A9)$$

with similar expressions for w .

Applying the transform, Eqs.(A1) to (A7) become

$$\frac{\partial G}{\partial \tau} = - \left(\frac{1}{4\beta} + \beta \alpha_n^2 \right) G + \beta \alpha_n \delta(\tau=0) - \frac{3\eta}{\kappa} \left(\frac{\partial g}{\partial \rho} \right)_{\rho=1} \quad (A10)$$

$$\kappa \frac{\partial g}{\partial \tau} = \frac{\partial^2 g}{\partial \rho^2} + \frac{2}{\rho} \frac{\partial g}{\partial \rho} \quad (A11)$$

$$G = 0, \text{ at } \tau = 0 \quad (A12)$$

$$g = 0, \text{ at } \tau = 0 \quad (A13)$$

$$\gamma \left(\frac{\partial g}{\partial \rho} \right)_{\rho=1} = G - (g)_{\rho=1} \quad (A14)$$

where α_n is a root of

$$\tan \alpha_n = -2\beta \alpha_n$$

When the Laplace transform is applied to Eqs.(A10) to (A14), the solutions are easily obtained as

$$g = \beta \alpha_n \sum_{j=1}^{\infty} \frac{\sinh(\rho \sqrt{\kappa s_j})}{\rho \left(\frac{dQ_n}{ds} \right)_{s_j}} e^{-s_j \tau} \quad (A15)$$

$$G = \beta \alpha_n \sum_{j=1}^{\infty} \frac{(1-\gamma) \sinh \sqrt{\kappa s_j} + \gamma \sqrt{\kappa s_j} \cosh \sqrt{\kappa s_j}}{\left(\frac{dQ_n}{ds} \right)_{s_j}} e^{-s_j \tau} \quad (A16)$$

where s_j is a root of

$$Q_n(s_j) = \left[\left(1 - \gamma \right) \left(\frac{1}{4\beta} + \beta \alpha_n^2 + s_j \right) - \frac{3\eta}{\kappa} \right] \sinh \sqrt{\kappa s_j}$$

$$+ \left[\gamma \left(\frac{1}{4\beta} + \beta \alpha_n^2 + s_j \right) + \frac{3\eta}{\kappa} \right] \sqrt{\kappa s_j} \cosh \sqrt{\kappa s_j} = 0 \quad (A17)$$

The bulk concentration is given by

$$C = 2e^{\frac{x}{2\beta}} \sum_{n=1}^{\infty} \frac{\sin \alpha_n x}{1 - \frac{\sin 2\alpha_n}{2\alpha_n}} G \quad (A18)$$

Substituting $\sqrt{\kappa s_j} = iz_j$,

$$C = 2\beta e^{\frac{x}{2\beta}} \sum_{n=1}^{\infty} \sum_{j=1}^{\infty} \frac{\alpha_n \sin \alpha_n x}{1 - \frac{\sin 2\alpha_n}{2\alpha_n}} \times \frac{(1-\gamma) \sin z_j + \gamma z_j \cos z_j}{A_{n,j} \sin z_j + B_{n,j} \cos z_j} e^{(-z_j^2 \tau / \kappa)} \quad (A19)$$

where $A_{n,j}$ and $B_{n,j}$ are defined in the text.

Note that when the bed may be assumed to be of infinite length, the following boundary condition is used

$$C = 0, \text{ at } X = \infty \quad (A20)$$

Only major modification is to apply a Fourier integral instead of using Eqs.(A8) and (A9). The solution is

$$C_e = \frac{2\beta D_p}{\pi L} \int_0^{\infty} \sum_{j=1}^{\infty} \alpha \sin \alpha \, d\alpha \frac{(1-\gamma) \sin z_j + \gamma z_j \cos z_j}{A_j \sin z_j + B_j \cos z_j} e^{-(2\beta z_j^2 \tau - \kappa) / (2\kappa \beta)} \quad (A21)$$

where A_j, B_j and z_j are, respectively, given by Eqs.(16), (17) and (19) provided that α_n is replaced by α .

Appendix B

Substitution of $s = iw$ into Eq.(21) gives

$$\bar{C}_e(iw) = Re(\bar{C}_e) + iIm(\bar{C}_e)$$

$$Re(\bar{C}_e) = e^{1/2\beta} [e^{b_c} \{M \cos b_s - N \sin b_s\} + e^{-b_c} \{(1-M) \cos b_s - N \sin b_s\}] \quad (B1)$$

$$Im(\bar{C}_e) = e^{1/2\beta} [e^{b_c} \{N \cos b_s + M \sin b_s\} - e^{-b_c} \{N \cos b_s + (1-M) \sin b_s\}] \quad (B2)$$

where $b_c = (\zeta^2 + \phi^2)^{1/4} \cos \left(\frac{1}{2} \tan^{-1} \frac{\phi}{\zeta} \right)$

$$b_s = (\zeta^2 + \phi^2)^{1/4} \sin \left(\frac{1}{2} \tan^{-1} \frac{\phi}{\zeta} \right)$$

$$\zeta = \left(\frac{1}{2\beta} \right)^2 + \frac{3\eta}{\kappa \gamma \beta} \left[1 - \frac{l \cos \theta \sinh \theta + m \sin \theta \cosh \theta}{l^2 + m^2} \right]$$

$$\phi = \frac{\omega}{\beta} - \frac{3\eta}{\kappa \gamma \beta} \left[\frac{l \sin \theta \cosh \theta - m \cos \theta \sinh \theta}{l^2 + m^2} \right]$$

$$l = \gamma \theta (\cos \theta \cosh \theta - \sin \theta \sinh \theta) + (1-\gamma) \cos \theta \sinh \theta$$

$$m = \gamma \theta (\cos \theta \cosh \theta + \sin \theta \sinh \theta) + (1-\gamma) \sin \theta \cosh \theta$$

$$\theta = \sqrt{\frac{\kappa \omega}{2}}$$

$$M = - \frac{F(D-F) + G(E-G)}{(D-F)^2 + (E-G)^2}$$

$$N = \frac{F(E-G) - G(D-F)}{(D-F)^2 + (E-G)^2}$$

$$D = \left[\left(\frac{1}{2\beta} + b_c \right) \cos b_s - b_s \sin b_s \right] e^{(1/2\beta) + b_c}$$

$$E = \left[\left(\frac{1}{2\beta} + b_c \right) \sin b_s + b_s \cos b_s \right] e^{(1/2\beta) + b_c}$$

$$F = \left[\left(\frac{1}{2\beta} - b_c \right) \cos b_s - b_s \sin b_s \right] e^{(1/2\beta) - b_c}$$

$$G = \left[\left(\frac{1}{2\beta} - b_c \right) \sin b_s + b_s \cos b_s \right] e^{(1/2\beta) - b_c}$$

Nomenclature

C	= dimensionless tracer concentration in bulk stream; (concentration)/(amount of tracer input/extraparticle void volume of bed)	[—]
C_e	= C at reactor exit	[—]
$\bar{C}_e$	= Laplace transform of C_e	[—]
c_i	= dimensionless tracer concentration in particle; (concentration)/(amount of tracer input/extraparticle void volume of bed)	[—]
D_{ax}	= axial fluid mixing coefficient based on extraparticle void area	[cm ² /sec]
D_e	= effective diffusivity in porous solid	[cm ² /sec]
D_p	= particle diameter	[cm]
D_v	= molecular diffusivity	[cm ² /sec]
k_f	= particle-fluid mass transfer coefficient	[cm/sec]
L	= packed-bed height	[cm]
N_{Pe}	= $\frac{D_p U}{D_{ax}}$	[—]
N_{Re}	= $\frac{D_p \varepsilon_b U}{\nu}$	[—]
N_{Sc}	= $\frac{\nu}{D_v}$	[—]
N_{Sh}	= $\frac{k_f D_p}{D_v}$	[—]
R	= particle radius	[cm]
r	= radial distance variable	[cm]
t	= time	[sec]
U	= interstitial fluid velocity	[cm/sec]
X	= axial distance variable	[cm]
x	= $\frac{X}{L}$	[—]
z_j	= root of Eq.(19)	[—]
α_n	= root of Eq.(18)	[—]
β	= $\frac{1}{N_{Pe}(\frac{L}{D_p})}$	[—]

γ	= $\frac{2(\frac{D_e}{D_v})}{N_{Sh}}$	[—]
δ	= Kronecker's delta function	[—]
ε_b	= extraparticle void fraction	[—]
ε_p	= intraparticle void fraction	[—]
η	= $\varepsilon_p \frac{1 - \varepsilon_b}{\varepsilon_b}$	[—]
κ	= $\frac{N_{Re} N_{Sc}(\frac{\varepsilon_p}{\varepsilon_b})}{4(\frac{D_e}{D_v})(\frac{L}{D_p})}$	[—]
μ_n	= n -th absolute moment	[—]
ν	= kinematic viscosity	[cm ² /sec]
ρ	= $\frac{r}{R}$	[—]
τ	= $\frac{U t}{L}$	[—]

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