CONTINUOUS SEPARATION OF GAS MIXTURE BY THERMAL DIFFUSION COLUMN*

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As the concentration of the mixture at the feed position (z = 0), C₀, is usually different from that of the feed gas, C_f, the transport equations were solved under the boundary condition of C = C₀ at z = 0, to obtain the separation performance in a thermal diffusion column. To check the calculated results, a thermal diffusion column which consists of two concentric vertical cylinders was used, and separation of the binary gas mixture He-Ar was made.

From the experimental results, it was found that concentrations of both product and waste (or top and bottom) depend sensitively on the cut, that is, the ratio of enriched product flow rate to the feed flow rate (β), but only slightly on feed position (ζₒ).

The experimental results were in good agreement with the calculated results under the boundary condition of C = C₀ at z = 0.

An engineering method is presented by which the separation characteristics will simply and graphically be obtained.

Introduction

When a temperature gradient is applied to a homogeneous mixture of gases, each component diffuses at a different velocity, dependent upon molecular mass and interaction between molecules. The component with the lower molecular weight is concentrated at the hot wall. A very effective method of utilizing thermal diffusion for the separation of mixtures was employed by Clusius and Dickel. When a Clusius-Dickel column is used to bring about separations in mixtures and operated in a continuous manner, it is very important to give information of the correlation between separations and conditions for apparatus and operations. The correlations were derived theoretically by Furry, Jones and Onsager, and reviewed and discussed theoretically and experimentally by Jones and Furry.

As transport equations for columns reviewed by Jones and Furry incorporate a large number of assumptions, some modified theories have been presented by a number of workers. These theories have been in agreement with experimental data. All of these experiments, however, have been carried out by using mixtures to bring about a small separation (less than 10%). Then the Jones-Furry theory and its modifications which pertain to the separation of isotopic mixtures are valid for the above data. But these theories do not apply to the estimation of the separation of mixtures to bring about a large separation.

In the present paper, from this point of view, separations of He-Ar mixture are measured in a thermal diffusion column which consists of two concentric tubes. As a result, it is found that new factors, β(= s_f/σ_f) and ζₒ(=L_s/L_F), are necessary to determine separations. In the range of the present experiments factor β is very important.

Transport Equations and Concentration Distribution in Column

Transport equations

The thermogravitational thermal diffusion column used in this investigation is of a concentric cylinder type. The ratio of inside diameter of the outer to outside diameter of the inner cylinder is 2.875. So the working space between the concentric cylinders may be approximately regarded as that between parallel plates. Therefore, theoretical treatments of a flat-plate column are discussed and are compared with experimental results.

If a temperature gradient is applied to a mixture across parallel plates, a heavier component diffuses away from the hot wall to a greater extent than does a lighter, by the thermal diffusion effect. The hot wall stream enriched by the lighter is carried by natural convection to the top of the column, and the lighter is concentrated at the top. On the other hand, the heavier concentrates at the bottom of the column.

Due to ordinary diffusion, the concentration gradient produced by the combined effects of thermal diffusion and convection acts to oppose thermal diffusion and limits the separation.
In the continuous-flow thermal diffusion column, forced convection is added to natural convection and transport phenomena in the column are considerably complicated. Now, a column theory is developed based on the following assumptions:

1) Gas mixture is binary and ideal. 2) Convective velocity is entirely in the vertical direction and independent of z. The flow is laminar. 3) Temperature gradient is determined by conduction alone. 4) The concentration dependence of physical properties is neglected. 5) The transverse difference of concentration is very small in comparison with the vertical one. 6) The temperature dependence of various quantities is neglected except the density in buoyant force.

From the above assumptions, transport equations become as follows (reference to Fig. 1):

\[ \sigma_E C_E = \sigma_E \tilde{C} + H_E \tilde{C}(1 - \tilde{C}) - K_E \frac{d\tilde{C}}{dz} \]  

(1)

In the enriching section

\[ -\sigma_S C_S = -\sigma_S \tilde{C} + H_S \tilde{C}(1 - \tilde{C}) - K_S \frac{d\tilde{C}}{dz} \]  

(2)

In the stripping section

Boundary condition is

\[ \tilde{C} = C_0 \text{ at } z = 0 \]  

(3)

\( C_0 \) must be determined so as to satisfy total material balance and material balance for the lighter component

\[ \sigma_F = \sigma_E + \sigma_S \]  

(4)

\[ \sigma_F C_F = \sigma_E C_E + \sigma_S C_S \]  

(5)

Column coefficients \( H \) and \( K \) are expressed as follows:

\[ H_E = H_F \left(1 - \frac{15}{8} \beta R \right) \]  

(6)

\[ H_S = H_F \left(1 - \frac{15}{8} (1 - \beta) R \right) \]  

(6')

\[ K_{CE} = K_{CF} \left(1 - \frac{81}{64} \beta^2 R^2 \right) \]  

(7)

\[ K_{CS} = K_{CF} \left(1 - \frac{81}{64} (1 - \beta)^2 R^2 \right) \]  

(7')

\[ K_d = K_{dF} = 2 W \rho D_{AB} \]  

(8)

\[ K_E = K_{CE} + K_d \]  

\[ K_S = K_{CS} + K_d \]  

(9)

where

\[ H_F = \left(4/15\right) \left(\sigma_F \rho_T g \beta_T (\Delta T)^{\gamma} / \eta_T T \right) \]  

(10)

\[ K_{CF} = \left(32/15\right) \left(2 W \rho_{D_{AB}} \right) G_{nat.} \]  

(11)

\[ G_{nat.} = \left[B \rho_T g \beta_T (\Delta T) / 192 \rho_T \right] \]  

(12)

\[ R = \sigma_F / G_{nat.} \]  

(13)

\[ H \text{ and } K \] are dependent on the net flow rate and different from \( H_F \) and \( K_{CF} \) presented by Jones-Furry. As experiments usually are conducted in the range of \( R < 1 \), values of \( H \) reduce faster than those of \( K \) as values of \( R \) increase. Separations calculated by using \( H \) and \( K \) will be estimated less than those by \( H_F \) and \( K_{CF} \).

**Concentration distribution**

Let us introduce the following dimensionless quantities

\[ \beta = \frac{\sigma_E}{\sigma_F}, \quad \zeta_0 = \frac{L_S}{L_T}, \quad \zeta = \frac{(z + L_S)}{L_T} \]  

(13)

\[ \Phi_H E = \frac{\sigma_F}{H_E}, \quad \Phi_{KE} = \frac{\sigma_F L_T}{K_E}, \quad \Phi_H S = \frac{\sigma_F}{H_S}, \quad \Phi_{KS} = \frac{\sigma_F L_T}{S K} \]  

(14)

Transport equations become, in the enriching section,

\[ \frac{d\tilde{C}}{d\zeta} - \Phi_{KE} \tilde{C}(1 - \tilde{C}) + \Phi_{KS}(1 - \beta)(\tilde{C} - C_0) = 0 \]  

(15)

and in the stripping section

\[ \frac{d\tilde{C}}{d\zeta} - \Phi_{HS} \tilde{C}(1 - \tilde{C}) + \Phi_{KS}(1 - \beta)(\tilde{C} - C_0) = 0 \]  

(16)

Boundary condition is

\[ \tilde{C} = C_0 \text{ at } \zeta = \zeta_0 \]  

(17)

The material balance for the lighter becomes

\[ C_F = \beta C_E + (1 - \beta) C_S \]  

(18)

Solutions to these equations become as follows:

enriching separation concentration is

\[ \tilde{C} = \frac{C_0 \left[ (\sqrt{D_E} + (1 + \beta \Phi_H E) \cdot \tanh(\sqrt{D_E} \cdot A_E)) \right]}{\left[ (\sqrt{D_E} + (\beta \Phi_H E + 2 C_0 - 1) \cdot \tanh(\sqrt{D_E} \cdot A_E)) \right]} \]  

(19)

and stripping separation concentration is

\[ C_S = \frac{1}{1 - C_0} \left[ (\sqrt{D_S} + [1 + (1 - \beta) \Phi_H S] \cdot \tanh(\sqrt{D_S} \cdot A_S)) \right]}{\left[ (\sqrt{D_S} + [(1 - \beta) \Phi_H S + 1 - 2 C_0] \cdot \tanh(\sqrt{D_S} \cdot A_S)) \right]} \]  

(20)

The value of \( C_0 \) included in the above solutions must be determined so as to satisfy Eq. (5). \( C_E \) and \( C_S \) were calculated by a digital computer.
Now, let us consider the case in which the vertical concentration gradient is small and that in the column, \( C(1 - C) \approx a \) (constant). Then, Eqs. \((15)\) and \((16)\) become respectively

\[
C_E - C_0 = a \frac{[1 - \exp(-\beta \Phi_{HE} \cdot A_E)]}{\Phi_{HE} \cdot A_E}
\]

\[
C_S - C_0 = -a \frac{[1 - \exp(-\beta \Phi_{HS} \cdot A_S)]}{\Phi_{HS} \cdot A_S}
\]

Eq. \((5)\) becomes

\[
\frac{(C_F - C_0)}{a} = \frac{[1 - \exp(-\beta \Phi_{HE} \cdot A_E)]}{\Phi_{HE} \cdot A_E} - \frac{[1 - \exp(-\beta \Phi_{HS} \cdot A_S)]}{\Phi_{HS} \cdot A_S}
\]

Jones-Furry assumed that the concentration at the feed position in the column was equal to that of the feed gas. In other words, the feed position must be chosen so that \( C_0 = C_F \). From this point of view, Powers et al. conducted all experiments under the conditions of \( a_S = \sigma_E = \sigma_F \) and \( L_S = L_E = L_F \). To satisfy the condition of \( C_0 = C_F \) in Eq. \((5)\)’, the following relation is necessary:

\[
\frac{\Phi_{HE}}{\Phi_{HS}} = \frac{1 - \exp(-\beta \Phi_{HE} \cdot A_E)}{1 - \exp(-\beta \Phi_{HS} \cdot A_S)}
\]

If \( H_E = H_S \) and \( K_E = K_S \), the above becomes

\[
1 - \beta = \xi_0 \quad \text{or} \quad \sigma_E/\sigma_S = L_S/L_E
\]

Eq. \((18)\) indicates that the experimental conditions specified by Powers et al. are enough to satisfy the relation of \( C_0 = C_F \). However, \( C_0 \) is not equal to \( C_F \) in the case where \( C(1 - C) \) has no constant value over the column because of large separations, and where the feed enters at an arbitrary height of the column, and top and bottom products are taken continuously at an arbitrary ratio to flow rate.

**Experimental Apparatus and Procedure**

The schematic diagram of the experimental apparatus is shown in Fig. 2. The column used for this investigation was constructed in the form of two concentric cylinders made of stainless steel, 14 mm being the outside diameter of the inner tube which was heated by nickel-chrome heater, and 40 mm the inside diameter of the outer tube which was cooled by water. The distance between the two tubes \((2w)\) was 13 mm, and the column height was 154.5 cm. To prevent heater from bending caused by thermal expansion, the top of the heater was supported tightly and the bottom inserted, keeping a spacing, in a short pipe welded to the bottom flange of the column. To change feed position and measure hot wall temperatures, the outer tube has five holes at intervals of nearly one-fifth the column height along the column. The feed gas entered the column through one of these holes. The top product was taken out along the hot tube and the bottom product along the cold tube. Three thermocouples contact the inner tube through the other holes to measure hot wall temperatures. A high flow rate of water is used to keep the cold-wall temperature rise minimum, and inlet and outlet temperatures of cooling water through the jacketed outer tube were nearly equal. Experiments were performed under atmospheric pressure.

At the beginning of a run, the column and all pipelines were purged completely with feed mixture supplied from a gas bomb, and the feed rate was set at a specified flow rate. Then the inner tube was heated. The time at which the tube began to be heated was zero elapsed time. Two capillary flow meters were used to set top and bottom product rates approximately at a desired rate, and the two product rates were measured exactly by soap film meters. Three controlling values for small flow rates were used, on leading vinyl tubes, to control feed rate and enriching and stripping flow rates. Considerable care was taken to maintain a constant feed rate and constant wall temperatures.

All experiments were performed with He-Ar mixture of 46.5 mole\% He(8.1 mass\%). The runs were made at heater temperature level of 100°C and 300°C, and at feed rates for the range of 0 to 12 × 10\(^{-3}\) g/sec.

Samples taken from both the product streams were analyzed by gas chromatography periodically, and feed mixture concentration was analyzed twice before and after experiments. Sampling ports were located after the capillary to eliminate any disturbances introduced by sampling procedures. Sufficient time was allowed for reaching steady-state separation.

**Experimental Results and Discussion**

Approach to steady-state separations. It is shown in Fig. 3 that the time required to reach steady-state separation was about a half hour to one hour in the present experiments. This is the same time required for heater temperature to approach steady-state. That is, it can be observed that the heating process of the heater is rate-determining in this column. Typical experimental results at steady-state are indicated in Table 1.
### Table 1 Comparison of experimental results with theories for steady state separation concentrations

<table>
<thead>
<tr>
<th>Run No.</th>
<th>Mass flow rate mean $\mu$</th>
<th>Ratio Calculated results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu \times 10^3$ [g/sec]</td>
<td>$\mu / \mu_0$</td>
</tr>
<tr>
<td>106</td>
<td>0.9523</td>
<td>0.9523 0.6950 0.2573 0.2702 0.1009 0.0378 0.2125 0.0331 0.1052 0.0154</td>
</tr>
<tr>
<td>107</td>
<td>0.9715</td>
<td>0.9715 0.4815 0.4900 0.5044 0.1185 0.0405 0.2340 0.0345 0.1431 0.0200</td>
</tr>
<tr>
<td>108</td>
<td>0.9591</td>
<td>0.9591 0.3356 0.6228 0.7889 0.1460 0.0470 0.2716 0.0361 0.2555 0.0342</td>
</tr>
<tr>
<td>109</td>
<td>0.9795</td>
<td>0.9795 0.2068 0.7277 0.8927 0.1911 0.0582 0.2955 0.0366 0.3877 0.0553</td>
</tr>
<tr>
<td>110</td>
<td>0.9336</td>
<td>0.9336 0.0722 0.8614 0.9227 0.2718 0.0688 0.2955 0.0366 0.3877 0.0553</td>
</tr>
<tr>
<td>121</td>
<td>3.943</td>
<td>3.943 3.264 1.723 0.6793 0.0908 0.0411 0.1248 0.0356 0.0922 0.0270</td>
</tr>
<tr>
<td>122</td>
<td>3.816</td>
<td>3.816 2.556 1.200 0.3302 0.1029 0.0426 0.1462 0.0414 0.1193 0.0354</td>
</tr>
<tr>
<td>123</td>
<td>3.839</td>
<td>3.839 2.063 1.740 0.4575 0.1137 0.0548 0.1678 0.0453 0.1748 0.0425</td>
</tr>
<tr>
<td>124</td>
<td>3.914</td>
<td>3.914 1.416 2.498 0.6382 0.1400 0.0542 0.2318 0.0487 0.2449 0.0554</td>
</tr>
<tr>
<td>125</td>
<td>3.718</td>
<td>3.718 0.501 1.377 0.8652 0.2255 0.0625 0.2318 0.0487 0.2449 0.0554</td>
</tr>
<tr>
<td>136</td>
<td>11.56</td>
<td>11.56 9.759 1.799 0.1556 0.0851 0.0549 0.0960 0.0417 0.0882 0.0414</td>
</tr>
<tr>
<td>137</td>
<td>11.47</td>
<td>11.47 8.152 3.915 0.2891 0.0906 0.0595 0.0990 0.0491 0.0932 0.0506</td>
</tr>
<tr>
<td>138</td>
<td>11.52</td>
<td>11.52 6.563 4.737 0.4192 0.0968 0.0595 0.1033 0.0547 0.0983 0.0572</td>
</tr>
<tr>
<td>139</td>
<td>11.61</td>
<td>11.61 4.330 7.278 0.6270 0.1086 0.0648 0.1145 0.0618 0.1088 0.0644</td>
</tr>
<tr>
<td>140</td>
<td>11.68</td>
<td>11.68 2.023 9.656 0.8268 0.1380 0.0682 0.1473 0.0661 0.1405 0.0685</td>
</tr>
</tbody>
</table>

* $H_F = 1.969 \times 10^{-8}$ g/sec
** $G_{exit} = 68.62 \times 10^{-8}$ g/sec

![Fig. 3](image3.png) Transient behavior of enriching and stripping separation concentrations, and heater temperature

In this table, Jones-Furry theory was solved by boundary condition $C=C_F$ at $z=0$, using $H_F$ and $K_F$ as $H$ and $K$, respectively. On the other hand, the authors' calculation was based on a more general condition $C=C_0$ at $z=0$, using $H_E$ and $H_S$, and $K_E$ and $K_S$ for enriching and stripping section, respectively. All physical properties were estimated with feed gas concentration and arithmetic mean temperature of heater and cooling water. Thermal diffusion constant $\kappa$ was taken to be 0.42.

**Effects of the cut(β) on separation concentrations**

From Fig. 4, two significant facts are found. The first is that enriching separation concentration $C_E$ decreases sharply to $C_F$ as $\beta$ approaches 1, while in the stripping section $C_S$ increases to $C_F$ as $\beta$ approaches zero. The second is that $\beta$ has a more sensitive effect on separation concentrations than feed rate.

As shown in Fig. 5 and Table 1, the experimental results agree with the calculated when $\beta$ approaches 1 in the enriching and 0 in the stripping, that is to say, when the difference between feed concentration and separation concentrations is small. There is a deviation from the calculated curves in the region of large separations, because the effects of concentration dependence of physical properties are disregarded.

**Relations between flow rates and separation concentrations**

In accordance with Jones-Furry theory, it is concluded that the flow rate dependence of $C_E$ (or $C_S$) should be determined only by $\sigma_E$ (or $\sigma_S$). In other words, if $\sigma_E(=\beta \sigma_F)$ value is constant even though separation experiments were conducted at various feed rates, $C_E$ should be constant. According to the present work, however, $C_E$ at constant values of $\sigma_F$ varies along fine solid lines in Fig. 6. It may be shown that the flow rate dependence of $C_E$ and $C_S$ is poorly predicted by...
Jones-Furry theory, but well by the present work. As flow rate increases, the variations of $C_E$ at constant values of $\sigma_F$ becomes smaller, and the present work is in agreement with Jones-Furry theory. However, the smaller $\sigma_E$ becomes, the larger become the variations of $C_E$.

Effects of feed position on separations

Figure 7 shows that the effects of feed position on separations is negligible within the region of ten percent of mean natural convection flow rate. This is in agreement with the view substantiated by A.L. Jones, if the internal convective circulation is large as compared with the superimposed flow, that separation should be independent of the exact feed position. It is found from the calculated curves in Fig. 8 that feed position has no influence on separation concentrations. But the curves deviate from the experimental results when $\beta$ is small in the enriching and large in the stripping. This may be due to negligence of the effects of concentration dependence of physical properties.

Concentration distribution in column

Experimental results and calculated concentration distribution are shown in Fig. 9. It is possible to estimate the concentration in the column by using the calculated curves, because separation concentrations at both ends are in agreement with experimental data. The concentration in the column at the feed position deviates remarkably from the feed gas concentration.

Graphic Estimation of Separation and Other Operating Variables

It is convenient for the design of the column to estimate graphically many operating variables required in separations. As expressed in Eqs. (15) and (16), it is necessary for the calculation of $C_E$ and $C_S$ to give values of seven variables $C_F, \beta, \zeta_0, \Phi_{HE}, \Phi_{HS}, \Phi_{KE}$ and $\Phi_{KS}$. If the ratio of the feed rate to the natural convection rate $R$ is smaller than 0.1, $H$ and $K$ are approximately equal to $H_S$ and $K_S$, respectively. So $\Phi_{HE}$ and $\Phi_{KE}$ have the same values as $\Phi_{HS}$ and $\Phi_{KS}$. When $R<0.1$, $C_E$ and $C_S$ may be calculated by using five variables $C_F, \beta, \zeta_0, \Phi_H$ and $\Phi_K$. Figures 10 a and b indicate the results of numerical calculation in the case of $C_F=0.081, \zeta_0=0.9288$ and $\beta=0.2$, taking $\Phi_H$ or $\Delta$ as a parameter.

Next, let us illustrate how to use the graphs by making use of an example.

Example

It is desired to separate a binary gas mixture A-B (8.1 mass% A, $C_F=0.081$) by a thermal diffusion column. Enriched product rate ($a_E$) is one-fourth of stripped product rate ($a_S$) ($\beta=0.2$) and feed position is located at a height of 92.88% of total column length from the bottom ($\zeta_0=0.9288$). Assume that the column coefficients $H_F$ and $K_F$ are $6.36 \times 10^{-3}$ g/sec and $164.2 \times 10^{-3}$ g-cm/sec, respectively.

Problem 1) When total column length is 154.5 cm and the mixture is fed at flow rate of $5 \times 10^{-3}$ g/sec into the column, estimate $C_E$ and $C_S$.

Problem 2) To raise $C_E$ to 30 mass% when feed rate is $8 \times 10^{-3}$ g/sec, what column length is necessary?

Problem 3) To obtain $C_E$ of 30 mass% at a column length of 154.5 cm, what feed rates are necessary?

Solution

1) Two quantities $\Phi_H$ and $\Phi_K$ are

$$\Phi_H=\sigma_F/H_F=(5 \times 10^{-3})/6.36 \times 10^{-3}=0.786$$

$$\Phi_K=\sigma_F L_T/K_F=(5 \times 10^{-3})(154.5)/164.2 \times 10^{-3}=4.71$$

The point at which $\Phi_H$ and $\Phi_K$ have the above values in Fig. 10-a and -b indicates $C_E$ to be 0.305 and $C_S$ to be...
0.024. Experimental results for \( C_E \) and \( C_S \) under the same conditions are 0.307 and 0.023, respectively, as shown in Figs. 6 and 4.

2) \( \Phi_H \) becomes \( \sigma_F/H_F=8 \times 10^{-3}/6.36 \times 10^{-3}=1.26 \). The point of intersection of \( \Phi_H=1.26 \) and \( C_P=0.3 \), in Fig. 10-a, indicates \( \Phi_K \) to be 14.0. Then, total length of column is estimated as follows:

\[
14.0=\sigma_F L_T/K_F=(8 \times 10^{-3})/(164.2 \times 10^{-3})
\]

therefore, \( L_T=287 \) cm.

The experimental data for \( L_T=154.5 \) cm, \( \sigma_F=8.0 \times 10^{-3} \) g/sec and \( \beta=0.2 \) indicates \( C_E \) to be 0.245 from Fig. 6. Therefore, nearly twice the column length used in the present work is necessary to raise \( C_E \) by 0.055 of the rest.

3) Neither \( \Phi_H \) or \( \Phi_K \) can be calculated as they contain the unknown feed rate \( \sigma_F \). But, as shown in Figs. 10-a and -b, it is convenient for solving the problem to use diagram pictured by taking the variable \( A \), defined by Eq. (19), as a parameter.

\[
A=\Phi_K/\Phi_H=H_F L_T/K_F \quad (19)
\]

\( A \) becomes \( A=(6.36 \times 10^{-3})(154.5)/164.2 \times 10^{-3}=5.99 \)

The point of intersection of \( C_E=0.3 \) and \( A=5.99 \) indicates \( \Phi_H \) as d \( \Phi_K \) to be 0.83 and 4.9, respectively.

Feed rate \( \sigma_F \) is estimated independently from \( \Phi_H \) and \( \Phi_K \) as follows:

\[
\Phi_H=\sigma_F/H_F=0.83
\]

therefore, \( \sigma_F=(0.83)(0.63 \times 10^{-3})=5.28 \times 10^{-3} \) g/sec.

\[
\Phi_K=\sigma_F L_T/K_P=4.9, \text{ therefore, } \sigma_F=(4.9)/(164.2 \times 10^{-3})/154.5=5.20 \times 10^{-3} \text{ g/sec.}
\]
Thus, \( \sigma_F \) is determined by using the dimensionless quantity \( A \). The corresponding experimental result shown \( \sigma_F \) to be 5.2 \times 10^{-3} g/sec in Fig. 6.

**Conclusion**

Jones-Furry theory is not valid for the estimation of separations of mixtures which bring about large separations in a thermal diffusion column. To estimate separations, total material balance over the column and consideration of how to determine the concentration at the feed position in the column are necessary. As a result, it was found that new factors \( \beta (=\sigma_E/\sigma_F) \) and \( \xi_0 (=L_S/L_T) \) are necessary to determine separations. The following facts were obtained from experimental results.

1) The cut(\( \beta \)) has a sensitive effect on separations, and it is larger than the effect of feed rate.

2) Feed gas position is not the major factor affecting separations in the range of \( R<0.1 \).

3) Experimental results are in agreement with calculated values. But a deviation is observed in the region of large separations. The deviation may be due to the ignorance of the concentration dependence of physical properties.

Lastly, a graphic method for estimation of separation characteristics was presented by making use of an example.

**Acknowledgement**

The authors wish to thank Prof. Shigefumi Fujita for very valuable discussions, and Mr. Keizo Ito for the construction of experimental apparatus. This work was partially supported by the Scientific Research Foundation of the Ministry of Education of Japan.

**Nomenclature**

\[
\begin{align*}
A_E &= H_S L_E/2K_E \\
A_S &= H_S L_S/2K_S \\
\alpha &= C(1-C)=\text{constant} \\
B &= \text{width of plate}
\end{align*}
\]

\[[-] \]
Approximate expressions of stream functions related to fluid spheres are obtained for external and internal fluids in non-Newtonian systems described by the power law model.

Motion and mass transfer of fluid spheres in non-Newtonian systems are considered by using these approximate stream functions. Drag coefficient is evaluated by the energetic approximate method. Unsteady mass transfer to fluid spheres is analyzed by use of similarity transformations.

Comparisons between the present analysis and experimental data available in the literature show good agreement.

1. Introduction

Because of their importance in many chemical engineering processes, motion and mass transfer of fluid spheres in Newtonian fluid systems have been studied extensively in the past. However, references for non-Newtonian fluid systems are few.

The problem of flow of a power-law fluid around a fluid sphere was analyzed at low or intermediate Reynolds number by Nakano and Tien. They used a technique which is a combination of Galerkin’s method and variational principle. Mhatre and Kintner, Fararoui and Kintner, Warshay et al., Marrucci et al., and Mohan et al. studied the shape and terminal velocity of drops moving through power-law fluids and/or correlated their experimental data. Motion of bubbles in non-Newtonian fluids was theoretically or experimentally studied by Astarita and Apuzzo, Hirose and Moo-Young, Galderbank et al., and Barnett et al. The problem of mass transfer from bubbles in non-Newtonian fluids was experimentally studied by Aiba et al., Hirose and Moo-Young, Calderbank et al., and Barnett et al. 432

MOTION AND MASS TRANSFER OF FLUID SPHERES IN NON-NEWTONIAN SYSTEMS*

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Presented at the 36th Annual Meeting of the Soc. of Chem. Engns., Japan, Tokyo, April 3, 1971

JOURNAL OF CHEMICAL ENGINEERING OF JAPAN