Conclusion

A method to determine rational design margin taking account of parameter uncertainty is developed for practical process system design. The design margin is determined by minimizing the deviation of a performance index under the allowable limitation of the deviation of output variables. An altered form of the performance index, however, may have to be considered in the future. The method proposed here is applied to design of a simple chemical reaction process with recycle. Numerical calculation shows that this method is more rational and practical.

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Nomenclature

A	= reactant	
B	= intermediate product	
ΔC_L	= allowable lower limit of the deviation of output vector	
ΔC_U	= allowable upper limit of the deviation of output vector	
C_i	= molar concentration of i -th component ($i=A, B, R, X, Y$)	[mol/l]
F_{A0}	= mole flow rate of fresh feed	[mol/hr]
F_i	= output flow rate from reactor	[mol/hr]
k_i	= reaction rate constant ($i=B, X, Y, R$)	[1/hr]
M	= m -dim. column vector of design variable	
$\bar{M}$	= nominal optimum value of M	
ΔM	= deviation of design variable from $\bar{M}$	
P	= p -dim. column vector of parameter	
$\bar{P}$	= nominal value of P	
ΔP	= deviation of parameter from $\bar{P}$	
R	= desired product	

u	= r -dim. column vector of manipulating variable	
$\bar{u}$	= nominal optimum value of u	
Δu	= deviation of u from $\bar{u}$	
V	= tank volume	[l]
$\bar{V}$	= nominal optimum value of V	[l]
ΔV	= design margin of V	[l]
x_i	= mol fraction of each component i to C_{A0} [$i=A, B, X, Y, R$]	[—]
X	= undesired product	
Y	= undesired product	
$\$b$	= hourly unit cost of reactor	[\$/l/hr]
$\$r$	= unit cost of recycle of A, B, X and Y	[\$/mol]
α	= recycle ratio of A and B	[—]
β	= recycle ratio of X and Y	[—]
ξ_i	= allowable upper or lower limit of output variable in Eq.(14) ($i=1, 2, 3$)	
η	= p -dim. vector of the range of parameter deviation	
η°	= relative range of parameter deviation in Eq.(13)	
η_k	= k -th element of vector η	
<Subscript>		
A	= reactant	
B	= intermediate product	
R	= desired product	
X	= undesired product	
Y	= undesired product	
<Superscript>		
-1	= inverse matrix	

Literature Cited

- 1) Levenspiel, O.: "Chemical Reaction Engineering", John Wiley Son., 136-137 (1962)
- 2) Rohrel, R. A. and M. Sobral, Jr.: *IEEE Trans. on AC*, **AC-10**, **1**, 43-48 (1965)
- 3) Rudd, D. F. and C. C. Watson: "Strategy of Process Eng.", Chapter 11-15, John Wiley Son (1968)
- 4) Takamatsu, T., I. Hashimoto and H. Ohno: *I. E. C. Process Des. & Dev.*, **9**, **3**, 368-378 (1970)
- 5) Takamatsu, T., I. Hashimoto and S. Shioya: *Systems and Control of Japan*, **15**, **2** (1971)

SHORT COMMUNICATIONS

A NOTE ON THIN FILM EVAPORATION — PREDICTION OF HEAT TRANSFER RATES* —

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Introduction and Analysis

Chun and Seban²⁾ experimentally investigated the falling-film evaporator under turbulent flow conditions of the liquid film. The data obtained are correlated by the following non-dimensional equation for the turbulent flow of the film.

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$$h\left(\frac{\nu_L^2}{gk^3}\right)^{1/3} = 3.8 \times 10^{-3} Re_L^{0.4} Pr_L^{0.65} \quad (1)$$

The data obtained are also compared to the analysis of Dukler³⁾ and it is observed that the deviation between the experimental results and the theoretical analysis is considerable. Dukler³⁾ obtained heat transfer rates in falling films by numerical integration, and the analysis did not offer any straightforward equation to facilitate ready computation. The present note attempts to establish an equation for heat transfer coefficient in explicit form making use of some of the available hydrodynamic aspects of falling liquid films. The

Fig. 1 Temperature profile

Fig. 2 Heat transfer coefficient vs. Reynolds number

equation obtained can be used to estimate approximately heat transfer rates for $Pr \geq 1$.

The physical model solved is the one corresponding to that of (1). The necessary heat for vaporization taking place at the vapor-liquid interface is assumed to be conducted through the liquid film.

Further, the liquid is assumed to be near the saturation temperature corresponding to the system pressure. Molecular diffusion of the vapor at the vapor-liquid interface is not accounted for in the analysis. Thus, we have for the turbulent flow conditions of the liquid film,

$$\frac{q}{\rho_L C_p} = -(\alpha + \epsilon_H) \frac{dt}{dy}$$

whence for steady-state transport conditions, neglecting the possible curvature effects, we have

$$\frac{d}{dy} \left\{ (\alpha + \epsilon_H) \frac{dt}{dy} \right\} = 0 \quad (2)$$

where ϵ_H refers to eddy diffusivity for heat. Eq.(2) implies that the convective component of the enthalpy in the liquid film is totally ignored. When $0.029 \leq Pr \leq 14.3$, it is valid to assume $\epsilon_H = \epsilon_M^{(6)}$. For thin films, i. e., $\delta^+ < 50$, the eddy diffusivity⁽⁹⁾ ϵ_H is given by

$$\frac{\epsilon_H}{\alpha} = Pr \cdot A^2 y^{+2} \quad (3)$$

where

$$A = 0.091$$

Even though the suggested limit of applicability of Eq. (3) is $\delta^+ = 50$, it is observed that the velocity profile obtained in a straight circular tube with the aid of

Eq.(3) can be favourably compared to the universal velocity profile of Von Karman up to a value of $y^+ \leq 100$. Substituting Eq.(3) in Eq.(2) we get

$$\frac{d}{dy} \left\{ \alpha (1 + Pr \cdot A^2 y^{+2}) \frac{dt}{dy} \right\} = 0$$

or

$$\frac{d}{dy^+} \left\{ (1 + Pr \cdot A^2 y^{+2}) \frac{dt^+}{dy^+} \right\} = 0 \quad (4)$$

The solution of Eq.(4) subjected to the boundary conditions, viz., at $y^+ = 0$, $T^+ = 0$; and at $y^+ = \delta^+$, $T^+ = 1$ can be obtained as

$$T^+ = \frac{\tan^{-1}(Pr^{1/2} \cdot A \cdot y^+)}{\tan^{-1}(Pr^{1/2} \cdot A \cdot \delta^+)} \quad (5)$$

The temperature profiles are shown in Fig. 1 for $Pr = 5$ and 10; and $\delta^+ = 30$ and 50. The heat transfer coefficient under non-nucleating conditions can be obtained from

$$q_w = -k \frac{dt}{dy} \bigg|_{y=0} \quad (6)$$

Thus from Eqs.(5) and (6) we get

$$\frac{h\nu}{k_L u_L^*} = \frac{Pr^{1/2} \cdot A \cdot \beta}{\tan^{-1}(Pr^{1/2} \cdot A \cdot \delta^+)} \quad (7)$$

Eq.(7) is the basic form in which the shear velocity and the liquid film thickness are unknown. To attribute any utility to Eq.(7), the relationship for the film thickness as a function of Reynolds number must be known a priori. Kosky⁽⁷⁾ gives the relationship for falling films as

$$\delta^+ = 0.0504 Re_L^{7/8} \quad (8)$$

The shear velocity can be connected to the liquid film thickness by a simple force balance applied to an elemental length of the evaporator tube. Thus, we get for zero shear condition at the vapor-liquid interface

$$u_L^* = \sqrt{g \cdot \delta (1 + \delta/D)} \approx \sqrt{g \delta} \quad \text{when} \quad \frac{\delta}{D} \ll 1 \quad (9)$$

and

$$\frac{g \delta^3}{\nu_L^2} \approx 0.0025 Re_L^{7/4} \quad (10)$$

Introducing Eqs.(8) and (9) in Eq.(7) and after manipulation we get

$$h \left(\frac{\nu_L^2}{g k^3} \right)^{1/3} = \frac{0.03685 \beta Re_L^{7/24} Pr^{1/2}}{\tan^{-1}(0.00459 Pr^{1/2} \cdot Re_L^{7/8})} \quad (11)$$

It should be borne in mind that the accuracy of Eq. (10) depends solely upon the correctness of Eq.(8). From a recent review⁽¹⁰⁾ on falling films by Wallis, one can find that the exponent to Re_L differs from author to author. The choice of Eq.(8) in preference to others is mainly based on the reasoning that this equation has been able to satisfy experimental data obtained from many sources.

Figure 2 gives a comparison of the various available equations^(2,11). As is obvious from the plot, the present theoretical analysis lies between the two equations. Even though Eq.(11) does not reveal exact coincidence

with either of the two equations, it can be made use of as an estimation tool for design purposes and it shows a maximum deviation of 15 per cent from that of Eq. (1). As pointed out earlier Eq. (7) preserves its generality, and it is very likely that the relationship may vary from system to system [for example, Fr may have significant influence on film thickness⁸⁾] and Eq. (11) may have a different form depending upon the type of relationship chosen for the film dynamics.

The applicability of the analogy between momentum and heat transfer is further tested for the falling-film evaporators. Correcting for Prandtl effects, as is generally done for turbulent flow problems under free convective conditions⁴⁾, the Reynolds analogy can be written as

$$\frac{q_w}{\tau_w} = \frac{C_p \Delta t}{u_i} Pr_L^{-2/3} \quad (12)$$

where $u_i \approx 8/7 \cdot u_m$ for a one-seventh velocity profile. The relationship proposed by Belkin *et al.*¹⁾ for falling films has the form

$$\frac{\delta \cdot g^{1/3}}{\nu_L^{2/3}} = 0.063 Re_L^{2/3} \quad (13)$$

Substituting Eqs. (10) and (13) in Eq. (12) we get

$$\frac{q_w}{\Delta t} \left(\frac{\nu_L^2}{g k^3} \right)^{1/3} = 0.01389 Pr_L^{1/3} Re_L^{1/3} \quad (14)$$

Defining

$$q_w = h(T_w - T_B) = h \cdot \beta(T_w - T_s) \quad (15)$$

where β is the temperature correction factor, we get

$$h \left(\frac{\nu^2}{g k^3} \right)^{1/3} = \frac{0.01389}{\beta} Pr_L^{1/3} Re_L^{1/3} \quad (16)$$

The value of β is approximated to 1.125 based on a simple assumption at this stage that the temperature profiles are akin to the velocity profiles. Eq. (16) is also shown plotted in Fig. 2, and the agreement of the present analysis with the equation of Chun and Seban²⁾ (of course with a deviation of 10–15 per cent) justifies the validity of the simplifying assumptions introduced therein. Thus, the proposed equation can be employed

as an estimation tool for engineering calculations.

Acknowledgement

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Nomenclature

C_p	= specific heat at constant pressure
g	= acceleration of gravity
h	= heat transfer coefficient
k	= thermal conductivity
Pr	= Prandtl number
q	= heat flux
Re	= film Reynolds number
T^+	= dimensionless temperature $= (T_w - T)/(T_w - T_s)$
t	= temperature
u	= velocity
u_L^*	= shear velocity
y	= ordinate measured normal to the wall
α	= thermal diffusivity of the liquid
δ^+	= non dimensional film thickness $(= u_L^* \delta / \nu_L)$
μ	= absolute viscosity
ν	= kinematic viscosity
ρ	= density
τ	= shear stress

<Subscripts>

B	= bulk mean
i	= interface
L	= liquid
m	= mean
s	= saturation
w	= wall

Literature Cited

- 1) Belkin, H. H., A. A. Macheod, C. C. Monrad and R. R. Rothfos: *AIChE J.*, **5**, 245 (1959)
- 2) Chun, K. R. and R. A. Seban: *Trans. ASME, J. of Heat Transfer, Series C*, **93**, 391 (1971)
- 3) Dukler, A. E.: *Chem. Eng. Prog., Symp. Ser.*, **56**(30), 1 (1960)
- 4) Eckert, E. R. G. and R. M. Drake, Jr.: "Heat and Mass Transfer", p. 324, McGraw-Hill, New York (1959)
- 5) Garwin, L. and E. W. Kelly: *Ind. Eng. Chem.*, **47**, 392 (1955)
- 6) Hughmark, G. A.: *AIChE J.*, **17**, 902 (1971)
- 7) Kosky, P. G.: *Int. J. of Heat & Mass Transfer*, **14**, 1220 (1971)
- 8) Portalski, S.: *Chem. Eng. Science*, **18**, Dec. (1963)
- 9) Sleicher, C. A.: *Trans. ASUE*, **80**, 693 (1958)
- 10) Wallis, G. B.: "One-dimensional Two-phase Flow", p. 332, McGraw-Hill, New York (1969)
- 11) Wilke, W.: "Wärmeübergang an Reiselfilme", Vol. 490, Ver. Deut., Inger. Forshungsh. (1962)

APPROXIMATION FOR PREDICTION OF VAPORIZATION PLATE EFFICIENCY*

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The great majority of work on distillation calcula-

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tion has dealt with the assumption of ideal stage. Recently, as it was assumed that the liquid on the plate is perfect mixed, plate efficiencies have been included in treatment of the distillation column by several investigators^{1,6,9)} in order to approach more closely