- 23) Toyama, A.: M. S. Thesis of Rice Univ. (1966)
 24) Wichterle, I. and R. Kobayashi: *J. Chem. Eng. Data*, **17**, 4 (1972)
 25) Wichterle, I. and R. Kobayashi: *J. Chem. Eng. Data*, **17**, 9 (1972)
 26) Yen, L. C. S. and J. J. Mcketta: *AIChE J.*, **8**, 501 (1962)
 27) Zellner, M. G., L. C. Claitor and J. M. Prausnitz: *Ind. Eng. Chem., Fundamentals*, **9**, 549 (1970)

PHENOMENAL TRANSITION OF LIQUID ATOMIZATION FROM DISK*

SHIRO MATSUMOTO, KUNIKI SAITO AND YOICHI TAKASHIMA
Research Laboratory of Nuclear Reactor, Tokyo Institute of Technology, Tokyo

The change of atomization mechanism at the edge of a rotating disk was studied based on theoretical consideration and various experimental data.

The flow rate at the transition was determined by seeing directly the transition phenomena with a stroboscope. Four kinds of liquids and five kinds of disks were used in this experiment, and the rotational number was varied from 500 to 4000 rpm.

As a result, it was seen that the transition accompanied a kind of hysteresis phenomenon. The flow rate at the transition from direct drop to ligament formation was a little more than that at the reverse transition, as given by Eqs.(27) and (28), respectively. Also, for the transition from ligament to film formation and the reverse transition, a similar hysteresis was obtained. Hinze and Milbourn's equation can be used for liquid of less than a few poises in viscosity for prediction of the flow rate at the transition from ligament to film formation, and the equation by Tanasawa *et al.* is applicable to more viscous liquids.

Introduction

Spinning-disk atomizers are widely used for gas-liquid contact processes such as spray drying, cooling, scrubbing and combustion. It is known that three different types of drop formation may take place around and beyond the edge of a rotating disk.

Figure 1 illustrates these three different types of disintegration. At a very low flow rate, direct drop formation occurs around the edge, as shown in Fig. 1-a. As the flow rate increases, the size of drops removed from the disk remains constant, and the frequency of drop formation increases. When the frequency rises above some critical value, the sudden transition to ligament formation from direct drop formation appears, as shown in Fig. 1-b. With further increase of the flow rate, the ligaments become thicker and longer and finally lose their capability of removing all the liquid from the edge, and a thin liquid film is formed around the disk, as shown in Fig. 1-c.

The transition between these phenomena of liquid disintegration by a spinning disk has been studied by a few investigators^{1,2,6)}. Tanasawa *et al.*⁶⁾ gave an experimental equation for giving the transition condition from direct drop formation to ligament formation. The transition flow rate Q_1 was given by

$$Q_1 = 2.8 \left(\frac{D}{N} \right)^{2/3} \left(\frac{\sigma}{\rho} \right) / \left\{ 1 + 10 \left(\frac{\mu}{\sqrt{\rho \sigma D}} \right)^{1/3} \right\} \quad (1)$$

where D is the diameter of disk, N the rotational number and ρ , μ and σ the density, the viscosity and the surface tension of liquid, respectively. Concerning the transition from ligament to film formation, Hinze and Milbourn²⁾ gave a semi-empirical equation on assuming that film formation would take place when the feed rate of liquid exceeds a certain flow rate within which the liquid can be removed through ligaments.

Fig. 1-a
 Direct drop formation
 $R = 6$ cm
 $N = 1000$ rpm
 $\mu = 0.598$ poise
 $\rho = 1.21$ g/cm³
 $\sigma = 63.0$ dyne/cm
 $Q = 2.0$ cm³/sec
 Fig. 1-b
 Ligament formation
 $R = 5$ cm
 $N = 1000$ rpm
 $\mu = 1.70$ poise
 $\rho = 1.23$ g/cm³
 $\sigma = 62.0$ dyne/cm
 $Q = 8.0$ cm³/sec
 Fig. 1-c
 Film formation
 $R = 6$ cm
 $N = 1000$ rpm
 $\mu = 0.598$ poise
 $\rho = 1.21$ g/cm³
 $\sigma = 63.0$ dyne/cm
 $Q = 16.8$ cm³/sec

Fig. 1 Photographs of three different types of atomization

* Received on July 20, 1973
 Presented at the 7th Autumn Meeting of The Soc. of Chem. Engrs., Japan (Nagoya, Oct. 1973)
 〒152 東京都目黒区大岡山2-12-1
 東京工業大学原子炉工学研究所 松本史朗

Fig. 3 Schematic curved boundary

Fig. 2 Diameter of drop produced by direct drop formation

This flow rate Q_2 was given by

$$\frac{\rho Q_2^2}{\sigma R^3} \left(\frac{\rho R^3 \omega^2}{\sigma} \right)^{0.6} \left(\frac{\mu^2}{\rho \sigma R} \right)^{1/6} = 4.56 \quad (2)$$

where R is the radius of the disk and ω the angular speed of the disk.

Tanasawa *et al.*⁶⁾ also gave semi-empirical equations. For viscous liquids ($D\rho/\mu < 30$), the transition flow rate Q_2 was given by

$$Q_2 = 5.3 \left(\frac{D}{N} \right)^{2/3} \left(\frac{\sigma}{\rho} \right) \left(\frac{\rho}{\mu} \right)^{1/3} \quad (3)$$

Fraser *et al.*¹⁾ assumed that the transition occurs when the contracting velocity of the film is larger than the radial velocity of liquid leaving the disk, and gave a semi-empirical equation as follows:

$$\left(\frac{\rho R^3 \omega^2}{\sigma} \right) \left(\frac{\bar{Q}_2}{R^3 \omega} \right) \left(\frac{R \mu}{\rho \bar{Q}_2} \right)^{0.19} = 0.636 \quad (4)$$

where $\bar{Q}_2$ is the transition flow rate from film to ligament formation.

In this paper, three dimensionless parameters relating to the two transitions of liquid disintegration have been derived theoretically from two simple models. Their transition flow rates have been correlated in terms of these dimensionless parameters experimentally. The experimental results have been compared with the above mentioned semi-empirical equations which are expressed by the three dimensionless parameters.

Theoretical Consideration

Firstly, we consider the transition from direct drop to ligament formation. Denoting the number of drops appearing on the disk edge n , the diameter of drop removed from the disk d , and the leaving frequency of a drop from the disk τ , the number of drops removed per second becomes $n\tau$ and from the mass balance,

$$\tau^+ = \frac{\tau}{\omega} = \frac{6Q_1}{\pi n d^3 \omega} \quad (5)$$

where τ^+ is the dimensionless frequency, Q_1 the flow rate and ω the angular velocity. It is assumed that the

transition from direct drop to ligament formation occurs when τ^+ is larger than some constant.

The diameter of drops produced by direct drop formation was obtained by Walton and Prewett⁷⁾.

$$\frac{d}{R} = 2.69 We^{-1/2} \quad (6)$$

This equation was confirmed by the authors' experimental results as shown in **Fig. 2**.

The number of local positions in which the drop grows along the disk edge will be same as that of ligaments when the transition occurs. The number of ligaments was studied by Hinze and Milbourn²⁾, and Kamiya³⁾. The semi-empirical equation of Hinze and Milbourn is given by

$$n = b We^{1/4} Re^{1/3} \quad (7)$$

where We and Re are defined by

$$We = \frac{\rho R^3 \omega^2}{\sigma}, \quad Re = \frac{R^2 \omega}{\nu} \quad (8)$$

From Eqs.(5), (6) and (7), the dimensionless transition flow rate Q_1^+ is given by

$$Q_1^+ = C Re^{5/6} We^{-5/4} \quad (9)$$

where Q_1^+ is defined by

$$Q_1^+ = \frac{Q_1}{2\pi R^2 \sqrt{\nu \omega}} \quad (10)$$

and ν is the kinematic viscosity of liquid.

However, the constant C must be determined experimentally.

Next, we consider the transition from film to ligament formation by assuming that the transition occurs when the surface force is overcome by the force of inertia and the centrifugal force at the free boundary beyond the edge of a rotating disk. Further, it is assumed that the free boundary forms a curved boundary corresponding to the radial velocity distribution at the edge of the disk, as shown in **Fig. 3**.

The force of inertia of liquid removed from the disk is given by

$$F_I = \int_0^{2\pi} \int_0^{z_0} \rho u^2 R d\theta dz \quad (11)$$

where u is the radial velocity of liquid and z_0 the surface position from the disk (film thickness).

The centrifugal force of liquid removed from the disk is given by

$$F_C = \int_0^{2\pi} \int_0^{z_0} \rho \frac{v^2}{R} \frac{u}{\omega} R d\theta dz \quad (12)$$

where v is the tangential velocity of liquid.

If the curved boundary of liquid removed from the disk is approximated by

$$r = R + \frac{u}{\omega} = R(1 + F) \quad (13)$$

Eq.(13) presents one radius of the curvature R_1 . The other radius R_2 is obtained by

$$R_2 = \left\{ 1 + \left(\frac{dz}{dr} \right)^2 \right\}^{3/2} \left/ \left| \frac{d^2r}{dz^2} \right| \right. \quad (14)$$

Also, the surface element, ds , on the curved boundary is obtained from

$$ds = 2\pi r \sqrt{1 + \left(\frac{dr}{dz} \right)^2} \quad (15)$$

Therefore, the surface force at the free boundary of liquid removed from the disk is given by

$$F_S = \int_0^{z_0} \sigma \left(\frac{1}{R_1} + \frac{1}{R_2} \right) ds dz \quad (16)$$

Previous dimensionless flow rate Q^+ is given by^{4,5)}

$$Q^+ = \frac{Q}{2\pi R^2 \sqrt{\nu \omega}} = \int_0^{\xi_0} F d\xi \quad (17)$$

where F is the dimensionless radial velocity divided by the rotating speed of the disk, $R\omega$, and ξ the dimensionless distance from the disk defined by

$$\xi = z \sqrt{\frac{\omega}{\nu}} \quad (18)$$

Thus, ξ_0 denotes the dimensionless film thickness on the disk.

By using the rotating speed and Eq.(18), Eqs.(11), (12) and (16) are transformed into the following equations.

$$F_I = 2\pi \rho R^3 \omega \sqrt{\nu \omega} \int_0^{\xi_0} F^2 d\xi \quad (19)$$

$$F_C = 2\pi \rho R^3 \omega \sqrt{\nu \omega} \int_0^{\xi_0} F G^2 d\xi \quad (20)$$

$$F_S = 2\pi \sigma \sqrt{\frac{\nu}{\omega}} \int_0^{\xi_0} \left\{ 1 + \frac{R^2 \omega}{\nu} (F')^2 \right\}^{3/2} + \frac{R^2 \omega}{\nu} |F''| F \frac{d\xi}{1 + \frac{R^2 \omega}{\nu} (F')^2} \quad (21)$$

where G is the dimensionless tangential velocity.

Thus the ratio, C' of both the force of inertia and the centrifugal force to the surface force is given by

$$C' = \frac{We \int_0^{\xi_0} (F^2 + FG^2) d\xi}{\int_0^{\xi_0} \frac{\{1 + Re(F')^2\}^{3/2} + Re|F''|F}{1 + Re(F')^2} d\xi} \quad (22)$$

Fig. 4 Schematic diagram of experimental apparatus

Fig. 5 Detail of disk

Therefore, it is assumed that if the ratio, C' , is smaller than some constant, the transition will take place from film to ligament formation, and if the ratio is larger than the constant, the reverse will occur. However, C' must be determined experimentally by calculating Eq.(22) under many experimental data. In this paper, we consider the functional correlation among the three dimensionless parameters, Re , We and ξ_0 . The numerator in Eq.(22) is a function of We and ξ_0 , the denominator a function of Re and ξ_0 . From Eq.(17), ξ_0 can be determined from Q^+ .

Therefore, C' can be given by the following functional correlation.

$$C' = f(Re, We, Q^+) \quad (23)$$

Thus the transition flow rate from film to ligament formation and the reverse may be correlated experimentally in terms of Re and We .

Eqs.(2), (3) and (4) can be rearranged by these parameters as follows:

Hinze and Milbourn:

$$Q_2^+ = 0.340 \frac{Re^{2/3}}{We^{0.883}} \quad (24)$$

Tanasawa *et al.*:

$$Q_2^+ = 0.297 \frac{Re^{6/5}}{We} \quad (25)$$

Fraser *et al.*:

$$\overline{Q}_2^+ = 0.108 \frac{Re^{2/3}}{We^{0.875}} \quad (26)$$

Experimental

A flow diagram of the experimental apparatus is shown in Fig. 4. The detail of disks is shown in Fig. 5. Both the flow rate and the rotational speed have been

Fig. 6 Transition from direct drop to ligament formation

Fig. 7 Transition from ligament to direct drop formation

varied to such an extent that three different types of disintegration have been achieved. The transition has been observed directly with a stroboscope.

The physical properties of liquid used in this experiment are given in **Table 1**. The density, the viscosity and the surface tension of liquid were measured by an ordinary hydrometer, a viscometer of Ubellode type and a Denouy tensiometer, respectively.

Result and Discussion

The representative results obtained in this experiment are listed in **Table 2**, where $\bar{Q}_1$ and $\bar{Q}_2$ are the transition flow rate from ligament to direct drop formation and from film to ligament formation, respectively.

The correlation expressed by Eq.(9) for the transition from direct drop to ligament formation gives good agreement with the experimental results. However, a little better correlation given by Eq.(27) has been obtained, as shown in **Fig. 6**.

In the case of the reverse process, the condition of transition happens at a slightly smaller flow rate, as

Table 1 Physical properties of liquid

Liquid		Density	Viscosity	Surface tension
		[g/cm ³]	[poise]	[dyne/cm]
A	Glycerine solution	1.23	1.70	62.0
B	Ethanol + Glycerine solution	1.09	0.495	31.0
C	Glycerine solution	1.21	0.598	63.0
D	Glycerine solution	1.16	0.115	64.2

Table 2 Experimental results of transition flow rate

Liquid	N [rpm]	R [cm]	Q ₁ [cm ³ /sec]	$\bar{Q}_1$ [cm ³ /sec]	Q ₂ [cm ³ /sec]	$\bar{Q}_2$ [cm ³ /sec]
A	1000	3	2.15	1.25	9.65	7.25
	2000	3	0.90	0.90	6.00	4.85
	2000	4	1.05	0.65	7.06	5.88
	3000	5	0.75	0.70	6.26	5.07
	2000	6	1.20	0.90	8.84	7.90
	3000	6	0.85	0.70	6.90	6.00
	3000	8	1.00	0.48	7.87	7.65
	4000	5	0.56	0.48	4.88	4.58
	4000	6	0.65	0.45	5.84	5.17
	300	5	9.53	8.00	34.2	28.1
C	500	3	3.48	2.00	17.1	15.5
	500	5	5.18	4.20	23.8	19.2
	500	6	6.40	5.10	26.2	20.1
	500	10	5.10	4.36	22.1	20.1
	1000	3	2.15	1.30	11.4	8.19
	1000	5	3.01	2.70	16.1	12.7
	1000	8	4.10	3.40	21.1	17.5
	2000	4	2.13	1.60	9.25	8.00
	2000	5	2.00	1.60	10.9	10.0
	500	3	10.2	9.00	22.4	21.2
D	800	5	9.50	5.70	25.7	19.6
	800	6	6.40	5.80	23.7	—
	1000	3	6.40	—	15.3	12.5
	1000	6	7.00	—	18.0	14.0
	2000	3	—	—	10.4	8.55
	2000	6	3.70	—	11.9	—
	1000	4	1.55	1.34	8.26	6.53
	2000	4	0.86	0.65	4.80	4.40
	1000	6	1.70	1.55	9.23	8.80
	2000	6	0.96	0.86	6.95	5.79
B	1000	8	1.80	1.58	12.2	11.7
	2000	8	1.14	0.94	8.13	7.30

shown in **Fig. 7**. The dash line in Fig. 7 represents the best fit line in Fig. 6.

Therefore, the recommendable experimental equations are

$$Q_1^+ = 0.096 \frac{Re^{0.95}}{We^{1.15}} \quad (27)$$

$$\bar{Q}_1^+ = 0.073 \frac{Re^{0.95}}{We^{1.15}} \quad (28)$$

Figure 8 shows the transition from ligament to film formation. The reverse transition is shown in **Fig. 9**. In these figures, the two-point dash line represents Eq.(24) and the dash line Eq.(26).

Hinze and Milborn's equation for the transition from ligament to film formation gives fairly good a-

Fig. 8 Transition from ligament formation to film formation

Fig. 9 Transition from film formation to ligament formation

greement with the authors' experimental results, as shown in Figs. 8 and 9. On the other hand, the semi-empirical equation by Fraser *et al.* gives lower values than the experimental results.

A comparison between the semi-empirical equation by Tanasawa *et al.* and the experimental results is shown in **Figs. 10 and 11**. In these figures, solid line represents Eq.(25). For the transition from film to

Fig. 10 Transition from ligament formation to film formation

Fig. 11 Transition from film formation to ligament formation

ligament formation, the effect of the viscosity of liquid was not sufficiently taken into consideration in Eq. (25), as shown in Fig. 11. However, the reverse transition flow rate gives fairly good agreement with the experimental results except for the data of low viscous liquid, as shown in Fig. 10. This equation is originally a semi-empirical equation made for the transition flow rate from ligament to film formation using highly viscous liquids.

However, Hinze and Milbourn's equation does not fit for very viscous liquid, though it does very well for liquid of less than a few poises in viscosity.

Previous three semi-empirical equations for the transition flow rate from ligament to film formation were compared with many experimental results. It

was seen that none of the equations could give good correlation among Re , We and Q^+ over a wide range of operating conditions.

Conclusion

The transition flow rate of three different types of disintegration were studied. Eqs. (27) and (28) were obtained from the experimental results at the transition from direct drop to ligament formation and at the reverse transition, respectively. On the other hand, three semi-empirical equations of transition flow rate from ligament to film formation or the reverse were rearranged by three dimensionless parameters, Re , We and Q^+ and compared with various experimental results. For the transition flow rate from ligament to film formation or the reverse, Hinze and Milbourn's equation can be used for liquid of less than a few poises in viscosity. For more viscous liquid, the equation by Tanasawa *et al.* is applicable for prediction of the transition flow rate.

Acknowledgment

One of the authors (S. Matsumoto) was granted a scholarship from the Sakkokai Foundation, and expresses his appreciation for the support.

Nomenclature

b	= constant	[—]
C	= constant	[—]
C'	= constant	[—]
D	= diameter of disk	[cm]
d	= diameter of drop	[cm]
F	= dimensionless radial velocity ($u/R\omega$)	[—]
F_I	= force of inertia	[dyne]
F_C	= centrifugal force	[dyne]

F_S	= surface force	[dyne]	ξ	= dimensionless coordinate	[—]
G	= dimensionless tangential velocity ($v/R\omega$)	[—]	ξ_0	= dimensionless film thickness	[—]
N	= rotational number	[—]	θ	= tangential coordinate	[—]
n	= number of ligaments	[—]	μ	= viscosity of liquid	[g/cm·sec]
Q_1	= transition flow rate from direct drop formation to ligament formation	[cm ³ /sec]	ν	= kinematic viscosity of liquid	[cm ² /sec]
$\bar{Q}_1$	= transition flow rate from ligament formation to direct drop formation	[cm ³ /sec]	ρ	= density of liquid	[g/cm ³]
Q_2	= transition flow rate from ligament formation to film formation	[cm ³ /sec]	σ	= surface tension of liquid	[dyne/cm]
$\bar{Q}_2$	= transition flow rate from film formation to ligament formation	[cm ³ /sec]	τ	= leaving frequency of a drop from disk	[1/sec]
R	= radius of disk	[cm]	τ^+	= dimensionless leaving frequency	[—]
R_1	= principal radius of curvature	[cm]	ω	= angular speed of disk	[radian/sec]
R_2	= principal radius of curvature	[cm]			
r	= radial coordinate	[—]			
Re	= Reynolds number ($R^2\omega/\nu$)	[—]			
s	= surface area	[cm ²]			
u	= radial velocity	[cm/sec]			
v	= tangential velocity	[cm/sec]			
We	= Weber number ($\rho R^3\omega^2/\sigma$)	[—]			
z	= axial coordinate	[—]			
z_0	= thickness of liquid film	[cm]			

Literature Cited

- 1) Fraser, R. P., N. Dombrowski and J. H. Routley: *Chem. Eng. Sci.*, **18**, 323 (1963)
- 2) Hinze, J. O. and H. Milbourn: *J. Appl. Mech.*, **17**, 145 (1950)
- 3) Kamiya, T.: *J. Chem. Eng. Japan*, **5**, 391 (1972)
- 4) Matsumoto, S., K. Saito and Y. Takashima: *Bull. Tokyo Inst. Tech.*, No. 109, 69 (1972), No. 116, 86 (1973)
- 5) Matsumoto, S., K. Saito and Y. Takashima: *J. Chem. Eng. Japan*, **6**, 503 (1973)
- 6) Tanasawa, Y., Y. Miyasaka and M. Umehara: *Trans. Soc. Mech. Eng. Japan*, **25**, 879, 888, 897 (1959)
- 7) Walton, W. H. and W. C. Prewett: *Proc. Phys. Soc.*, **62**, 341 (1949)

SOLID-LIQUID MASS TRANSFER IN FALLING LIQUID FILMS ON SINGLE SPHERES*

TSUTOMU HIROSE, YASUNORI MORI** AND YUJI SATO
Department of Chemical Engineering, Kyushu University, Fukuoka

Mass transfer equations are derived and compared with dissolution experiments of $C_6H_5COOH-H_2O$ and $Cu-H_2SO_4-K_2Cr_2O_7$ systems on single spheres (6 different sizes from 3/4" to 3"). In region I (long contact time with smooth laminar film), Sh converges

$$Sh = 1.608 (GaSc/Pe^*)^{1/3} \text{ or } k = 2.01 D/\bar{\delta}$$

($\bar{\delta}$: mean film thickness) and increases with decreasing volumetric flow rate Q . In region II (short contact time with smooth laminar film), Sh converges

$$Sh = 0.716 Pe^{*1/3} (GaSc)^{2/9}$$

The transition from region I to II occurs around $Pe^*/(GaSc)^{1/4} \approx 10$. In region III (short contact time with wavy film surface), Sh increases with Pe^* to the 1/3–1/2 power and is substantially higher than that in region II. A generalized short contact time equation using the measured local film thickness δ

$$Sh = 0.501 Pe^{*1/3} \left\{ \int_0^\pi (d/\delta) \sin\theta d\theta \right\}^{2/3}$$

is valid in this region. Thus, main cause for enhancement in Sh is the reduction in film thickness relevant to the wave formation. The II–III transition is somewhere between wave inception and wave coverage flow rates. In the above, $Sh = kd/D$, $Pe^* = Q/dD$ and $Ga = gd^3/\nu^2$, in which k is based on a log-mean driving force.

1. Introduction

Gas-liquid mass transfer in spherical falling film has been extensively investigated both theoretically and experimentally^{3,4,8,14,23} and the results have been used to interpret gas absorption rate in packed column absorbers. It is also recognized that there are many industrial practices where solid-liquid mass transfer in

falling film on particles is equally important. For example, Satterfield, Pelossof and Sherwood²⁰ discussed mass transfer limitation in a trickle-bed catalytic reactor in which reacting gas and liquid components must be transferred to the surface of packing catalyst through falling film on it. In a trickling filter, mass transfer of substrate to the biological film adhering to supporting particles is important¹.

Only one analytical paper on solid-liquid mass transfer in spherical falling film has appeared, just recently, for diffusivity measurement¹⁵, although some^{7,10,11,17,25} have appeared for inclined flat falling film (mainly in relation to heat transfer problem). Falling film on particles is expected to have different character-

* Received on August 23, 1973

Presented at the 37th Annual Meeting of The Soc. of Chem. Engrs., Japan, C-116, at Nagoya, April 3, 1972

** Toyo Engineering Corp.

〒812 福岡市東区箱崎

九州大学工学部化学機械工学科 広瀬 勉