plates is most of the total pressure drop in the column and can be interpreted by the orifice equations.

Both the gas holdup and pressure drop with disk vibration can be estimated as an amount proportional to $(A\nu - A\nu_c)$ added to the respective values without vibration.

Nomenclature

A	= amplitude of vibration	[mm]
C_g	= flow coefficient for gas phase	[—]
C_l	= flow coefficient for liquid phase	[—]
d_d	= diameter of vibrating disk	[mm]
d_i	= inner diameter of column (50 mm)	[mm]
$d_{i,eq}$	= hydraulic diameter of column	[mm]
d_h	= hole diameter of partition plate	[mm]
$d_{h,eq}$	= hydraulic diameter of partition plate hole	[mm]
g	= gravitational constant	[cm/s ²]
g_c	= conversion factor	[g·cm/G·sec]
h_l	= column height of one stage	[mm]
l	= level difference between taps	[mm]
m	= fraction of free hole area of partition plate	[—]
n_d	= disk number	[—]
N_s	= stage number	[—]
ΔP	= pressure difference defined by Eq.(6)	[mmH ₂ O/stage]
ΔP_g	= gas-phase pressure drop defined by Eq.(4)	[mmH ₂ O/stage]
ΔP_l	= liquid-phase pressure drop defined by Eq.(5)	[mmH ₂ O/stage]

ΔP_{tp}	= two-phase pressure drop defined by Eq.(6)	[mmH ₂ O/stage]
$\Delta P_{tp,0}$	= two-phase pressure drop defined by Eq.(7)	[mmH ₂ O/stage]
$\Delta P_{tp,v}$	= two-phase pressure drop defined by Eq.(8)	[mmH ₂ O/stage]
R_g	= coefficient defined by Eq.(7)	[—]
R_l	= coefficient defined by Eq.(7)	[—]
u_g	= gas superficial velocity	[cm/sec]
u_l	= liquid superficial velocity	[cm/sec]
ν	= vibrating frequency	[1/sec]
ρ_g	= density of gas phase	[gr/cm ³]
ρ_l	= density of liquid phase	[gr/cm ³]
ϕ_g	= gas holdup defined by Eq.(6)	[—]
$\phi_{g,0}$	= gas holdup defined by Eq.(1)	[—]
$\phi_{g,v}$	= gas holdup defined by Eq.(2)	[—]

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MASS TRANSFER IN A MULTISTAGE VIBRATING DISK COLUMN WITH COCURRENT GAS-LIQUID FLOW*

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Mass transfer characteristics of a cocurrent multistage vibrating disk column are studied by the absorption of pure carbon dioxide into water. Axial mixing in the liquid phase is taken into account by using a back-flow model.

Disk vibration definitely enhances the mass transfer rate at low gas velocity and high liquid velocity. On the other hand, at high gas velocity and low liquid velocity, disk vibration does not noticeably improve the mass transfer characteristics, and even worsens them in some cases.

The increase in absorption rate by disk vibration can be well estimated by information on the gas holdup characteristics, the mass transfer characteristics without vibration, the gas bubble size data and the axial mixing characteristics in the liquid phase.

Introduction

Various types of mechanical agitation have been used to improve the mass transfer characteristics in liquid-liquid and gas-liquid contacting operations. Not a few workers have dealt with rotating disk liquid-

liquid extractors^{5,10,12}, pulsed extraction columns^{1,2} and gas-liquid or liquid-liquid contactors with rotating impellers^{4,6}.

On the other hand, only a few studies^{3,9,11} have been published about the vibrating disk or perforated plates types of contactor, although this type of agitation is found to be particularly suitable for gas-liquid reaction with suspended solid catalysts, and is expected to be more widely applied to various mass transfer operations.

Prochazka *et al.*⁹ have investigated the performance

* Received on July 13, 1973

Presented at the 37th Annual Meeting of The Soc. of Chem. Engrs., Japan, April, 1972

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of a reciprocating plate extractor and reported the high separation efficiency of the extractor. Employing a multistage vibrating disk column, Takeba *et al.*⁽¹¹⁾ have discussed its application to a gas-liquid reaction. In our previous papers^(8,13), the axial mixing characteristics in the liquid phase and the gas holdup characteristics of a multistage vibrating disk column (M.V.D.C.) were studied in a gas-liquid cocurrent system.

The present paper describes the results of experimental studies on the gas-liquid mass transfer characteristics of the M.V.D.C. for the absorption of pure carbon dioxide gas into water. Axial mixing in the liquid phase is taken into account by means of a back-flow model.

Experimental

The experimental apparatus is shown schematically in **Fig. 1**. The details of the M.V.D.C. have been described in the previous paper⁽⁸⁾. The disk vibrates sinusoidally.

Pure carbon dioxide gas is absorbed in water flowing upward cocurrently in the column. Each stage of the column has a sampling tap for the measurement of the CO₂ concentration profiles of aqueous phase along the column height.

The CO₂ concentration in the liquid leaving the top of the column was monitored by an electric conductivity meter. After the meter indication becomes steady, the liquid samples are taken from each of the stages through the sampling taps. The sampling rate is controlled so that the sampled liquid does not contain any gas bubbles. The concentration of CO₂ absorbed in the sample is determined by a titration method.

All the experiments are carried out at atmospheric pressure and a constant temperature of 30 ± 0.5 °C.

The experimental conditions and the main parameters are listed in **Table 1**.

Results and Discussion

(1) Mathematical model

For evaluation of the overall mass transfer characteristics of a M.V.D.C., a mathematical model of the column should be established by taking account of axial mixing. It has been shown that the liquid-phase mixing characteristics of the column can be expressed by a back-flow model consisting of *N* equi-volume perfectly mixed stages⁽⁸⁾.

For the absorption of pure CO₂ gas, the material balance equations of solute gas in the liquid phase at steady state are given by Eqs.(1) to (3):

$$(1 + \alpha + K)\xi_1 - \alpha\xi_2 = K \quad (1)$$

for the first stage,

$$-(1 + \alpha)\xi_{i-1} + (1 + 2\alpha + K)\xi_i - \alpha\xi_{i+1} = K \quad (2)$$

for the *i*-th stage (*i* = 2, 3, ..., *N* - 1) and

$$-(1 + \alpha)\xi_{N-1} + (1 + \alpha + K)\xi_N = K \quad (3)$$

for the *N*-th stage,

Fig. 1 Experimental apparatus

Table 1 Experimental conditions

Number of stages	$N_s = 7, 9$	[-]
Vib. disk diameter	$d_d = 30, 36$	[mm]
Hole diameter	$d_h = 12, 20$	[mm]
Inside dia. of column	$d_i = 50$	[mm]
Vib. frequency	$\nu = 0-420$	[cpm]
Vib. amplitude	$A = 0-12$	[mm]
Liquid velocity	$u_l = 0.16-0.79$	[cm/sec]
Gas velocity	$u_g = 1.15-6.20$	[cm/sec]
Temperature	$T = 30 \pm 0.5$	[°C]

where $K = K_L a V / F$, $\xi_i = (C_i - C_0) / (C^* - C_0)$ and α is the back-flow ratio of the back-flow model. In the above equations, it has been assumed that the average gas holdup, the back-flow ratio and the overall liquid-phase mass transfer capacity coefficient are constant throughout the column. Eqs.(1), (2) and (3) can be solved analytically, and the solution is given by Eq.(4):

$$\xi_i = 1 - (A\lambda_1^i + B\lambda_2^i) \quad (4)$$

where

$$\lambda_1 = (1 + 2\alpha + K + \sqrt{(K+1)^2 + 4\alpha K}) / 2\alpha$$

$$\lambda_2 = (1 + 2\alpha + K - \sqrt{(K+1)^2 + 4\alpha K}) / 2\alpha$$

$$A = [\lambda_1 \{ (1 + \alpha + K - \alpha\lambda_1) + \left(\frac{\lambda_1}{\lambda_2} \right)^{N-2} (1 + \alpha + K - \alpha\lambda_2) \}]^{-1}$$

$$B = A(\lambda_1/\lambda_2)^{N-1}$$

$$\times \{1 + \alpha - (1 + \alpha + K)\lambda_1\} / \{(1 + \alpha + K)\lambda_2 - 1 - \alpha\}$$

If the liquid-phase mixing characteristics of the column are known from another experiment, the rate of absorption in terms of *K* can be determined from Eq. (4) by trial and error method for the experimental values of the CO₂ concentrations at the inlet and outlet of the column.

(2) Mixing characteristics

It has been shown that the liquid-phase mixing characteristics of the column can be expressed by the back-flow model and that the model parameter α is a function of the dimensionless variable $\{(A\nu\varepsilon/u_l)(d_h/d_i)^3\}$ ⁽⁸⁾. In the range of the present experiments dealing with the high vibrating speed range, the ratio α can be calculated by Eq.(5).

$$\alpha = \frac{4A\nu\varepsilon}{u_l} \left(\frac{d_h}{d_i} \right)^2 \left(\frac{d_d}{d_i} \right)^2 \quad (5)$$

In **Fig. 2**, a comparison is made between the experi-

Fig. 2 Concentration profiles along the column

Fig. 4 Effect of gas and liquid velocity on volumetric mass transfer coefficient in low vibrating speed range

mental concentration profiles along the column height and those calculated from Eqs.(4) and (5). Good agreement between them shows that Eq.(5) is acceptable for the estimation of the liquid-phase axial mixing in the M.V.D.C.

(3) Gas holdup

The gas holdup in the M.V.D.C. can be estimated by Eq.(6) within an accuracy of $\pm 12\%$ ^{13).}

$$\phi_{g,v} = \phi_{g,0} + 0.0034 n_d \left(\frac{d_{h,eq}}{d_{i,eq}} \right)^{-0.56} \left(\frac{d_d}{d_{i,eq}} \right)^{2.0} (A\nu - A\nu_c) \quad (6)$$

where $d_{i,eq}$ and $d_{h,eq}$ are the hydraulic diameter of the column and the partition plate hole respectively, and

$$\phi_{g,0} = 0.074 u_g^{0.7} u_l^{-0.08} \left(\frac{d_d}{d_{i,eq}} \right)^{0.5} \left(\frac{d_{h,eq}}{d_{i,eq}} \right)^{-0.15} \left(\frac{h}{d_{i,eq}} \right)^{-0.3}$$

$$A\nu_c = 2.4 \text{ cm/sec.}$$

(4) Mass transfer rate

The effect of the disk vibration speed $A\nu$ on the CO_2 concentration in the outlet liquid phase is shown in Fig. 3 for a few typical cases. At high liquid velocity, disk vibration has a favorable effect on column performance when $A\nu$ exceeds 2–3 cm/sec. On the contrary, as the liquid velocity u_l decreases, the effect of disk vibration becomes less appreciable and finally becomes unfavorable at a very low liquid velocity. These

Fig. 3 Effect of disk vibration on absorption rate

Fig. 5 Correlation for $K_L a$ in low vibrating speed range

facts show that the mixing characteristics of the liquid phase should be reflected in the evaluation of column performance. It is further noted from Fig. 3 that the mass transfer rate can be scarcely enhanced by disk vibration until the vibrating speed $A\nu$ exceeds about 3 cm/sec.

Figure 4 shows the effects of the superficial gas and liquid velocities, u_g and u_l , on the overall liquid-phase mass transfer capacity coefficient $K_L a$ in the low vibrating speed range. In this figure, the data obtained in a multistage pulsed column by Kubo *et al.*⁷⁾ are also included for reference.

In the low vibrating speed range, the data of $K_L a$ for the M.V.D.C. can be correlated with an accuracy of 15% by Eq.(7);

$$\frac{u_l}{K_L a} = \frac{1}{1 - \phi_{g,0}} \left[0.35 \left(\frac{u_l}{u_g} \right)^{1.09} + 0.075 \right] \quad (7)$$

as shown in Fig. 5.

As the vibrating speed $A\nu$ becomes higher, the mass transfer rate increases significantly, as shown in Fig. 3. This enhancement in the mass transfer rate can be considered to be caused mainly by an increase in the contacting area by the vibration of the disks. The overall liquid-phase mass transfer capacity coefficient is proportional to the surface area of the bubbles per unit volume a , which is dependent on the average bubble size d_m , as

Fig. 6 Effect of vibrating speed on average bubble diameter

Fig. 8 Comparison of experimental results with calculated ones

$$a = 6\phi/d_m \quad (8)$$

if the shape of the bubbles is assumed to be spherical. The average bubble diameter d_m has been measured by a photographic method in these experiments, and the following relation has been obtained, as shown in Fig. 6.

$$\begin{cases} d_m = d_0 (=0.55 \text{ cm}) & (A\nu < A\nu_c) \\ d_m = 0.0262A\nu + d_0 & (A\nu \geq A\nu_c) \end{cases} \quad (9)$$

From Eqs. (6), (7), (8) and (9), the enhancement factor E , defined as the ratio of the dimensionless capacity coefficient with disk vibration K_v to that without vibration K_0 , can be derived as

$$E \equiv \frac{K_v}{K_0} = \frac{\phi_{g,0} + 0.0034n_d \left(\frac{d_{h,eq}}{d_{i,eq}} \right)^{-0.56} \left(\frac{d_d}{d_{i,eq}} \right)^{2.0} (A\nu - A\nu_c)}{\phi_{g,0} (d_m/d_0)} \quad (10)$$

The overall liquid-phase mass transfer coefficient K_L depends on the bubble size in general. But it is assumed in this analysis that the coefficient is independent of bubble size because of the slight increase in size caused by disk vibration.

In Fig. 7, the enhancement factors for the case of $d_h/d_i = 0.24$ and $d_d/d_i = 0.72$ are plotted against the vibrating speed for various values of gas holdup without vibration of the disk $\phi_{g,0}$, as an example.

In Figs. 8 and 9, the experimental CO_2 concentrations in the outlet liquid phase are compared with

Fig. 7 Effect of vibrating speed on enhancement factor

Fig. 9 Comparison of experimental results with calculated ones

those calculated by Eqs. (4), (5), (6), (7), (9) and (10), and good agreement between them has been obtained within an accuracy of $\pm 7\%$.

Conclusion

It has been shown that the liquid-phase mixing characteristics should be taken into consideration for precise evaluation of the mass transfer performance of the M.V.D.C. for pure CO_2 gas absorption. In other words, the overall mass transfer rate in the M.V.D.C. is influenced by two main factors, i.e., the gas holdup and the axial mixing caused by disk vibration. At low liquid velocity and high gas velocity, the axial mixing becomes noticeable and the overall mass transfer is not promoted appreciably by disk vibration. Indeed, the vibration affects the mass transfer efficiency unfavorably in some cases.

On the other hand, at high liquid velocity and low gas velocity, disk vibration definitely enhances the overall mass transfer efficiency. This fact shows that the M.V.D.C. may be particularly suitable for contact of an expensive gas.

The effect of disk vibration on the mass transfer rate can be quantitatively interpreted by using the enhancement factor of Eq. (10) and the axial mixing result in terms of the back-flow ratio of Eq. (5).

Acknowledgment

All the numerical calculations in this study have been carried out at the Computer Center, University of Osaka Prefecture.

Nomenclature

A	= amplitude of vibration	[mm]
a	= interfacial area per volume	[cm ²]
$A\nu_c$	= critical vibrating speed of the disk	[cm/sec]
C_i	= CO ₂ concentration in the liquid phase in the i -th stage	[mole/cc]
C_0	= inlet CO ₂ concentration in the liquid phase	[mole/cc]
C^*	= solubility of CO ₂ gas in water	[mole/cc]
d_d	= diameter of a vibrating disk	[mm]
d_i	= inner diameter of a column	[mm]
$d_{i,eq}$	= hydraulic column diameter ($=d_i-d_s$)	[mm]
d_h	= hole diameter of a partition plate	[mm]
$d_{h,eq}$	= hydraulic hole diameter of partition plate ($=d_h-d_s$)	[mm]
d_m	= mean bubble size calculated by Eq.(9)	[cm]
d_0	= bubble size without vibration	[cm]
d_s	= vibrating shaft diameter	[mm]
E	= enhancement factor defined by Eq.(10)	[—]
F	= flow rate of liquid	[cm ³ /sec]
K	= dimensionless capacity coefficient	[—]
K_L	= overall liquid-phase mass transfer coefficient	[cm/sec]
K_0	= dimensionless capacity coefficient at $A\nu=0$	[—]
K_v	= dimensionless capacity coefficient at $A\nu>A\nu_c$	[—]
K_{La}	= overall liquid-phase mass transfer capacity coefficient	[1/sec]
n_d	= number of disks	[—]
N_s	= number of stages	[—]
u_g	= gas superficial velocity	[cm/sec]
u_l	= liquid superficial velocity	[cm/sec]
V	= volume of the column	[cc]
Z	= column height	[cm]

α	= back flow ratio	[—]
ϵ	= liquid phase holdup ($=1-\phi_{g,v}$)	[—]
ν	= vibrating frequency	[1/sec]
ξ	= dimensionless concentration	[—]
$\phi_{g,v}$	= gas holdup defined by Eq.(6)	[—]
$\phi_{g,0}$	= gas holdup at $A\nu<A\nu_c$	[—]

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