TERMINAL VELOCITY OF TWO-PHASE DROPLET*

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1. Introduction

A new spray column extractor, in which the solute in liquid droplets combined with bubbles (=two-phase droplets) formed by a double tube nozzle is extracted with continuous-phase liquid, was proposed²⁾ and rate of coalescence between droplets, mass transfer rate and the effect of the coalescence on mass transfer rate were studied in previous paper^{3,4)}.

The present investigation is undertaken to observe the terminal velocity of a two-phase droplet, which is an important factor for studies of the coalescence rate and mass transfer rate in the new extractor, and to provide basic information for the practical design of the new extractor.

2. Experimental Apparatus and Procedure

Experimental apparatus

Figure 1 shows the experimental apparatus schematically. The column was the same as a spray column having 41.9 mm inside diameter and 1000 mm length. It was made of glass, with an outer, square water jacket to facilitate photographs and to maintain the experimental temperature at 25°C.

The spray nozzle was a double tube. The gas (=air) was injected from the inside tube into liquid droplets flowing in the annular space of the double tube, and two-phase droplets were formed at the tip of a nozzle.

Table 1 shows the size of nozzles.

Procedure

Table 2 shows the systems used and the properties of the systems. For the dispersed-phase liquid, toluene and toluene + CCl₄ (=mixed liquid A and B of nearly the same density of continuous-phase liquid (=water)) were used. Oil of a certain fixed acetic acid concentration (about 0.1 mol/l) and air were fed into the lower part of the column as shown in Fig. 1. Volumetric flow rate of oil was fixed at about 0.06 [cc/sec].

The two-phase droplets formed in the way described above rose in water flowing from the top of the column, of which volumetric flow rate was fixed at 8.4 cc/sec, and the rising times of about twenty droplets were

measured with a stopwatch at several column heights of 10, 30, 50, 70 and 90 cm. Photographs of the two-phase droplets were taken at a column height of 15 cm and about thirty two-phase droplets chosen at random in the photographs were measured, assuming that the two-phase droplets and the bubbles in the droplets were in the form of ellipsoid gyration, to determine the total volume and the bubble volume in the two-phase droplet.

3. Results and Discussion

An example of the experimental results is shown in Fig. 2, which shows plots of column heights (y) versus rising times of droplets (t), and is represented by straight lines other than initial heights before reaching the terminal velocities. The straight lines were independent of the solute concentration of oil in two-phase droplets, which changed with column height, and the initial concentration of solute within the investigated range of 0.0 (no mass transfer) to 0.7 mol/l by the preliminary experiment. The plotting of the terminal velocities (u_{dt}) against the volume ratios of gas to oil in two-phase droplets (β) is shown in Fig. 3. It shows that the terminal velocity depended on β rather than droplet diameter (d) (0.275–1.17 cm) varied by using three kinds of nozzles, and increased abruptly with increasing β .

Under steady-state conditions, the drag force acting on the volume equivalent sphere of two-phase droplet is exactly balanced by the buoyant force acting upward on the droplet, and the force balance gives Eq.(1).

$$v \cdot \Delta \rho \cdot g = C_D \cdot \frac{\pi d^2}{4} \cdot \frac{u_{dt}^2 \rho_w}{2} \quad \dagger \quad (1)$$

where the density difference between two-phase droplet and continuous-phase liquid (=water) is represented by the equation

$$\Delta \rho = \rho_w - \rho_o / (1 + \beta) \quad \dagger \quad (2)$$

The drag coefficients (C_D) are calculated by Eq.(1). In Fig. 4 the drag coefficients of two-phase droplets are

Table 1 Size of nozzles used

Outside tube		Inside tube	
Inner diameter [mm]	Outer diameter [mm]	Inner diameter [mm]	Outer diameter [mm]
0.50	0.80	0.20	0.30
1.10	1.51	0.20	0.30
2.00	2.80	0.20	0.30

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†The volume of bubble in two-phase droplet changes with the static head, therefore, v , d , β and $\Delta \rho$ change slightly with column height. In this study, these variables were taken as the arithmetical means of values at the top and the bottom of the spray column.

Table 2 Physical properties of the systems used

Dispersed phase	Liquid Continuous phase	Viscosity [g/cm·sec]		Surface tension [dynes/cm]		Interfacial tension [dynes/cm]	Density [g/cm ³]		Density difference [g/cm ³]
		Dispersed phase μ_0	Continuous phase μ_w	Dispersed phase σ_0	Continuous phase σ_w		Dispersed phase ρ_0	Continuous phase ρ_w	
Toluene	Water	0.00553	0.00903*	27.40	71.97*	26.0	0.8611	0.997	0.1359
Mixed liquid A toluene + 23.0wt% CCl ₄	Water	0.00581	0.00903*	27.0	71.97*	27.0	0.9624	0.997	0.0346
Mixed liquid B: toluene + 24.5wt% CCl ₄	Water	0.00580	0.00903*	27.0	71.97*	27.1	0.9752	0.997	0.0218

* by International Critical Tables

AC: Air chamber
AI: Air injector
J: Jacket
N: Nozzle
SC: Spray column

Fig. 1 Experimental apparatus

Fig. 2 Column heights (y) vs. rising times of droplets (t)

plotted against Reynolds number (Re), with the previous data¹⁾ for air bubbles. This experimental range was $400 \leq Re \leq 1000$, within which zigzag or spiral motion in rising of droplets or bubbles are observed. For the correlation of terminal velocities of two-phase droplets, the parameters of X ($=Re/P_{Hu}^{0.15} + 0.75$) and Y ($=C_D \cdot We \cdot P_{Hu}^{0.15}$), which had been proposed by Hu and Kintner⁵⁾ for the study of terminal velocity of liquid droplet without mass transfer, were used in this study, and the results are shown in **Fig. 5**. It shows that the present data appeared approximately 15% above Hu's equation⁵⁾, but were correlated with the following two equations satisfactorily.

$$\left. \begin{aligned} Y &= 1.59X^{1.28} & (6 \leq X \leq 21.6) \\ Y &= 0.0553X^{2.37} & (21.6 < X \leq 30) \end{aligned} \right\} * \quad (3)$$

*The interfacial tension (σ) in the parameters of X and Y was taken as the mean value calculated by the following equation.

$$\sigma = \sigma_w f + \sigma \cdot (1-f)$$

Here, $f[-]$ is area fraction of water-air surface in total interface of two-phase droplet, assuming that a bubble in two-phase droplet is in the symmetrical form with respect to the horizontal plane (AB) cutting the droplet as shown in Fig. 6, because it seemed that a film of dispersed-phase liquid did not cover the whole surface of a two-phase droplet.

Fig. 3 Dependence of u_{dt} on β

Fig. 4 Variation of drag coefficient with Reynolds number

Fig. 5 Correlation of terminal velocities of two-phase droplets

Fig. 6 Simplified form of two-phase droplet

Hence, the exponents (1.28 and 2.37) in Eq.(3) equal those in Hu's equation⁵⁾ respectively.

For 84% of all the data obtained in the experiment, the terminal velocities (4.70–22.0 cm/sec) of two-phase droplets with mass transfer could be estimated by Eq.(3) within $\pm 4\%$ error.

Nomenclature

C_D	= drag coefficient	[—]
d	= volume equivalent sphere diameter	[cm]
d_{NI}	= inner diameter of outside tube	[mm]
d_{NO}	= outer diameter of outside tube	[mm]
f	= area fraction of water-air surface in total interface of two-phase droplet	[—]
g	= acceleration due to gravity	[cm/sec ²]

P_{Hu}	= parameter used by S. Hu <i>et al.</i> , = $\delta^3 \cdot \rho_w^2 / g \cdot \mu^4 \cdot \Delta \rho$	[—]
Re	= Reynolds number, = $d \cdot \rho_w \cdot u_{dt} / \mu_w$	[—]
t	= rising time of droplet	[sec]
u_{dt}	= terminal velocity of droplet	[cm/sec]
v	= volume of droplet	[cm ³]
We	= Weber number, = $d \cdot u_{dt}^2 \cdot \rho_w / \sigma$	[—]
X	= dimensionless number, = $Re / P_{Hu}^{0.5} + 0.75$	[—]
y	= column height	[cm]
Y	= dimensionless number, = $C_D \cdot We \cdot P_{Hu}^{0.15}$	[—]
β	= volme ratio of gas to oil in two-phase droplet	[—]
μ	= viscosity	[g/cm sec]
ρ	= density	[g/cm ³]
σ	= interfacial tension	[dynes/cm]
σ_w	= surface tension of water	[dynes/cm]
δ	= average interfacial tension, = $\sigma_w f + \sigma \cdot (1-f)$	[dynes/cm]

<Subscripts>

i	= initial
o	= oil
w	= water

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JETTING VELOCITY OF A LIQUID JET IN NON-NEWTONIAN FLUID SYSTEMS*

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In the low flow rate region drops form directly at the nozzle tip. As the flow rate through a nozzle is increased, a jet of liquid issues from the nozzle, and drops form from a laminar cylindrical jet. Therefore, the size of drops can no longer be predicted by the low-velocity correlation. To define the region of applicability of the low-velocity correlation and to select the operating condition for the flow rate of the dispersed phase, it is necessary to predict the jetting velocity above which a jet issues from the nozzle.

Theory

For characterization of the non-Newtonian behavior of fluids, the power law model is used.

$$\tau_j^i = -K \left(\frac{1}{2} e_k^m e_m^k \right)^{(n-1)/2} e_j^i \quad (1)$$

We consider that a jet forms if there is a sufficient upward force at the nozzle exit to form a jet. Jet formation is thus governed by a resulting force

$$F_K + F_P = F_\sigma \quad (2)$$

To form a jet the net upward force at the nozzle tip must overcome the interfacial tension force

$$F_\sigma = \pi \sigma d_N \cos \theta \quad (3)$$

The upward forces consist of the kinetic force of the fluid exiting from the nozzle, given by

$$F_K = \frac{3n'+1}{2(2n'+1)} U_N'^2 \rho' \frac{\pi}{2} d_N^2 \quad (4)$$

The pressure of the liquid leaving the nozzle must be greater than that in the continuous phase. This excess force necessary to sustain a spherical drop is

$$F_P = \frac{\pi \sigma d_N^2}{D_F} \quad (5)$$

Substituting Eqs.(3)–(5) into Eq.(2), we obtain

$$U_J = \left\{ \frac{4(2n'+1)\sigma}{(3n'+1)\rho' d_N} \left(\cos \theta - \frac{d_N}{D_F} \right) \right\}^{1/2} \quad (6)$$

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