TURBULENT PIPE FLOW OF DILUTE POLYMER SOLUTIONS*

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An analysis similar to the van-Driest damping factor model for Newtonian fluids results in a viscoelastic damping factor for turbulent pipe flow of dilute polymer solutions. The turbulent relaxation time $\lambda$, which must be used in the viscoelastic damping factor model, is defined and correlated empirically. The present analysis is tested with the measured velocity distribution in the unsaturated drag reduction region, and friction factor in the unsaturated and saturated drag reduction regions. A comparison of the present model with two other well-known models, the elastic sublayer model of Virk et al. and the thickened laminar sublayer model of Seyer and Metzner, is made at the maximum drag reduction asymptote. From these comparisons it is concluded that the present model is valid to predict the friction factor and the velocity distribution in both the unsaturated and the saturated drag reduction regions.

1. Introduction

A remarkable reduction of friction factor is observed in turbulent flow of a dilute solution of linear macromolecular polymer. This phenomenon, called drag reduction or Toms phenomenon, has been studied experimentally and theoretically. But, because of the lack of appropriate constitutive equations that are valid in turbulent flow fields, and because of the complexity of this phenomenon, its mechanism has not so far been fully understood.

It is generally accepted from previous work that there are effects of several sources on drag reduction. These effects are of the following four types.

i) concentration effect
ii) polymer molecular weight effect
iii) pipe diameter effect
iv) saturation effect

The purpose of this paper is to explain these various effects in drag-reducing systems theoretically and experimentally.

2. Analysis

A damping factor model similar to that of van-Driest10, which is one of the simplest analyses in turbulent flow of Newtonian fluids, will be derived in an analytical procedure.

Maxwell model will be used as a constitutive equation for viscoelastic liquid:

$$\tau + \lambda \frac{\partial \tau}{\partial t} = \mu \frac{\partial u}{\partial y} \quad (1)$$

Let us consider the second problem of Stokes shown in Fig. 1.

Equation of motion in this system can be written as

$$\rho \frac{\partial u}{\partial t} = \frac{\partial \tau}{\partial y} \quad (2)$$

From Eqs. (1) and (2) one obtains the basic equation of this analysis.

$$\lambda \frac{\partial^2 u}{\partial t^2} + \frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} \quad (3)$$

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6) Eguchi, Y.: Kagaku Kōgaku, 35, 1006 (1971)
The boundary conditions are

\[ u(0, t) = U \cos \omega t \quad \text{at} \quad y = 0 \quad (4a) \]
\[ u(\infty, t) = 0 \quad \text{at} \quad y \to \infty \quad (4b) \]

The solution of Eq. (3) with Eqs. (4a-b) is given as

\[ u = U e^{-\frac{Gy}{2}} \cos \left( \omega t - \frac{Py}{2} \right) \quad (5) \]

where

\[ G = \left[ \left( \frac{\omega}{2} \right)^2 \left( -a + \sqrt{a^2 + 1} \right) \right]^{1/2} \]
\[ P = \left[ \left( \frac{\omega}{2} \right)^2 \left( a + \sqrt{a^2 + 1} \right) \right]^{1/2} \]

According to this solution the damping factor for the amplitude of oscillation is given as

\[ \exp \left[ - y \left( \left( \frac{\omega}{2} \right)^2 \left( -a + \sqrt{a^2 + 1} \right) \right) \right]^{1/2} \]

By an argument similar to van-Driest's the damping factor of turbulent pipe flow in drag-reducing systems becomes

\[ DF = 1 - \exp \left[ - y \left( \left( \frac{\omega}{2} \right)^2 \left( -a + \sqrt{a^2 + 1} \right) \right) \right]^{1/2} \]

If \( \lambda \) is put to zero, the damping factor becomes

\[ DF_1 = 1 - \exp \left[ - y^+ \right] \quad (6) \]

where

\[ a = \left( 2 \lambda / \nu \right) \left( u^+ / 2 \lambda \right) \]

Then the friction factor of turbulent pipe flow in a drag-reducing system can be derived from this damping factor. Employing Prandtl's mixing length theory, the total shear stress of turbulent flow is expressed as

\[ \frac{\tau}{\nu} = dt_+ dy^+ + I^+ \left( \frac{dt_+}{dy^+} \right)^2 \quad (9) \]

The distribution of shear stress is linear over the pipe cross section, that is:

\[ \frac{\tau}{\nu} = 1 - \frac{y^+}{R^+} \quad (10) \]

As the mixing length of viscoelastic fluids, the following expression, which corresponds to the semiempirical expression of Nikuradse\(^9\) in the Newtonian case, is adopted:

\[ I^+ = \left( 0.4 y^+ - 0.44 \frac{y^+}{R^+} + 0.26 \frac{y^+}{R^+} \right) \]

\[ \times \left[ 1 - \exp \left( - \frac{y^+}{26} \left( -a + \sqrt{a^2 + 1} \right) \right) \right]^{1/2} \quad (11) \]

From Eqs. (9), (10) and (11) we obtain the velocity distribution as

\[ u^+ = \int_0^{y^+} 2 \left( 1 - \frac{y^+}{R^+} \right) dy^+ \quad \left( 1 + \sqrt{1 + 4 \exp \left( \frac{y^+}{R^+} \right) \left( 1 - \frac{y^+}{R^+} \right) \right) \right) \quad (12) \]

3. Experimental Apparatus and Procedure

The flow diagram of experimental apparatus is shown in Fig. 2. To avoid mechanical degradation of the polymers, the test liquids were forced to flow only by the difference of height between the head tank and the test tube, which was about 190 cm. The test sections are circular tubes of 1'' dia. and 1/2'' dia. with smooth walls.

The pressure drop in the test section was measured with a U-tube manometer of CCl\(_4\), and the velocity distribution over the cross section was measured by a pitot tube of 1 mm dia. In the case of high-concentration polymer solution, the velocity distributions measured by the pitot tube are not correct because of the viscoelasticity of the polymer solutions. This anomalous behavior was checked by comparing the integrated flow rate with the directly measured flow rate by the weight-time method. When the error between these two methods was less than ±1.5%, the
The polymer species used in this study are PEO (polyethylene oxide) and PAA (polyacrylamide). The intrinsic viscosity of the polymer solutions was measured by an Ostwald viscometer. The molecular weight of the polymer was calculated for polyethylene oxide by the following equation given by Shin

\[ \eta = 1.03 \times 10^{-4} M^{0.76} \]  

(16)

and for polyacrylamide by the following equation given by Collinson

\[ \eta = 6.80 \times 10^{-4} M^{0.66} \]  

(17)

The relaxation time for a laminar simple shear flow was calculated by the following equation, which was derived from the linear viscoelastic theory by Rouse

\[ \lambda_i = \frac{2}{5} \frac{\eta}{\gamma} \bar{y}^2 \frac{M_c}{RT} \]  

(18)

None of the dilute polymer solutions used in this study showed a non-Newtonian viscosity, but they showed a weak elasticity.

4. Results and Discussion

(a) Unsaturated region

If the relaxation time \( \lambda \) and the friction velocity \( u^* \) are given, the damping factor is calculated by Eq. (8), but when the value of \( \lambda_i \) in Eq. (18) is used as the relaxation time of turbulent pipe flow, the value of \( \alpha \) in Eq. (8) becomes so small that no effect of viscoelasticity appears in the damping factor. Thus from the experimental results of pressure drop the relaxation time was calculated by using Eqs. (8), (12) and (13). The relaxation time obtained in this way is defined as the turbulent relaxation time \( \lambda_t \). The correlation of \( \lambda_t \) thus obtained was found to be

\[ \frac{\lambda_t}{\lambda_i} = 3.76 \times 10^8 W e^{1.34} \]  

(19)

where

\[ W e = \lambda_i u^*/D \]

A comparison of this equation with the experimental results is shown in Fig. 3. This plot includes all the effects in the unsaturated region explained in Section 1.

Before investigating each of the effects, the validity of the viscoelastic damping factor model must be shown, so the velocity profiles of dilute polymer solutions in turbulent pipe flow were calculated from Eqs. (8), (12) and (19), and compared with experimental results as shown in Fig. 4. It can be concluded from this diagram that the viscoelastic damping factor model can predict the actual velocity distribution fairly well in the low drag reduction region.

Next, the effects of various sources on drag reduction in the unsaturated region will be confirmed in Figs. 5~7. The concentration effect is shown in Fig. 5. The curves of friction factor calculated from Eqs. (8), (12), (13) and (19) fit well with experimental results. In the same manner the predicted curves of the molecular weight effect are represented in Fig. 6. Figure 7 shows that the viscoelastic damping factor model is valid for pipe diameter effect. From these diagrams it is clear that the present model is valid to predict the dependence of friction factor on concentration, molecular weight and pipe diameter effects in the unsaturated drag reduction region. Moreover the applicability of this model to other polymer species was tested for the case of PAA. The friction factor data of PAA solution showed good agreement with the predicted value.

Finally, it must be emphasized that the onset phenomena of drag reduction, which have been studied by some previous authors, are not observed in the results obtained from the damping factor model. Drag reduction seems to occur as soon as the flow field becomes turbulent, although the amount of drag reduction is very small. The amount of drag reduction depends on \( \alpha \), which depends on \( \lambda_i \) and \( u^* \) or on the Weissenberg number and the Reynolds number. Therefore, the position of the point at which the deviation of the friction factor curve from the Newtonian one becomes significant is not at one point, the so--
called “onset point of drag reduction”. The position varies with the above-mentioned effects on drag reduction.

(b) Maximum drag reduction region

It has been indicated previously by many authors and is generally accepted that there exists a maximum drag reduction asymptote in Toms phenomena. Virk\(^{16}\) recently summarized many data at the maximum drag reduction, and pointed out that the friction factor at this asymptote was independent of the polymer concentration, the polymer species, the molecular weight of the polymer and the pipe diameter.

The experimental result of this study coincides with that of Virk at maximum drag reduction. The value of \(a\) can be calculated from the friction factor data. The values of \(a\) calculated at the maximum drag reduction asymptote have nearly the same values, i.e. 40~60, and they are independent of the Reynolds number and the Weissenberg number. The scatter of \(a\) is believed to be caused by polymer degradation. Since the error caused by degradation has a tendency to make the value of \(a\) smaller, the value 60 is adopted for \(a\) as a universal value independent of the Weissenberg number and Reynolds number.

Employing this value of \(a\), the friction factor and the velocity profiles at the maximum drag reduction asymptote are calculated, and the results are shown in Fig. 8. Two other models for maximum drag reduction, the elastic sublayer model (Virk\(^{16}\)) and the modified thickened sublayer model (Seyer and Metzner\(^{14}\)), are shown in the same diagram. In the elastic sublayer model, the turbulent core of the pipe flow disappears at the maximum reduction asymptote, and the whole region except the laminar sublayer becomes the elastic sublayer, which has a mixing length different from that of the Newtonian flow. On the other hand, the thickened laminar sublayer model shows a parallel shift of the velocity distribution of the turbulent core, so the laminar sublayer is thickened. The mixing length has the same value as that for Newtonian flow in the core region. The viscoelastic damping factor model of this work results in a velocity profile similar to that obtained from Seyer-Metzner’s model at higher Reynolds numbers, but shows a profile similar to that of Virk at lower Reynolds numbers. In the higher Reynolds number range, the damping factor becomes nearly unity and does not effect the velocity profile in the central part of the pipe. On the other hand, at lower Reynolds numbers, the effect of viscoelastic damp is very significant over the whole cross section of pipe, so the velocity profiles become similar to that of the elastic sublayer model. In Fig. 8 the experimental mean velocity profiles obtained by Seyer and Metzner, which are the only available experimental results at max-
maximum drag reduction, are included. These velocity profiles obtained by means of the bubble tracer method agree very well with the present viscoelastic damping factor model. Summarizing the results mentioned above, it is concluded that in the maximum drag reduction region the elastic sublayer reaches about $y^+ \approx 200$. Since the value of $R^+$ is larger than such a value as 200 at higher Reynolds numbers, there exists a turbulent core, and since the value of $R^+$ becomes smaller than 200 at lower Reynolds numbers, the turbulent core disappears and the whole cross section consists of two regions, the laminar sublayer and the elastic sublayer.

It is convenient to adopt the Prandtl coordinates for the comparison of friction factor data with previous results. Figure 9 shows the present data and the curves predicted from the above-mentioned three models for maximum drag reduction. In the region of Reynolds numbers smaller than $10^6$, both the present viscoelastic damping factor model and the elastic sublayer model predict nearly equal friction factor values at maximum drag reduction asymptote, and both are in good agreement with experimental results. The asymptote of Seyer and Metzner is only valid at higher Reynolds numbers because of the assumptions made for the derivation of this model. Thus the difference of these three models must be discussed at very high Reynolds numbers, which correspond to the region of $Re^{1/3} > 2 \times 10^6$. But to date no friction factor data are available because experiments at higher Reynolds numbers are very difficult. The question of which model gives the more correct prediction for maximum drag reduction will be answered when friction factor data at higher Reynolds numbers or more precise measurements of turbulent characteristics for drag-reducing systems become available.

The eddy diffusivity for dilute polymer solutions can be calculated from Eq. (15). Both the predicted curves and the experimental results are shown in Figure 10. The eddy diffusivity in the turbulent core region at unsaturated drag reduction is calculated from

$$\delta_M = 0.07 \frac{R^+}{\nu}$$

(20)

which is part of the eddy diffusivity model of Mizushina and Ogino\(^8\) for Newtonian fluids, and which is acertained by the experimental results of Laufer\(^6\) and Reichardt\(^11\). The predicted curves for the unsaturated region show good agreement with experimental results. It is shown from this diagram that, in the unsaturated drag reduction region, the eddy diffusivity of dilute polymer solutions decreases strongly near the wall. Since the effect of elasticity does not reach the center part of pipe flow in the lower drag reduction region, the eddy diffusivity of polymer solutions is equal to that of a Newtonian fluid in the turbulent core region.

5. Conclusions

1. The viscoelastic damping factor model was derived.
2. The correlation of turbulent relaxation time to be used in this model for the unsaturated region was given by Eq. (19). It was shown that this viscoelastic damping factor model gave a good prediction of the effects of various sources on drag reduction.
3. At maximum drag reduction, the universal value of $a$ in Eq. (8) was obtained experimentally as 60, and using this value the friction factor and velocity profiles were calculated. These results were found to agree fairly well with experimental results.

Nomenclature

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<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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<tr>
<td>$c$</td>
<td>polymer concentration</td>
<td>[wpmm]</td>
</tr>
<tr>
<td>$D$</td>
<td>pipe diameter</td>
<td>[cm]</td>
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<tr>
<td>$DF$</td>
<td>viscoelastic damping factor</td>
<td>[-]</td>
</tr>
<tr>
<td>$DF_0$</td>
<td>damping factor of van Driest</td>
<td>[-]</td>
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<tr>
<td>$f$</td>
<td>Fanning friction factor</td>
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<tr>
<td>$l$</td>
<td>mixing length of polymer solution</td>
<td>[cm]</td>
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1. Introduction

The discovery that partial demixing can be achieved by the application of a temperature difference to fluid mixtures, usually referred to as "separation by thermal diffusion" is attributed to Ludwig\(^1\) or Soret\(^2\). Until 1938 when Clusius and Dickel\(^3\) applied the thermogravitational effect to improve separations produced by the phenomenon, studies were restricted to simple cells. The thermogravitational effect is utilised by the provision of a vertical space usually narrow (of the order of 1 mm), across which the temperature difference is maintained. In principle as the hotter fluid rises due to convection currents, it becomes enriched with one component or components as a result of the effects of thermal diffusion; at the same time the colder stream becomes enriched in the remaining component or components. In this way a gravitational effect is achieved.

The behaviour of three binary mixtures benzene-\(\alpha\)-heptane, \(\alpha\)-heptane-\(\alpha\)-propyl iodide and dodecane-quinoline, in a thermogravitational diffusion column have been studied. Results covering a wide range of initial composition are recorded. It has been found that the ideal parabolic curve relating separation and initial mixture composition is distorted; greater distortion occurring when "ring" compounds are present. The non-ideal behaviour is attributed to molecular "mobility" differences.

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SEPARATION AND INITIAL COMPOSITION IN LIQUID THERMAL DIFFUSION*

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The behaviour of three binary mixtures benzene-\(\alpha\)-heptane, \(\alpha\)-heptane-\(\alpha\)-propyl iodide and dodecane-quinoline, in a thermogravitational diffusion column have been studied. Results covering a wide range of initial composition are recorded. It has been found that the ideal parabolic curve relating separation and initial mixture composition is distorted; greater distortion occurring when "ring" compounds are present. The non-ideal behaviour is attributed to molecular "mobility" differences.

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