A LOGIC APPROACH IN THE SIMPLEX METHOD OF LINEAR PROGRAMMING*

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The purpose of this paper is to show a logic of direct approach for the simplex method on L.P.

From the characteristics of the objective function and the constraints, L.P. problems can be classified as follows:

Form	Objective function	Constraint
A) Standard	Max $c^T x$	$Ax < b, x > 0$
B) Nonstandard		
Type I	Max $c^T x$	$Ax > b, x > 0$
Type II	Min $c^T x$	$Ax < b, x > 0$
Type III	Min $c^T x$	$Ax < b, x > 0$
Type IV	Max $c^T x$	$Ax \geq b, x > 0$
Type V	Min $c^T x$	$Ax \geq b, x > 0$

Preliminaries: A linear programming model has been constructed from three parts. The influence of characteristics of each part can be considered as in the following order: objective function, R.H.S. (right-hand side) and left-hand side. The proposed method involves two concepts with respect to criteria. *The first concept is that the direction of objective function should be always kept in an objective policy, and is characterized by the coefficient of objective function. The second concept is related to the determination of the optimal condition. These concepts are connected with the definition of the key capacity factor Ψb .*

$$\Psi b \equiv \text{value of R.H.S.} / \sum_{j=1}^m a_{ij}$$

where j is a constant chosen by characteristics of the objective function. These concepts are the main preliminaries of this method. Furthermore, additional preliminaries are shown as follows: **1)** Let b be a capacity for the optimum feasible solution. **2)** The criterion of decision for each iteration is that all the capacities must be used for most profitable decision, and each decision is changed to the next one. **3)** Let Δc be the profit difference between two decisions. **4)** The key capacity factor, which is a possibility to use the present capacities for the deci-

sion, also helps to calculate the new capacities. **Table 1** shows how to apply the main criteria of the concepts proposed, as well as the policy to solve all type of L.P. problems.

Steps of calculation procedure: **1-** Convert the inequalities to equalities with nonnegative slack variables. **2-** Determine the first feasible solution. The variables which make this feasible solution are called the basic set. **3-** Choose the policy to solve the problems by Table 1. This is the first speciality of the logical way of the present paper. **4-** The feasible solution is now tested for optimality. This is the second speciality, which relates to the key capacity. The profit difference Δc_j is evaluated by the following calculation.

$$\Delta c_j = c_j - \sum_{i=1}^m a_{ij} c_i \quad (j=1, \dots, n)$$

If one or more of Δc_j is positive, the feasible solution is nonoptimal for the standard form, and Type I, IV of nonstandard form. If all Δc_j are negative, the optimum solution has been found, and the solution terminates. If one or more of Δc_j is negative, the feasible solution is nonoptimal for Type II, III and V of nonstandard form. If all Δc_j are positive, the solution is found, and it terminates. **5-** Determine new capacities and the coefficients of the problem in step 4. **6-** Return to step 3, and continue until the optimum solution is obtained.

Examination

Example 1¹⁾

$$\begin{aligned} &\text{Max } (x_1 - x_2) \\ &\text{subject to} \\ &2x_1 - x_2 < 4 \\ &x_1 - 2x_2 < 2 \quad x_1, x_2 > 0 \\ &x_1 + x_2 > 5 \end{aligned}$$

[Solution] This problem is solved by the standard form of L.P. mentioned in Table 1. **1-** Step 1 leads

$$\begin{aligned} 2x_1 - x_2 + z_1 &= 4 \\ x_1 - 2x_2 + z_2 &= 2 \\ x_1 + x_2 + z_3 &= 5 \end{aligned}$$

2- Tableau I shows the first feasible solution. **3-** Choose the policy as shown in Tableau I denoting the mark $\uparrow$. **4-** This problem means that all Δc_j must be negative or zero, but there is one positive

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Standard form of L.P. (A)	Table 1 Entering policy of the LP problem				
	Nonstandard form of L.P. (B)				
	a) Type I	b) Type II	c) Type III	d) Type IV	e) Type V
Lowest value of the key capacity factor	highest value of the key capacity factor	highest value of the key capacity factor	lowest value of the key capacity factor	key capacity factor must be on valid area for the present capacity	
biggest Δc_j	biggest Δc_j	smallest Δc_j	smallest Δc_j	biggest Δc_j	smallest Δc_j

Δc_j in the tableau I. Therefore, this feasible solution is not the optimum solution. 5- From steps 4 and 5, **Tableau II** must be formed by choosing the key capacity factor as

$$4/2=2, \quad 2/1=2, \quad 5/1=5, \quad \Psi b=2$$

Consequently, as mentioned in Tableau II the value marked by * is the key capacity factor. Then the capacities (b) are rechosen by

$$2, \quad 2-2 \times 1=0, \quad 5-2 \times 1=3$$

Thus, the coefficients of variables are as follows:

$$x_1=2z_1+z_2+z_3 \rightarrow z_1=(1/2)x_1-(1/2)z_2-(1/2)z_3$$

$$x_2=-z_1-2z_2+z_3$$

$$=-(1/2)x_1+(1/2)z_2+(1/2)z_3-2z_2+z_3$$

$$=-(1/2)x_1-(3/2)z_2+(3/2)z_3$$

Tableau II gives the optimum solution, because there is no positive Δc_j in it. The solution is: $x_1=2$, $x_2=0$. The solution agrees with that by Amundson¹, however, this method does not need any resolution.

Example 2

$$\text{Max } (3x_1-2x_2)$$

subject to

$$-x_1+2x_2 < 8$$

$$x_1+10x_2 > 10 \quad x_1, x_2 > 0$$

$$(1/3)x_1+(1/2)x_2 < 5$$

[Solution] This problem is solved by Type IV of the nonstandard form. 1- Step 1 leads

$$-x_1+2x_2+z_1=8$$

$$x_1+10x_2-z_2=10$$

$$(1/3)x_1+(1/2)x_2+z_3=5$$

2- Thus, **Tableau III** shows the feasible solution.

3- Choose the policy as shown by mark $\uparrow$ in Tableau III. 4- In this problem all Δc_j must be negative or zero. But there is one positive Δc_j in it. Therefore this feasible solution is not optimum.

5- From steps 4 and 5, Tableau IV must be formed by choosing the key capacity factor as

$8 \times (-1) = -8$, $(10/1) = 10$, $5 \times (3/1) = 15$, $\Psi b = 15$
Thus, only $\Psi b = 15$ is in valid area for all conditions of the problem (See Fig. 1). The new capacities (b) are as follows

$$8 - (-1)(15) = 23$$

$$10 - (1)(15) = -5$$

$$\Psi b = 15$$

Then

$$z_3 = 3x_1 + 3z_1 - 3z_2$$

Fig. 1 Choosing of Ψb for Example 2

Tableau I							
c_i	z_i	x_1	x_2	z_1	z_2	z_3	C
0	z_1	2	-1	1	0	0	4
0	z_2	1	-2	0	1	0	2
0	z_3	1	1	0	0	1	5
	c_j	1	-1	0	0	0	
	$\sum_{j=1}^m a_{ij}c_i$	0	0	0	0	0	
	Δc_j	1	-1	0	0	0	
$\uparrow$							

Tableau II							
c_i	z_i	x_1	x_2	z_1	z_2	z_3	C
1	z_1	1	-1/2	1/2	0	0	2*
0	z_2	0	-3/2	-1/2	1	0	0
0	z_3	0	3/2	-1/2	0	1	3
	c_j	1	-1	0	0	0	
	$\sum_{j=1}^m a_{ij}c_i$	1	-1/2	1/2	0	0	
	Δc_j	0	-1/2	-1/2	0	0	

Tableau III							
c_i	z_i	x_1	x_2	z_1	z_2	z_3	C
0	z_1	-1	2	1	0	0	8
0	z_2	1	10	0	-1	0	10
0	z_3	1/3	1/2	0	0	1	5
	c^T	3	-2	0	0	0	
	$\sum_{j=1}^m a_{ij}c_i$	0	0	0	0	0	
	Δc_j	3	-2	0	0	0	

Tableau IV							
c_i	z_i	x_1	x_2	z_1	z_2	z_3	C
0	z_1	0	7/2	1	0	3	23
0	z_2	0	3/2	0	-1	-3	-5
3	z_3	1	-2	0	0	3	15
	c^T	3	-2	0	0	0	
	$\sum_{j=1}^m a_{ij}c_i$	3	9/2	0	0	0	
	Δc_j	0	-13/2	0	0	-3	

C = capacity

$$x_2 = (7/2)z_1 + (17/2)z_2 + (3/2)x_1$$

Tableau IV gives optimum solution, since no positive Δc_j is in it. Finally, $x_1=15$, $x_2=0$

Hitherto two different approaches to solve L.P. problem by the simplex method involving the resolution of the degeneracy problem have been developed by several investigators²⁻⁴. However, the present paper emphasises that when one uses the proposed logic way one never meets any degeneracy problem involving resolution procedure. Generally, the ordinary simplex method of L.P. is a dynamic trial and error way, the number of trials is almost the same as that of extreme points, and the solution converges between m and $2m$, where m is the number of constraints. The logical way presented follows dynamic trial and error calculation procedure, but the number of iterations is always less than that by the ordinary simplex method.

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sity of Istanbul.

Nomenclature

a_{ij}	= coefficient of x_{ij}	[—]
b_i	= parameter of constraint i related to capacity	[—]
c_i	= coefficient of objective function of z_i	
c_j	= coefficient of objective function of x_j	[—]
Δc_j	= difference between c_j and $\sum_{j=1}^m a_{ij}c_i$	[—]
m	= number of constraints	[—]
n	= number of real variables	[—]
x_j	= real variable	[—]
z_j	= slack variable	[—]

<Subscripts>

i	= constraint
j	= real variable

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No.	Page	Column	Line	erroneous	should read
(b: bottom, t: top)					
1	1	2	6b	spectrat	spectra
	4	2	2b	elucidated	elucidate
	7	1	8b	Walti	Wälti
2	137	2	14b	(1)	(13)
3	158	abstract	2t	$W = W_0 \exp [-(A/E_0)^n]$	$W = W_0 \exp [-(A/E)^n]$
	161	2	Eq. (4)	$P_s = P_o \exp \left(\frac{T-T_o}{T} \cdot \frac{\Delta H_0}{RT_o} \right)$	$p_s = p_o \exp \left(\frac{T-T_o}{T} \cdot \frac{\Delta H_0}{RT_o} \right)$
	163	1	Eq. (11)	$0.26 \frac{y^{+3}}{R^{+2}}$	$0.24 \frac{y^{+2}}{R^{+2}}$
	163	2	Eq. (13)	$f = \tau_w (\rho \bar{u}^2 / 2)$	$f = \tau_w / (\rho \bar{u}^2 / 2)$
	201	2	9t	almostp article-free	almost particle-free
	209	2	Fig. 2	$k_{ob} a_b / \varepsilon_b$ [sec]	$k_{ob} a_b / \varepsilon_b$ [sec ⁻¹]
	212	2	11t	(197)	(1973)
	230	2	4 & 6t	$\sqrt{[\eta + \beta\eta + C_e(i\pi/2\omega_0)^2]^2 - C_e(i\pi/\omega_0)^2}$	$\sqrt{[\eta + \beta\eta + C_e(i\pi/2\omega_0)^2]^2 - C_e\eta(i\pi/\omega_0)^2}$