α_0	= initial amplitude of disturbance	[cm]
μ	= viscosity of liquid	[g/cm·sec]
ρ	= density of liquid	[g/cm ³]
σ	= surface tension	[dyn/cm]
ω	= angular velocity of disk	[sec ⁻¹]

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GROUP THEORETIC APPROACH TO SIMILARITY SOLUTIONS OF CHEMICAL BOUNDARY LAYER

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In this paper group theoretic approach to similarity solution of chemical boundary layer has been employed. We have seen that if $n=1$ then in general a similarity solution is possible when the flow is accompanied by n th-order homogeneous volume reaction.

Chemical boundary layer may be described by the following equations¹⁾:

$$\phi_1 \equiv \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - \frac{\partial^2 u}{\partial y^2} = 0 \quad (1)$$

$$\phi_2 \equiv \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

$$\phi_3 \equiv \frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} - \frac{1}{Sc} \frac{\partial^2 c}{\partial y^2} - \frac{L^2 K}{\nu} c^n = 0 \quad (3)$$

Where Sc =Schmidt's number, ν =kinematic viscosity c =concentration of the solute, L =characteristic length k =a suitable constant.

It can easily be verified that the expressions ϕ_1 , ϕ_2 , ϕ_3 remain constant conformally invariant under the group of transformation^{2,3)}

$$\left. \begin{aligned} \bar{x} &= a^{\alpha_1} x, \quad \bar{y} = a^{\alpha_2} y, \quad \bar{t} = a^{\alpha_2} t \\ \bar{u} &= a^{\alpha_1 - 2\alpha_2} u, \quad \bar{v} = a^{-\alpha_2} v, \quad \bar{c} = a^{2\alpha_2/(1-n)} c \end{aligned} \right\} \quad (4)$$

Thus we see that $n \neq 1$.

Let us choose η_1 and η_2 as follows:

$$\eta_1 = n^{-p} y, \quad \eta_2 = x^{-2p} t \quad (5)$$

where $p = \alpha_1/\alpha_2$.

It can easily be verified that η_1 and η_2 are absolute invariants under the group of transformations (4).

The dependent variables can similarly be transformed as follows:

$$\left. \begin{aligned} u &= x^{1-2p} F_1(\eta_1, \eta_2) \\ v &= x^{-p} F_2(\eta_1, \eta_2) \\ c &= x^{2p/(1-n)} F_3(\eta_1, \eta_2) \end{aligned} \right\} \quad (6)$$

With the help of relations (5) and (6) Eqs. (1), (2) and (3) respectively become

$$\begin{aligned} \frac{\partial F_1}{\partial \eta_2} + F_1 \left\{ (1-2p)F_1 - p \left(\eta_1 \frac{\partial F_1}{\partial \eta_1} + 2\eta_2 \frac{\partial F_1}{\partial \eta_2} \right) \right\} \\ + F_2 \frac{\partial F_1}{\partial \eta_1} - \frac{\partial^2 F_1}{\partial \eta_1^2} = 0 \end{aligned} \quad (7)$$

$$(1-2p)F_1 - p \left\{ \eta_1 \frac{\partial F_1}{\partial \eta_1} + 2\eta_2 \frac{\partial F_1}{\partial \eta_2} \right\} + \frac{\partial F_2}{\partial \eta_1} = 0 \quad (8)$$

$$\begin{aligned} \frac{\partial F_3}{\partial \eta_2} + F_1 \left\{ \frac{2p}{(1-n)} F_3 - p \left(\eta_1 \frac{\partial F_3}{\partial \eta_1} + 2\eta_2 \frac{\partial F_3}{\partial \eta_2} \right) \right\} \\ + F_2 \frac{\partial F_3}{\partial \eta_1} - \frac{1}{Sc} \frac{\partial^2 F_3}{\partial \eta_1^2} - \frac{L^2 K}{\nu} F_3^n = 0 \end{aligned} \quad (9)$$

Using the independent variables η_1 , η_2 and the dependent variables F_1 , F_2 , F_3 we set up a new one-parameter transformations group as follows:

$$\begin{aligned} \bar{\eta}_1 &= b^{\epsilon_1} \eta_1, \quad \bar{\eta}_2 = b^{2\epsilon_1} \eta_2, \quad \bar{F}_1 = b^{-2\epsilon_1} F_1 \\ \bar{F}_2 &= b^{-\epsilon_1} F_2, \quad \bar{F}_3 = b^{2\epsilon_1/(1-n)} F_3 \end{aligned} \quad (10)$$

It can easily be verified that the left-hand sides of Eqs. (7), (8) and (9) remain constant conformally invariant under the transformations (10). It can be verified that transformation of the independent variables given by

$$\eta = \eta_1 \eta_2^{-1/2} \quad (11)$$

is an absolute invariant under the group of transformations (10).

The transformations involving the dependent variables are as follows:

$$F_1 = \eta_2^{-1} H_1(\eta), \quad F_2 = \eta_2^{-1/2} H_2(\eta), \quad F_3 = \eta_2^{1/(1-n)} H_3(\eta) \quad (12)$$

Employing the transformation (12) and the relation (11) in Eqs. (7), (8), (9) we get respectively:

$$\{H_1 + (1/2)\eta H_1'\} - H_1^2 - H_2 H_1' + H_1'' = 0 \quad (13)$$

$$H_1 + H_2' = 0 \quad (14)$$

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$$\left[\frac{1}{(1-n)} H_3 - \frac{1}{2} \eta H_3' \right] + H_2 H_3' - \frac{1}{Sc} H_1'' - \frac{L^2 K}{\nu} H_3^n = 0 \quad (15)$$

Thus we see that the chemical boundary layer Eqs. (1), (2) and (3) are reduced respectively to Eqs. (13), (14) and (15), which are ordinary differential equations. Hence similarity solution of chemical boundary layer equations is possible by group theoretic approach. We have already shown that the above results are valid when $n \neq 1$.

The boundary conditions for H_1 , H_2 , H_3 can be fixed at this stage and the Eqs. (13)–(15) can be solved for H_1 , H_2 , H_3 with the help of the boundary conditions.

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MINIMUM WETTING RATE ON WETTED-WALL COLUMN —IN THE ABSENCE OF MASS AND HEAT TRANSFER—

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Introduction

In falling-film mass or heat transfer units, it is important to make a stable liquid flow over the wall to ensure the wetted area contributing to the transfer process. To date, regarding wetted-wall and packed columns with or without mass and/or heat transfer, several workers have measured the minimum wetting rates (M.W.R.) of two cases, i.e. one where the flow rate is reduced until the uniform film just starts to rupture (Case I), and another where the rate is increased until initially dry surface is wetted completely (Case II). Also, a few workers have theoretically and experimentally presented equations to predict the M.W.R. These equations are shown in

Table 1. Nevertheless, even for the isothermal condition there is a great discrepancy in the measured values, and theoretical treatments of the film rupture have not necessarily been successful. The present work attempts to give satisfactory reasons for such inconsistency.

Experimental Apparatus and Procedure

Experiments have been done for water, *n*-propanol-water mixture (8.62 mole %) and ethyl oxalate in the two cases mentioned above by use of 3.5 cm-O.D. smooth glass tubes. The apparatus is shown in **Figs. 1a** and **1b**. Six specimen tubes have been prepared, each of which has a different top shape, as shown in **Fig. 2**. The tube was cleaned with chromic acid mixture, and then washed with distilled water before the start of an experiment. This pretreatments was given to all tubes.

A liquid from the head tank was supplied to the

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Table 1 Summarization of equations for M.W.R.

Workers	Equation		Basis of derivation of the Equation	Applicability
Hartley and Murgatroyd ³⁾	$\Gamma/\mu = 0.803 (\rho\sigma^3/\mu^4 g)^{0.2}$	(1)	Minimum total energy rate, theoretical	Case I, II
Ponter, Davies, Ross and Thornley ⁴⁾	$\Gamma/\mu = 1.116 (1 - \cos \theta)^{0.6} (\rho\sigma^3/\mu^4 g)^{0.2}$	(2)	Momentum balance, theoretical	Case I, II
Hobler and Czajka ⁴⁾	$4\Gamma/\mu = 0.6513 (1 - \cos \theta)^{0.6} (\rho\sigma^3/\mu^4 g)^{0.325} (\sin \beta)^{2/3}$	(3)	Correlation of experimental data, semitheoretical	Case II
Iijima and Kuzuoka ⁵⁾	$m\rho^2/\mu^{0.4} = 0.11 \log(\sigma/7.0)$ [CGS]	(4)	Correlation of experimental data, empirical	Case I