$$\left[\frac{1}{(1-n)} H_3 - \frac{1}{2} \eta H_3' \right] + H_2 H_3' - \frac{1}{Sc} H_1'' - \frac{L^2 K}{\nu} H_3^n = 0 \quad (15)$$

Thus we see that the chemical boundary layer Eqs. (1), (2) and (3) are reduced respectively to Eqs. (13), (14) and (15), which are ordinary differential equations. Hence similarity solution of chemical boundary layer equations is possible by group theoretic approach. We have already shown that the above results are valid when $n \neq 1$.

The boundary conditions for H_1 , H_2 , H_3 can be fixed at this stage and the Eqs. (13)–(15) can be solved for H_1 , H_2 , H_3 with the help of the boundary conditions.

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MINIMUM WETTING RATE ON WETTED-WALL COLUMN —IN THE ABSENCE OF MASS AND HEAT TRANSFER—

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Introduction

In falling-film mass or heat transfer units, it is important to make a stable liquid flow over the wall to ensure the wetted area contributing to the transfer process. To date, regarding wetted-wall and packed columns with or without mass and/or heat transfer, several workers have measured the minimum wetting rates (M.W.R.) of two cases, i.e. one where the flow rate is reduced until the uniform film just starts to rupture (Case I), and another where the rate is increased until initially dry surface is wetted completely (Case II). Also, a few workers have theoretically and experimentally presented equations to predict the M.W.R. These equations are shown in

Table 1. Nevertheless, even for the isothermal condition there is a great discrepancy in the measured values, and theoretical treatments of the film rupture have not necessarily been successful. The present work attempts to give satisfactory reasons for such inconsistency.

Experimental Apparatus and Procedure

Experiments have been done for water, *n*-propanol-water mixture (8.62 mole %) and ethyl oxalate in the two cases mentioned above by use of 3.5 cm-O.D. smooth glass tubes. The apparatus is shown in **Figs. 1a** and **1b**. Six specimen tubes have been prepared, each of which has a different top shape, as shown in **Fig. 2**. The tube was cleaned with chromic acid mixture, and then washed with distilled water before the start of an experiment. This pretreatments was given to all tubes.

A liquid from the head tank was supplied to the

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Table 1 Summarization of equations for M.W.R.

Workers	Equation		Basis of derivation of the Equation	Applicability
Hartley and Murgatroyd ³⁾	$\Gamma/\mu = 0.803 (\rho\sigma^3/\mu^4g)^{0.2}$	(1)	Minimum total energy rate, theoretical	Case I, II
Ponter, Davies, Ross and Thornley ⁴⁾	$\Gamma/\mu = 1.116 (1 - \cos \theta)^{0.6} (\rho\sigma^3/\mu^4g)^{0.2}$	(2)	Momentum balance, theoretical	Case I, II
Hobler and Czajka ⁴⁾	$4\Gamma/\mu = 0.6513 (1 - \cos \theta)^{0.6} (\rho\sigma^3/\mu^4g)^{0.325} (\sin \beta)^{2/3}$	(3)	Correlation of experimental data, semitheoretical	Case II
Iijima and Kuzuoka ⁵⁾	$m\rho^2/\mu^{0.4} = 0.11 \log(\sigma/7.0)$ [CGS]	(4)	Correlation of experimental data, empirical	Case I

Fig. 1 Experimental apparatus for measurement of minimum wetting rate

Fig. 2 Various types of top of wetted-wall column

top of the wetted-wall column through adjustable cock or valve and made to flow downward in a filmwise manner on the outside of the tube suspended vertically. The flow rates were measured at the column bottom with measuring cylinders in both apparatus. Appearance of the liquid film was observed for all experiments.

Discussion of Results

In the first place, as a preliminary experiment the M.W.R. were measured for 15, 30, 50 and 75 cm-L. tubes to investigate the effect of tube length. But no effect was noted, so the following measurements were carried out for 15 cm-L. tubes with various shapes of wetted-wall top only.

Data obtained at room temperature in the two cases are shown in Fig. 3.

First, in Case I the M.W.R. were, as expected, very low; for *n*-propanol-water mixture and ethyl oxalate in particular they were nearly zero (below 0.0004 g/cm·sec) and the film formed on the glass tubes held uniform for a short time even when the liquid flows were stopped. In explanation of this phenomenon, it is recalled that Richards and Carver⁹⁾ and other investigators have reported that the contact angle between glass and liquid, such as water and some organic compounds, is zero provided the glass is

Fig. 3 Minimum wetting rates for smooth glass tubes with various types of column top at room temperature

already covered by a film of the liquid. If this fact is introduced in Eq. (2) (see Table 1) the M.W.R. reduces to zero.

On the contrary, for water the M.W.R. were greater than zero. This is probably due to the presence of an adsorbed material, such as grease, on the glass surface. One can hardly wash it off completely in practice, and water is less familiar than other organic compounds and organic solutions. For water, in fact, measurement of the M.W.R. with accuracy was very difficult.

Moreover, difference of the top shape of the wetted-wall column produces a difference in the M.W.R. This denotes that the liquid distribution varies according to each type of tube and, in turn, the M.W.R. is subject to the end-effect.

In this figure are also plotted the values for water reported by Friedman²⁾, Norman⁶⁾ and Bond¹⁾ and the values for all liquids derived from Eqs. (1) and (4) together. The minimum value obtained for water for Type F is twice Friedman's value, though the higher values obtained for the tube of Type B are almost consistent with Norman's and Bond's data. Further, the result for Type E is close to the value predicted from Eq. (4). For water a much lower value, which agrees quite well with Friedman's, was obtained for Type B in a preliminary test. But such a value could not be reproduced in the present experiment.

Second, in Case II much higher liquid flow rates were required than that in Case I. This is caused by the fact that the contact angle for an advancing liquid film over the dry surface is greater than for a receding film over the wet surface. It is obvious from this figure that there is a great disagreement in this case between the values for each type of tube as well as in Case I and that in some cases there are less reproducible data. The value for Type A is the greatest, and is four times or more the value for Type F, which is the lowest for any liquid in this experiment.

In comparison with the values predicted from Eqs.

(1), (2) and/or (3), which are plotted in the same figure, for water and *n*-propanol-water mixture the experimental value for Type A is close, though most of the results for water and *n*-propanol-water mixture and all results for ethyl oxalate measured in the present work are lower.

An attempt was made to correlate the experimental results based on the equations shown in Table 1. **Fig. 4** shows the result plotted as (Γ/μ) against $(\rho\sigma^3/\mu^4g)$. The equations are also superposed on this figure by taking the contact angle parameter in Eqs. (2) and (3), by putting $\beta=90^\circ$ (vertical surface) in Eq. (3) and by transforming Eq. (4) as follows:

$$\frac{\Gamma}{\mu} = \left[\frac{0.11^3}{3} \left(\frac{g^2}{\rho^7\sigma} \right)^{0.6} \left\{ \log \left(\frac{\sigma}{7.0} \right) \right\}^3 \right] \left(\frac{\rho\sigma^3}{\mu^4g} \right)^{0.2} = K \left(\frac{\rho\sigma^3}{\mu^4g} \right)^{0.2} \quad (4)'$$

Here, when in Eq. (4)' the physical properties of water at 20°C are substituted in the square brackets, the coefficient *K* becomes about 0.14, and this value is little altered even if the physical properties of water at other temperatures are substituted. It is obvious that a difference in top shape provides a different result. Nevertheless, in Case II the data for each type of tube are roughly resolved into a single line with slope 0.2. Hence, the following relation is obtained between two dimensionless numbers:

$$\Gamma/\mu = C(\rho\sigma^3/\mu^4g)^{0.2} \quad (5)$$

This result is consistent with Eqs. (1) and (2). Eq. (5), however, has a different numerical constant, *C*, for each type of tube.

In Case I, only the data for water are plotted in this figure, and no correlation can be obtained, because the data for *n*-propanol-water mixture and ethyl oxalate are very low and cannot appear in this log-log plot. But those data are not so important in practice, for commercial equipment should be operated in a range above the value in Case II for reliability.

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Fig. 4 Relation between (Γ/μ) and $(\rho\sigma^3/\mu^4g)$

Nomenclature

<i>C</i>	= numerical constant defined in Eq. (5)	[—]
<i>g</i>	= acceleration due to gravity	[cm/sec²]
<i>K</i>	= coefficient defined in Eq. (4)'	[—]
<i>m</i>	= Nusselt film thickness	[cm]
β	= angle of inclination of plate	[—]
Γ	= minimum wetting rate	[g/cm·sec]
θ	= contact angle	[—]
μ	= viscosity of liquid	[g/cm·sec]
ρ	= density of liquid	[g/cm³]
σ	= surface tension of liquid	[dyne/cm]

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