quality of mixedness is presented for the case in which the transient response method is used to find the degree of mixing.

According to this definition, the quality of mixedness varies from zero for the complete separate state or "piston flow" to values approaching unity for the final mixing state or the complete mixing state.

The practical application of the newly defined quality of mixedness is explained with two examples.

Nomenclature

C	= noticed solute concentration	[g-mol/cm ³]
$E(\tau)$	= dimensionless residence time distribution function	
H	= entropy	[dit]
k	= tank number	[—]
M	= quality of mixedness	[—]

N	= total amount of noticed solute	[g-mol]
n	= region number	[—]
P	= probability	[—]
p	= probability density	[—]
t	= time	[sec]

τ	= dimensionless time	[—]
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CONCENTRATION RESPONSES IN HIGH VISCOSITY LIQUID STIRRED TANKS

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The mixing characteristics of high-viscosity liquid in stirred tanks were investigated from measurement of the concentration response curves by injecting tracer in the tanks, and it was confirmed, by comparing the results of the concentration response curves calculated from the circulation time distribution, that the circulation time distribution can be used to predict the concentration response curve that represents the mixing performance. The effect of diffusive mixing was found to be significant in the concentration response curve obtained experimentally, particularly in the case of paddle or turbine impellers in the range of Reynolds number less than 10. The characteristic values of mixing both observed and calculated showed a similar tendency and it was concluded that paddle and turbine impellers are preferable for mixing in the Reynolds number range greater than 20, and helical ribbon and helical screw impellers for less than 20. An attempt was made to correlate the parameters characterizing the circulation time distribution with operating conditions, and a method of estimating the characteristic values of mixing from the parameters of the circulation time distribution was obtained.

Introduction

The mixing of viscous liquid in a stirred tank is primarily achieved by convective mixing and shear mixing. It has been shown^{9,11)} that the mixing characteristics of the stirred tank, where convective mixing is controlling, can be predicted by use of circulation time distribution, which may be obtained by the tracing particle method. This method of calculation does not, however, take into consideration the effect of stagnant region which is not subject to convective mixing.

In the present work, therefore, it was attempted to make a direct measurement of the concentration response curve^{4,5)} and compare with the calculated results from the circulation time distribution. At the same time, the relationships between the parameters^{9,11)} characterizing circulation time distribution and mixing characteristics were investigated.

Concentration Response Curve

Fig. 1 shows examples of the concentration response curves of helical ribbon and helical screw impellers calculated from the circulation time distribution in accordance with the method, as reported in the previous paper¹⁰⁾, based on the following equa-

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Fig. 1 Concentration response curve of mixing (calculated)

Fig. 2 Measuring apparatus

Fig. 3 Concentration response curve of mixing (observed)

Fig. 4 Concentration response curve of mixing (observed)

tion.

$$\frac{C(t)}{C(\infty)} = \frac{1}{C(\infty)} \int_0^t C(T) y(t-T) dT + \delta(t) \quad (1)$$

Fig. 5 Concentration response curve of mixing (observed)

Fig. 6 Comparison of observed and calculated curves

Since the circulation time in the helical screw is relatively sharply distributed, the corresponding concentration response curve shows the characteristics of oscillation.

The measurement of the concentration response curve was carried out with use of an electric conductivity device by injecting a small amount of tracer having the same viscosity as the bulk liquid in the tank but different electric conductivity. A pair of platinum wire electrodes of 1 mm diameter and 15 mm length with a gap of 5 mm was installed in the test zone, and alternating current of 1 KC and 10~30 V was introduced. As the tracer, 10~25 cc of the same liquid as the bulk in the tank but containing 1~2 N of KCl was injected in the test zone and the change of electric conductivity in the course of time was detected by the electrodes and recorded. Fig. 2 shows the structure of the electrodes and the measurement devices. The electric conduction data were converted into concentration change based on the value at the time when mixing was achieved ($|C(t) - C(\infty)|/C(\infty) = 0.01$).

The observed results of the concentration response curve are shown in Figs. 3, 4 and 5. It can be noted

Fig. 7 Comparisons of frequency and amplitude ratio

that the observed results as well as the calculated results show characteristics peculiar to the type of impellers.

In case of paddle or turbine impellers, there is a tendency of becoming more oscillatory with decrease of Reynolds number or increase of liquid viscosity, which corresponds to the fact that the circulation time distribution curve becomes steeper. As seen in Fig. 3, in the range of Reynolds number less than 10, the effect of diffusive mixing, which is not taken into account in the response curve calculated from the circulation time distribution, becomes significant. It is seen, as expected from the calculation of the response curve from the circulation time distribution, that the response curve of a helical ribbon is less oscillatory compared with that of a helical screw. Though it is difficult to make a direct comparison of the observed and calculated response curves, Fig. 6 shows an example of the comparison of both results obtained under rather similar conditions. Taking into account experimental errors, for example, in the process of injection of the tracer, it may be considered that both results agree fairly well. However, there is a general tendency that the observed results have more complicated features. Since it is considered more practical to compare some characteristic values rather than to compare the response curves directly, the frequency and the amplitude ratio were compared, as shown in Fig. 7, where the significant correspondence between the observed and calculated results can

Fig. 8 Mixing time

Fig. 9 Mixing time index

be seen. Here, the frequency is the number of waves per unit time (in seconds) in the response curve and the amplitude ratio is defined as the ratio of two continuing spontaneous deviations of $C(t)/C(\infty)$, ε_1 and ε_2 , as shown in Fig. 1.

The characterizing values of mixing defined previously¹⁰⁾ were obtained both from the calculated and observed results and are compared in Figs. 8 and 9, where the dimensionless mixing time, $NT_M\bar{\theta}$, which is the same as those defined previously^{5-8,12)}, and the mixing time index, $NM_i\bar{\theta}$, are shown. Since the average circulation time was not measured in the present experiment, differently from the previous experiments⁹⁻¹¹⁾, the mixing time was represented as the product of $TM\bar{\theta}$ and this term was made dimensionless by multiplying by N . The other terms, $M_i\bar{\theta}$, $T_c\bar{\theta}$ and $T_d\bar{\theta}$, were treated in the same manner. It is seen in Fig. 8 that the mixing times of paddle and turbine impellers are smaller compared with that of

Fig. 10 Convective and diffusive mixing time constants

Fig. 11 Ratio of mixing time to power number

screw ribbon impellers in the range of Reynolds number greater than 20, but this relation is reversed in the smaller range of Reynolds number. It may be concluded that the helical ribbon impeller is preferable in the low Reynolds number range and the anchor type can be suitably used in the larger Reynolds number range. In comparison of the calculated and observed results of mixing time, there can be seen a similar tendency while the observed results are of larger values. Since the calculated response curves were obtained only from the convective circulation time distribution, ignoring the effect of the diffusive mixing which is significant in the low Reynolds number range, the observed results are considered to be more realistic. The same tendency is seen also in the mixing time index shown in Fig. 9. Generally speaking, it is considered that the paddle and turbine impellers are suitable in the range of Reynolds number larger than 20 and the helical ribbon and screw impellers are preferable in the smaller range of Reynolds number, while the anchor impellers are not efficient in either range.

Fig. 10 shows the experimental results of mixing time constant obtained for the paddle and turbine

impellers. The convective mixing time constant increases remarkably in the range of Reynolds number less than 10 where the time constant of diffusive mixing predominates over the time constant of convective mixing and becomes rate-determining in the overall mixing process.

Generally speaking, an effort of decreasing the mixing time of high-viscosity liquid causes higher power requirements. When the ratio of mixing time to power consumption is small, the mixing is considered to be efficient. Therefore the ratio of the dimensionless mixing time and the dimensionless power number, NT_M/N_p , was calculated and plotted against Reynolds number as shown in Fig. 11, where it is seen that helical screw, helical ribbon and anchor impellers are preferable in the low Reynolds number range under 20 while paddle and turbine impellers are efficient in the higher Reynolds number range. Since the power consumptions were not measured in the present experiments, the power number was calculated for the present experimental conditions in accordance with previous works, such as data of Calderbank³⁾ for paddle and turbine impellers, the method of Bourne²⁾ for helical ribbon, the method of Beckner¹⁾ for anchor and data of Nagata *et al.*⁷⁾ for helical screw.

As shown previously^{9,11)}, the circulation time distribution can be represented with seven parameters, $\alpha, \beta, \gamma, a, b, c$ and n from the following two equations:

$$y_1(t) = \beta(t - \gamma) \sin \{\pi(t - \gamma)/\alpha\} \quad (2)$$

$$y_2(t) = a(t - c)^n \exp \{-b(t - c)\} \quad (3)$$

Since n is significant only under limited conditions of large Reynolds number with anchor or helical ribbon impellers, the circulation time distribution under ordinary conditions can be defined with six parameters. There are two equations, Eqs. (4) and (5), representing the relations among the parameters, which are, however, not enough to determine each value of the parameters.

$$a/b^2 + \alpha^2 \beta / \pi = 1 \quad (4)$$

$$ac/b^2 + 2a/b^3 + \alpha^3 \beta (4/\pi - 1/\pi) + \alpha^2 \beta \gamma / \pi = 1 \quad (5)$$

It was found from the experimental results that some combined values of the parameters such as $\alpha^2 \beta / \pi$, α , γ/α and bc are functions of N_{Re} , as shown in Figs. 12 (a), (b), (c) and (d). It is, therefore, possible to determine the said six parameters with the use of the four figures in addition to Eqs. (4) and (5). Furthermore, it is possible with aid of Fig. 12 (e), representing the relationship between $N\bar{\theta}$ and N_{Re} , to convert the normalized circulation time distribution to the non-normalized one.

In Figs. 12, the parameters $\alpha, \beta, \gamma, a, b, c$ and the average circulation time $\bar{\theta}$ were calculated from previous experiments^{9,11)}, and the curves were obtained

Fig. 12 Parameters vs. Reynolds number

Fig. 13 Relation between M_i and parameters

by plotting the calculated data.

Fig. 13 shows an example of correlation of the mixing characteristics directly from the parameters of the circulation time distributions, where it is seen that the value of $\alpha\beta\gamma + ac/b$ should be preferably less than 3. The terms of $\alpha\beta\gamma$ and ac/b represent respectively the ratios of the dispersion coefficients to the areas of the curves $y_1(t)$ and $y_2(t)$, i.e. σ_{y1}/m_{y1} and σ_{y2}/m_{y2} . It can be noted that such circulation time distributions as of small values in the ratio, in other words broadly distributed, are preferable for mixing performance.

Conclusion

The mixing characteristics of high-viscosity liquid in stirred tanks were investigated by measuring the concentration response curves and the following conclusions were obtained.

(1) The concentration response curves experimentally obtained agreed well with those calculated from the circulation time distribution, and it was confirmed that the prediction of the response curve from the circulation time distribution is reasonable.

(2) The mixing characteristics obtained by experiments and calculations show a similar tendency. In

the range of Reynolds number greater than 20, paddle or turbine impellers are suitable and in the smaller range, helical ribbon and helical screw impellers are preferable, while anchor type impellers are not recommended under the conditions of the present experiments.

(3) It was found that in the experimental results of the concentration response curve, the effect of diffusive mixing is significant. Particularly in the case of paddle or turbine impellers in the range of Reynolds number less than 10, the time constant of diffusive mixing becomes greater than that of convective mixing and diffusive mixing controls the overall rate of the mixing process.

(4) An attempt was made to correlate the param-

eters characterizing the circulation time distribution with Reynolds number, and it was found that the characteristic values of mixing are small, or mixing is achieved efficiently when the value of the term of $\alpha\beta\gamma + ac/b$ is less than 3.

Nomenclature

a, b, c	= parameter	[—]
$C(t)$	= concentration of liquid at t	[mole/m ³]
$C(\infty)$	= concentration of liquid at $t = \infty$	[mole/m ³]
D	= diameter of vessel	[mm]
H	= liquid depth	[mm]
k_c, K_d	= constant	[—]
M_i	= mixing time index, $\int \left \frac{C(t) - C(\infty)}{C(\infty)} \right dt$	[—]
my_1, my_2	= mean value of $y_1(t), y_2(t)$	[—]
N	= rotation speed	[sec ⁻¹]
N_p	= power number	[—]
N_{Re}	= Reynolds number, $\rho ND^2/\mu$	[—]
n	= parameter	[—]
T, t	= time	[—]
T_c	= (convective) mixing time constant $\varepsilon_c = k_c \exp(-t/T_c)$	[—]
T_d	= (diffusive) mixing time constant $\varepsilon_d = k_d \exp(-t/T_d)$	[—]
T_M	= mixing time, the time required for $\varepsilon \leq \varepsilon_a$ in the response curve	[—]
$y(t), y_1(t), y_2(t)$	= circulation time distribution	[—]
α, β, γ	= parameter	[—]

$\delta(t)$	= delta function	[—]
$\bar{\theta}$	= average circulation time	[sec]
$\sigma y_1, \sigma y_2$	= variance of $y_1(t), y_2(t)$	[—]
$\varepsilon_1, \varepsilon_2$	= deviations of response curve $C(t)/C(\infty)$ from unity	[—]
ε_a	= approved deviation	[—]
$\varepsilon_c, \varepsilon_d$	= deviations of circumscribed curve of response curve $C(t)/C(\infty)$ from unity respectively in convective and diffusive mixing	[—]
μ	= viscosity	[poise]
ρ	= density	[g/mm ³]

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LAW OF THE WALL AND VELOCITY DEFECT LAW IN FULLY TURBULENT NON-BAFFLED AGITATED VESSEL

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The characteristic friction velocity in an agitated vessel is defined, based on the assumption that the flow field is affected by the condition at impeller tip. The velocity profiles are well correlated in terms of the dimensionless variables defined by using this friction velocity. The correlation equations are given for the wall vicinity, potential and impeller regions, respectively. Two of them may be regarded as law of the wall and velocity defect law in agitated vessels.

Introduction

The importance of agitated vessels in chemical pro-

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cesses has prompted many researchers to study their performance, and many excellent results have been reported. However, most papers were based on dimensional analysis, while those based on the phenomenological point of view are few.

Recently, a study of transport phenomena at the