quality of mixedness is presented for the case in which
the transient response method is used to find the
degree of mixing.

According to this definition, the quality of mixed-
ess varies from zero for the complete separate state
or "piston flow" to values approaching unity for the
final mixing state or the complete mixing state.

The practical application of the newly defined
quality of mixedness is explained with two examples.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>noticed solute concentration</td>
<td>[g-mol/cm³]</td>
</tr>
<tr>
<td>E(t)</td>
<td>dimensionless residence time distribution function</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>entropy</td>
<td>[dit]</td>
</tr>
<tr>
<td>k</td>
<td>tank number</td>
<td>[--]</td>
</tr>
<tr>
<td>M</td>
<td>quality of mixedness</td>
<td>[--]</td>
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</tbody>
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Literature Cited


CONCENTRATION RESPONSES IN HIGH VISCOITY LIQUID STIRRED TANKS

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The mixing characteristics of high-viscosity liquid in stirred tanks were investigated from
measurement of the concentration response curves by injecting tracer in the tanks, and it was
confirmed, by comparing the results of the concentration response curves calculated from the
circulation time distribution, that the circulation time distribution can be used to predict the
concentration response curve that represents the mixing performance. The effect of diffusive
mixing was found to be significant in the concentration response curve obtained experimentally,
particularly in the case of paddle or turbine impellers in the range of Reynolds number less
than 10. The characteristic values of mixing both observed and calculated showed a similar
tendency and it was concluded that paddle and turbine impellers are preferable for mixing in the
Reynolds number range greater than 20, and helical ribbon and helical screw impellers for less
than 20. An attempt was made to correlate the parameters characterizing the circulation time
distribution with operating conditions, and a method of estimating the characteristic values of
mixing from the parameters of the circulation time distribution was obtained.

Introduction

The mixing of viscous liquid in a stirred tank is
primarily achieved by convective mixing and shear
mixing. It has been shown⁶⁻¹¹) that the mixing char-
acteristics of the stirred tank, where convective mixing
is controlling, can be predicted by use of circulation
time distribution, which may be obtained by the
tracing particle method. This method of calculation
does not, however, take into consideration the effect of
stagnant region which is not subject to convective mixing.

In the present work, therefore, it was attempted to
make a direct measurement of the concentration re-
sponse curve⁴⁻⁵) and compare with the calculated results
from the circulation time distribution. At the same
time, the relationships between the parameters⁴⁻¹¹)
characterizing circulation time distribution and mixing
characteristics were investigated.

Concentration Response Curve

Fig. 1 shows examples of the concentration re-
sponse curves of helical ribbon and helical screw
impellers calculated from the circulation time distri-
bution in accordance with the method, as reported
in the previous paper¹⁹), based on the following equa-
Since the circulation time in the helical screw is relatively sharply distributed, the corresponding concentration response curve shows the characteristics of oscillation.

The measurement of the concentration response curve was carried out with use of an electric conductivity device by injecting a small amount of tracer having the same viscosity as the bulk liquid in the tank but different electric conductivity. A pair of platinum wire electrodes of 1 mm diameter and 15 mm length with a gap of 5 mm was installed in the test zone, and alternating current of 1 KC and 10−30 V was introduced. As the tracer, 10−25 cc of the same liquid as the bulk in the tank but containing 1−2 N of KCl was injected in the test zone and the change of electric conductivity in the course of time was detected by the electrodes and recorded. Fig. 2 shows the structure of the electrodes and the measurement devices. The electric conduction data were converted into concentration change based on the value at the time when mixing was achieved \( (C(t)−C(∞))/C(∞) = 0.01 \).

The observed results of the concentration response curve are shown in Figs. 3, 4 and 5. It can be noted
that the observed results as well as the calculated results show characteristics peculiar to the type of impellers.

In case of paddle or turbine impellers, there is a tendency of becoming more oscillatory with decrease of Reynolds number or increase of liquid viscosity, which corresponds to the fact that the circulation time distribution curve becomes steeper. As seen in Fig. 3, in the range of Reynolds number less than 10, the effect of diffusive mixing, which is not taken into account in the response curve calculated from the circulation time distribution, becomes significant. It is seen, as expected from the calculation of the response curve from the circulation time distribution, that the response curve of a helical ribbon is less oscillatory compared with that of a helical screw. Though it is difficult to make a direct comparison of the observed and calculated response curves, Fig. 6 shows an example of the comparison of both results obtained under rather similar conditions. Taking into account experimental errors, for example, in the process of injection of the tracer, it may be considered that both results agree fairly well. However, there is a general tendency that the observed results have more complicated features. Since it is considered more practical to compare some characteristic values rather than to compare the response curves directly, the frequency and the amplitude ratio were compared, as shown in Fig. 7, where the significant correspondence between the observed and calculated results can be seen. Here, the frequency is the number of waves per unit time (in seconds) in the response curve and the amplitude ratio is defined as the ratio of two continuous spontaneous deviations of $C(i)/C(\infty)$, $\varepsilon_i$ and $\varepsilon_2$, as shown in Fig. 1.

The characterizing values of mixing defined previously$^{10}$ were obtained both from the calculated and observed results and are compared in Figs. 8 and 9, where the dimensionless mixing time, $NT_m\theta$, which is the same as those defined previously$^{9-12}$, and the mixing time index, $NM\theta$, are shown. Since the average circulation time was not measured in the present experiment, differently from the previous experiments$^{9-11}$, the mixing time was represented as the product of $TM\theta$ and this term was made dimensionless by multiplying by $N$. The other terms, $M\theta$, $T\theta$ and $T_{\theta}$, were treated in the same manner. It is seen in Fig. 8 that the mixing times of paddle and turbine impellers are smaller compared with that of
Fig. 10 Convective and diffusive mixing time constants

Fig. 11 Ratio of mixing time to power number

screw ribbon impellers in the range of Reynolds number greater than 20, but this relation is reversed in the smaller range of Reynolds number. It may be concluded that the helical ribbon impeller is preferable in the low Reynolds number range and the anchor type can be suitably used in the larger Reynolds number range. In comparison of the calculated and observed results of mixing time, there can be seen a similar tendency while the observed results are of larger values. Since the calculated response curves were obtained only from the convective circulation time distribution, ignoring the effect of the diffusive mixing which is significant in the low Reynolds number range, the observed results are considered to be more realistic. The same tendency is seen also in the mixing time index shown in Fig. 9. Generally speaking, it is considered that the paddle and turbine impellers are suitable in the range of Reynolds number larger than 20 and the helical ribbon and screw impellers are preferable in the smaller range of Reynolds number, while the anchor impellers are not efficient in either range.

Fig. 10 shows the experimental results of mixing time constant obtained for the paddle and turbine impellers. The convective mixing time constant increases remarkably in the range of Reynolds number less than 10 where the time constant of diffusive mixing predominates over the time constant of convective mixing and becomes rate-determining in the overall mixing process.

Generally speaking, an effort of decreasing the mixing time of high-viscosity liquid causes higher power requirements. When the ratio of mixing time to power consumption is small, the mixing is considered to be efficient. Therefore the ratio of the dimensionless mixing time and the dimensionless power number, $NT_M/N_P$, was calculated and plotted against Reynolds number as shown in Fig. 11, where it is seen that helical screw, helical ribbon and anchor impellers are preferable in the low Reynolds number range under 20 while paddle and turbine impellers are efficient in the higher Reynolds number range. Since the power consumptions were not measured in the present experiments, the power number was calculated for the present experimental conditions in accordance with previous works, such as data of Calderbank for paddle and turbine impellers, the method of Bourne for helical ribbon, the method of Beckner for anchor and data of Nagata et al. for helical screw.

As shown previously, the circulation time distribution can be represented with seven parameters, $\alpha$, $\beta$, $\gamma$, $a$, $b$, $c$ and $n$ from the following two equations:

$$y_1(t) = \beta(t-\gamma) \sin \left[\frac{\pi(t-\gamma)}{\alpha}\right]$$

$$y_2(t) = a(t-c)^n \exp \left[-b(t-c)\right]$$

Since $n$ is significant only under limited conditions of large Reynolds number with anchor or helical ribbon impellers, the circulation time distribution under ordinary conditions can be defined with six parameters. There are two equations, Eqs. (4) and (5), representing the relations among the parameters, which are, however, not enough to determine each value of the parameters.

$$\frac{a}{b^2} + \frac{\alpha^2 \beta}{\pi} = 1$$

$$\frac{ac}{b^2} + \frac{2a}{b^3} + \frac{a^3 \beta(4/\pi - 1/\pi)}{\pi} + \frac{a^2 \beta \gamma}{\pi} = 1$$

It was found from the experimental results that some combined values of the parameters such as $\alpha^2 \beta/\pi$, $\alpha$, $\gamma/\alpha$ and $bc$ are functions of $N_{Re}$, as shown in Figs. 12 (a), (b), (c) and (d). It is, therefore, possible to determine the said six parameters with the use of the four figures in addition to Eqs. (4) and (5). Furthermore, it is possible with aid of Fig. 12 (e), representing the relationship between $N_T$ and $N_{Re}$, to convert the normalized circulation time distribution to the non-normalized one.

In Figs. 12, the parameters $\alpha$, $\beta$, $\gamma$, $a$, $b$, $c$ and the average circulation time $\bar{\theta}$ were calculated from previous experiments, and the curves were obtained.
by plotting the calculated data.

Fig. 13 shows an example of correlation of the mixing characteristics directly from the parameters of the circulation time distributions, where it is seen that the value of $a\tilde{y} + ac/b$ should be preferably less than 3. The terms of $a\tilde{y}$ and $ac/b$ represent respectively the ratios of the dispersion coefficients to the areas of the curves $y_1(t)$ and $y_2(t)$, i.e. $\sigma_1/m_{y_1}$ and $\sigma_2/m_{y_2}$. It can be noted that such circulation time distributions as of small values in the ratio, in other words broadly distributed, are preferable for mixing performance.

**Conclusion**

The mixing characteristics of high-viscosity liquid in stirred tanks were investigated by measuring the concentration response curves and the following conclusions were obtained.

(1) The concentration response curves experimentally obtained agreed well with those calculated from the circulation time distribution, and it was confirmed that the prediction of the response curve from the circulation time distribution is reasonable.

(2) The mixing characteristics obtained by experiments and calculations show a similar tendency. In the range of Reynolds number greater than 20, paddle or turbine impellers are suitable and in the smaller range, helical ribbon and helical screw impellers are preferable, while anchor type impellers are not recommended under the conditions of the present experiments.

(3) It was found that in the experimental results of the concentration response curve, the effect of diffusive mixing is significant. Particularly in the case of paddle or turbine impellers in the range of Reynolds number less than 10, the time constant of diffusive mixing becomes greater than that of convective mixing and diffusive mixing controls the overall rate of the mixing process.

(4) An attempt was made to correlate the param-
Nomenclature

\[ a, b, c = \text{parameter} \]
\[ C(t) = \text{concentration of liquid at } t \] [mole/m³]
\[ C(\infty) = \text{concentration of liquid at } t = \infty \] [mole/m³]
\[ D = \text{diameter of vessel} \] [mm]
\[ H = \text{liquid depth} \] [mm]
\[ k_e, K_d = \text{constant} \] [–]
\[ M_t = \text{mixing time index}, \left( \frac{[C(t) - C(\infty)]}{C(\infty)} \right) dt \] [–]
\[ m_{y_1}, m_{y_2} = \text{mean value of } y_1(t), y_2(t) \] [–]
\[ N = \text{rotation speed} \] [sec⁻¹]
\[ N_p = \text{power number} \] [–]
\[ N_{Re} = \text{Reynolds number}, \frac{pND^2}{\mu} \] [–]
\[ n = \text{parameter} \] [–]
\[ T, t = \text{time} \] [–]
\[ T_c = \text{(convective) mixing time constant} \]
\[ \frac{k_e}{k_2} \exp \left( -\frac{t}{T_c} \right) \] [–]
\[ T_d = \text{(diffusive) mixing time constant} \]
\[ \frac{k_2k_4}{k_4} \exp \left( -\frac{t}{T_d} \right) \] [–]
\[ T_M = \text{mixing time, the time required for} \]
\[ t \leq t_4 \text{ in the response curve} \] [–]
\[ y(t), y_1(t), y_2(t) = \text{circulation time distribution} \] [–]
\[ \alpha, \beta, \gamma = \text{parameter} \] [–]
\[ \delta(t) = \text{delta function} \] [–]
\[ \theta = \text{average circulation time} \] [sec]
\[ \sigma y_1, \sigma y_2 = \text{variance of } y_1(t), y_2(t) \] [–]
\[ \epsilon_1, \epsilon_2 = \text{deviations of response curve } C(t)/C(\infty) \]
\[ \text{from unity} \] [–]
\[ \varepsilon_a = \text{approved deviation} \] [–]
\[ \varepsilon_{y_1}, \varepsilon_{y_2} = \text{deviations of circumscribed curve of} \]
\[ \text{response curve } C(t)/C(\infty) \text{ from unity respectively in convective and diffusive mixing} \] [–]
\[ \mu = \text{viscosity} \] [poise]
\[ \rho = \text{density} \] [g/mm³]

Literature Cited


LAW OF THE WALL AND VELOCITY DEFECT LAW IN FULLY TURBULENT NON-BAFFLED AGITATED VESSEL

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The characteristic friction velocity in an agitated vessel is defined, based on the assumption that the flow field is affected by the condition at impeller tip. The velocity profiles are well correlated in terms of the dimensionless variables defined by using this friction velocity. The correlation equations are given for the wall vicinity, potential and impeller regions, respectively. Two of them may be regarded as law of the wall and velocity defect law in agitated vessels.

Introduction

The importance of agitated vessels in chemical processes has prompted many researchers to study their performance, and many excellent results have been reported. However, most papers were based on dimensional analysis, while those based on the phenomenological point of view are few.

Recently, a study of transport phenomena at the