BUBBLE FORMATION IN VISCOELASTIC LIQUIDS

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The injection of gas into a viscoelastic liquid is important in many industrial operations. However, there have been few investigation of bubble formation in viscoelastic liquids. This paper presents an analysis for predicting the size of bubbles formed directly at the nozzle tip. A gas jet or a thin gas film issuing from an orifice or a nozzle into a viscoelastic liquid breaks up into bubbles. In this paper, stability analyses for a cylindrical gas jet and a thin gas film in a viscoelastic liquid are also considered by use of linear perturbation theory.

1 Bubble Formation at Low Velocities in Viscoelastic liquids

An analysis is based on a two-stage bubble formation mechanism¹. The expression for the final volume may be written as

$$V_F = V_S + Q t_e \quad (1)$$

In the first stage, when only the inflation of the bubble takes place, the static bubble volume, V_S , is given by the equation

$$V_S = \pi d_N \sigma / (\rho - \rho^g) g \quad (2)$$

In the second stage, when the bubble rises and gets detached from the nozzle, a differential force balance on the bubble can be written as follows:

$$d(mv^*)/dt = F_g + F_K - F_D - F_a \quad (3)$$

$$F_g = V_F(\rho - \rho^g)g, F_a = \pi d_N \sigma, F_K = 4Q^2 \rho^g / \pi d_N^2$$

$$F_D = 3\pi \mu D_P v^* - (4\pi \mu v^{*3} / D_P) \{0.016(\lambda_1 - \lambda_2)(3\lambda_1 - \lambda_2)\}$$

Eq. (3) is used to determine the detachment time t_e . The drag force of a bubble in viscoelastic liquids is replaced with that of a solid sphere in liquids characterized by the Oldroyd three-constant model^{1,3}. Eq. (3) is linearized with the following assumptions^{2,5}.

(a) Mass is constant and is equal to the mass of the bubble at the end of the first stage except in the buoyancy force.

(b) The drag force due to expansion remains constant and is equal to the drag force on the bubble at the end of the first stage.

(c) The additional drag force due to the upward motion of the bubble is negligible compared to drag

force due to expansion.

Then Eq. (3) reduces to

$$dv/dt = At + B + C + D + E \quad (4)$$

$$A = \frac{Q(\rho - \rho^g)g}{m}, \quad B = -\frac{6\pi\mu}{m} \frac{Q}{4\pi(3/4\pi)^{1/3}} V_F^{-1/3}$$

$$C = 4Q^2 \rho^g / m \pi d_N^2, \quad D = Q^2 V_F^{-5/3} / 6\pi(3/4\pi)^{2/3}$$

$$E = \frac{2\pi\mu}{m} \frac{Q^3}{(4\pi)^3} (3/4\pi)^{-7/3} V_F^{-7/3} \{0.016(\lambda_1 - \lambda_2)(3\lambda_1 - \lambda_2)\}$$

It is assumed as the detachment condition that the length of the neck is equal to the diameter of the static bubble^{1,5}. This condition is employed to solve Eq. (4), which reduces to

$$(6V_S/\pi)^{1/3} = At^3/6 + (B + C + D + E)/2 \cdot t^2 \quad (5)$$

The elasticity of a surrounding liquid decreases bubble volumes as shown in Fig. 1.

2 Bubble Formation at High Velocities in Viscoelastic Liquids (Stability Analyses for a Gas Jet and a Thin Gas Film in Viscoelastic Liquids)

The equations of motion are

$$\rho(\partial \mathbf{v} / \partial t) = -\nabla p - [\nabla \cdot \boldsymbol{\tau}] + \rho \mathbf{g} \quad (6)$$

$$(\nabla \cdot \mathbf{v}) = 0 \quad (7)$$

We assume that the gas is incompressible and inviscid, and that the rheological properties of the viscoelastic liquid are described by the following constitutive equation:

$$\boldsymbol{\tau} + \lambda_1 (\partial \boldsymbol{\tau} / \partial t) = \mu \{ \boldsymbol{\Delta} + \lambda_2 (\partial \boldsymbol{\Delta} / \partial t) \} \quad (8)$$

The amplitude ζ of any resulting wave is given by the equation

$$\zeta = \zeta_0 e^{ikz + \alpha t} \quad (9)$$

where ζ_0 is the initial amplitude of the disturbance.

2.1 The stability of a cylindrical gas jet in a viscoelastic liquid

The dispersion relation between the growth rate of wave α and the wave number k for this case is obtained from the balance of forces at the interface of the two fluids:

$$\tau_{rr} - \tau_{\theta\theta} + p_a = 0 \quad \text{at } r \approx a \quad (10)$$

Levich's method⁴ for Newtonian jets surrounded by a flowing gas is used to obtain the forces affected on the interface, and the dispersion equation is obtained as follows:

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$$\alpha^2 + \frac{2\bar{\nu}k^2}{K_0(ka)} \left\{ -\dot{K}_1(ka) + \frac{2kl}{k^2+l^2} \frac{K_1(ka)}{K_1(la)} \dot{K}_1(la) \right\} \alpha$$

$$= \frac{\sigma k}{\rho a^2} (1-k^2 a^2) \frac{K_1(ka)}{K_0(ka)} \frac{l^2-k^2}{l^2+k^2}$$

$$+ \frac{\rho^g k^2 U^2}{\rho} \frac{I_0(ka)}{I_1(ka)} \frac{K_1(ka)}{K_0(ka)} \frac{l^2-k^2}{l^2+k^2} \quad (11)$$

where

$$l^2 = k^2 + \alpha/\bar{\nu} \quad \text{and} \quad \bar{\nu} = (1 + \alpha\lambda_2)/(1 + \alpha\lambda_1) \cdot \nu$$

When the modified kinematic viscosity of the liquid, $\bar{\nu}$, is small but not completely neglected and the wavelength is short, Eq. (11) is approximated by

$$\alpha^2 + 2\bar{\nu}k^2\alpha + (\sigma k - \rho^g U^2)/\rho \cdot k^2 = 0 \quad (12)$$

This result is quite similar to the dispersion equation for a viscoelastic liquid jet in air.

The relationship of growth rate and wave number of disturbance is shown in **Fig. 2**. It shows that the curve of α vs. k has a maximum and hence there is a disturbance at some wave number which grows most rapidly. Values of the dominant wave number $(ka)^*$ and the corresponding disturbance growth rate α^* decrease with increasing $\bar{\mu}$. Since for most fluids $\lambda_1/\lambda_2 > 1$, $\bar{\mu}/\mu$ is smaller than unity. Hence the elasticity of the surrounding liquid increases values of $(ka)^*$ and α^* . The breakup length of the jet is given by

$$L = \frac{U}{\alpha^*} \ln \frac{a}{\zeta_0} \quad (13)$$

This indicates that the effect of elasticity of the surrounding liquid is to destabilize a gas jet.

The diameter of bubbles formed from a gas jet is predicted by the following equation⁷⁾.

$$D_P = \{12a^3\pi/(ka)^*\}^{1/3} \quad (14)$$

It is seen from this equation that bubbles formed from a gas jet in an elastic liquid are smaller than those in a non-elastic liquid.

2.2 The stability of a cylindrical gas jet in a rotating viscoelastic liquid

This problem corresponds to that for the injection of gas into a stirred tank filled with a viscoelastic liquid.

$$\alpha^2 \left\{ 1 - \frac{2kl}{k^2+l^2} \frac{K_0(la)}{K_1(la)} \frac{K_1(ka)}{K_0(ka)} \right\} + \frac{2\bar{\nu}k^2}{K_0(ka)} \left\{ -\dot{K}_1(ka) + \frac{2kl}{k^2+l^2} \frac{K_1(ka)}{K_1(la)} \dot{K}_1(la) - \frac{l(k^2-l^2)}{k(k^2+l^2)} \frac{K_0(la)}{K_1(la)} K_1(ka) \right\} \alpha$$

$$= \left\{ ka\omega^2 + \frac{\sigma k}{\rho a^2} (1-k^2 a^2) \right\} \frac{l^2-k^2}{l^2+k^2} \frac{K_1(ka)}{K_0(ka)} + \frac{\rho^g k^2 U^2}{\rho} \frac{I_0(ka)}{I_1(ka)} \frac{K_1(ka)}{K_0(ka)} \frac{l^2-k^2}{l^2+k^2} \quad (18)$$

It is assumed that the Coriolis terms in the equation of motion are equal to zero, and that the wavelengths of deformations are large compared to the unperturbed radius of the jet at high viscoelasticity. The equations of motion then become

$$(E^2 - \alpha \bar{R}e)\Phi(r) = 0 \quad (15)$$

$$E^2(E^2 - \alpha \bar{R}e)\Psi(r) = 0 \quad (16)$$

where $E^2 = d^2/dr^2 - (1/r)d/dr - k^2$ and $\bar{R}e = a^2\omega/\bar{\nu}$.

The dispersion equation for this case is obtained

Fig. 1 Effect of elastic behavior of liquids on the size of bubbles formed directly at the nozzle tip

Fig. 2 Growth rate of wave number (a gas jet in a viscoelastic liquid)

from the following normal stress boundary condition on the interface⁸⁾:

$$\tau_{rr} - \zeta d\bar{p}/dr - \tau_{rr}^g + p_e = 0 \quad \text{at } r \approx a \quad (17)$$

where $d\bar{p}/dr$ is the mean pressure gradient. Then the dispersion equation is

The effect of rotation on stability is to increase the growth rate, to reduce the breakup length of the jet and to form smaller bubbles than in the non-rotating case.

For the non-rotating case, Eq. (18) becomes identical to Eq. (11).

2.3 The stability of a gas film in a viscoelastic liquid

A gas film moving in a liquid breaks up into bubbles.

Consider a sheet of thickness $2h$ moving with velocity U relative to the surrounding liquid, with the x -axis

Fig. 3 Growth rate of wave number (a gas film in a viscoelastic liquid, $We_g=10$)

along its centre line and the y -axis perpendicular to it. The boundary condition used to obtain the dispersion equation is

$$\tau_{yy} - \tau_{yy}^g + p_\sigma = 0 \quad \text{at } y \approx \pm h \quad (19)$$

The stability analysis for a very thin inviscid film investigated by Squire⁶⁾ is applied to that for this case.

Using Levich's method⁴⁾ for waves on the surface of a viscous liquid, we get the dispersion equation for the symmetrical wave.

$$\alpha^2 + 2\nu k^2 \left(1 - \frac{2lk}{l^2 + k^2} \right) \alpha = \left(\frac{\rho^g k^2 U^2}{\rho} \tanh kh - \frac{\sigma}{\rho} k^3 \right) \frac{l^2 - k^2}{l^2 + k^2} \quad (20)$$

For the anti-symmetrical oscillation, $\tanh kh$ in Eq. (20) is replaced by $\coth kh$.

The relationships of growth rate and wave number of disturbance in this case are shown in **Fig. 3**. Values of α^* for a symmetrical disturbance is larger than those for an anti-symmetrical disturbance. The effect of elasticity of the surrounding liquid is to destabilize a gas film and to reduce the size of bubbles formed from a gas film.

The formation of bubbles at the nozzle tip and the breakups of a gas jet and a gas film in a viscoelastic liquid were analyzed. It was concluded that the size of bubbles decreases with the elasticity of the surrounding liquid. In future, this problem will be studied experimentally.

Nomenclature

A	= constant as defined in Eq. (4)	[cm/sec ³]
a	= radius of jet	[cm]
B, C, D, E	= constants as defined in Eq. (4)	[cm/sec ²]
D_P	= diameter of bubble	[cm]
d_N	= diameter of nozzle	[cm]
F_D	= drag force	[dyn]

F_g	= buoyancy force	[dyn]
F_K	= force due to inflowing gas	[dyn]
F_σ	= force due to surface tension	[dyn]
g	= acceleration of gravity	[cm/sec ²]
$2h$	= thickness of film	[cm]
$I_n(x)$	= modified Bessel function of the first kind of order n	
$K_n(x)$	= modified Bessel function of the second kind of order n	
k	= wave number of disturbance	[cm ⁻¹]
L	= breakup length of jet	[cm]
l	= $(k^2 + \alpha/\nu)^{1/2}$	[cm ⁻¹]
m	= mass of bubble	[g]
p	= pressure	[g/cm ² ·sec ²]
$\hat{p}$	= mean pressure at the free surface	[g/cm ² ·sec ²]
p_σ	= stress due to surface tension	[g/cm ² ·sec ²]
Q	= volumetric flow rate	[cm ³ /sec]
Re	= Reynolds number ($= a^2 \omega / \nu$)	[—]
r	= radial coordinate direction	[cm]
t	= time	[sec]
t_c	= time of detachment	[sec]
U	= velocity	[cm/sec]
V_F	= volume of the final bubble	[cm ³]
V_S	= volume of the force balance bubble	[cm]
$\mathbf{v}$	= velocity tensor	[cm/sec]
v	= velocity of the centre of bubble	[cm/sec]
v^*	= velocity	[cm/sec]
We	= Weber number ($= 2aU^2\rho/\sigma$)	[—]
y	= distance measured normal to film	[cm]
Z	= Ohnesorge number ($= \mu/\sqrt{2a\rho\sigma}$)	[—]

α	= growth rate of disturbance	[sec ⁻¹]
$\mathbf{A}$	= rate of deformation tensor	[g/cm ² ·sec ²]
ζ	= amplitude of disturbance	[cm]
ζ_0	= initial amplitude of disturbance	[cm]
λ_1	= relaxation time	[sec]
λ_2	= retardation time	[sec]
μ	= viscosity	[g/cm ² ·sec]
ν	= kinematic viscosity	[cm ² /sec]
ρ	= density	[g/cm ³]
σ	= surface tension	[dyn/cm]
τ	= stress tensor	[g/cm ² ·sec ²]
Φ	= radial function in azimuthal velocity function	[cm ² /sec]
Ψ	= radial function in stream function	[cm ³ /sec]
ω	= angular velocity	[sec ⁻¹]

<Superscripts>

g	= dispersed phase
$-$	= modified by $(1 + \alpha\lambda_2)/(1 + \alpha\lambda_1)$
$\cdot$	= differential
$*$	= maximum growth rate

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