The model reduction methods based on the modal analysis approach offer such an important advantage that the inherent dynamic characteristics in the original system are successfully retained in the reduced models. Among these methods, the three model reduction techniques represented by Davison method, Marshall method, and the method of steady state approximation of fast mode state variable are not only simple but convenient for practical use. To demonstrate the applicability of these three model reduction techniques, the dynamic characteristics of the twelfth-order linearized model for a binary distillation column are investigated to compare with those of second-order reduced models.

Three Simple Model Reduction Techniques

Davison method is based on the assumption that the insignificant eigenvalues of the original system can be neglected and thus only the dominant eigenvalues of the original system are retained in the reduced model. The well known shortcoming of this method is that the reduced model usually shows an incorrect steady state. This shortcoming has been corrected by Chidambara whose method is proved to be identical to Marshall method.

In Marshall method, the original system is reduced to a lower-order model by neglecting the dynamics of the fast mode modal variables but by retaining the dominant eigenvalues of the original system in the reduced model. This approach offers the advantage of simplicity and zero steady-state error in the reduced model. Wilson et al. extended this idea proposed by Marshall to establishing the low-order control law for discrete dynamical systems.

The most important advantage of Davison and Marshall methods is of simplicity in the model reduction process. More sophisticated but complicated methods for determining reduced models based on the modal analysis approach are proposed by Gupta et al., Takamatsu et al., and Kuppurajulu et al. In these methods, the eigenvalues of the reduced models are not related directly to those of the original system but are determined by other criteria.

Another simple method for reducing the higher-order original system to a lower-order model is performed by using the notion of steady-state approximation of fast mode state variables. Though the model reduction process by this method, which is often used in the dynamic analysis of chemical processes, is as simple as the Davison and Marshall methods, the eigenvalues of the original system are not retained in the reduced model but the dynamics of the reduced model is directly related to the steady-state characteristics of the original system.

Numerical Examples

Three simple model reduction methods explained in the previous section are applied to determining the reduced dynamic models of a binary distillation column for which the following assumptions are made:

1) ideal stage for each tray and reboiler;
2) equimolar flow rates of liquid and vapor in both the enriching and stripping sections;
3) constant equimolar liquid holdup in each tray and negligible vapor holdup throughout the column;
4) constant composition and flow rate of boiling-state liquid feed.

The basic differential equations describing the dynamics of such an idealized binary distillation column as mentioned above are linearized around steady-state operating condition. Combining these linearized basic equations with a linear vapor-liquid equilibrium relation yields

$$\dot{x} = Ax + Bu$$  \hspace{1cm} (1)
\[ x = (x_{d1}, x_{d2}, x_{d3}, x_{e1}, x_{e2}, x_{e3}, x_{e4}, x_{e5}, x_{e6}, x_{e7}, x_{e8}, x_{e9}, x_{e10}); u = (\Delta L, \Delta V)^T \]

\[ A = H^{-1} \]

\[ B = H^{-1} \]

\[ H = \text{diag}(H_D, H_E, H_H, H_R, H_S, H_T) \]

Table 1: Eigenvalues of the twelfth-order original system described by Eqs. (1) and (2) and its second-order reduced models

<table>
<thead>
<tr>
<th></th>
<th>(H_D = 5)</th>
<th>(H_W = 10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original system</td>
<td>(-0.7989E00) (-0.8253E00) (-0.1273E01) (-0.1293E01)</td>
<td>(-0.5250E01) (-0.5143E02) (-0.1397E02) (-0.1385E03)</td>
</tr>
<tr>
<td>Reduced model by Davison or Marshall method</td>
<td>(-0.5085E02) (-0.3069E03) (-0.4237E02) (-0.4231E03)</td>
<td>(-0.5272E02) (-0.5270E03) (-0.7681E02) (-0.7656E03)</td>
</tr>
</tbody>
</table>
| Reduced model by SSA method | \(-0.7989E00\) \(-0.8253E00\) \(-0.1273E01\) \(-0.1293E01\) | \(-0.2524E04\) \(-0.2524E04\) |}

The step response characteristics of the second-order reduced models determined by the three methods are compared with those of the twelfth-order original system for the case shown in the first column of Table 1. Two examples of these comparisons are shown in Figs. 1 and 2, in which SSA is the abbreviation for the term “steady state approximation of fast mode state variables”. As obviously shown in these two figures, the step response characteristics of the reduced models based on Marshall and SSA methods are much closer to those of the original system than those based on Davison method.

A series of model reductions for the binary distillation column of 4 to 10 trays have confirmed that Marshall method usually leads to much more satisfactory results to the other two methods regardless of the size of the dimension of the original system. By increasing the dimension of the original system, the step response characteristics of the reduced models based on SSA method come nearer to those based on Marshall method, which approach more closely the step response characteristics of the original system. On the other hand, the Davison method, which has the inherent shortcoming of leading to an incorrect steady state, has been found to yield the most unfavorable step response characteristics even in the transient state.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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<tbody>
<tr>
<td>(F)</td>
<td>flow rate of feed to the column</td>
<td>[mol/min]</td>
</tr>
<tr>
<td>(H)</td>
<td>liquid holdup</td>
<td>[mol]</td>
</tr>
<tr>
<td>(L)</td>
<td>flow rate of liquid in the column</td>
<td>[mol/min]</td>
</tr>
<tr>
<td>(M)</td>
<td>number of trays in the stripping section</td>
<td></td>
</tr>
<tr>
<td>(N)</td>
<td>number of trays in the enriching section</td>
<td></td>
</tr>
<tr>
<td>(u)</td>
<td>control variable vector</td>
<td></td>
</tr>
<tr>
<td>(V)</td>
<td>flow rate of vapor in the column</td>
<td>[mol/min]</td>
</tr>
<tr>
<td>(x)</td>
<td>state variable vector</td>
<td></td>
</tr>
<tr>
<td>(x)</td>
<td>liquid composition of binary mixture</td>
<td>[mol fract]</td>
</tr>
</tbody>
</table>
Fig. 1 Comparison of step responses of second-order reduced models to the twelfth-order original system described by Eqs. (1) and (2); $\Delta x_D$ and $\Delta x_W$ to the positive step change of 0.1875 in $dL_S$

$A = \text{perturbation from a steady-state condition}$

$l(i=1,5) = \text{the number of trays in enriching or stripping section}$

$D, E, F, S, W = \text{the states in condenser, enriching section, feed, stripping section, and reboiler, respectively}$

**Literature Cited**


Fig. 2 Comparison of step responses of second-order reduced models to the twelfth-order original system described by Eqs. (1) and (2); $\Delta x_D$ and $\Delta x_W$ to the negative step change of 0.6 in $dV$
