TURBULENCE ENERGY SPECTRA IN BAFFLED MIXING VESSELS

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Turbulence energy spectra were measured for longitudinal and lateral components of fluctuating velocity, in baffled mixing vessels keeping geometrical similarity, by using a hot-film anemometer. From the comparison of the spectra, it was confirmed that the turbulence in the impeller stream deviated largely from both homogeneity and isotropy, especially in the lower wave-number range. However, these deviations were so small for the turbulence in other regions that the assumption of local isotropy would be applicable and that the homogeneity was implied to exist in the higher wave-number range. Energy spectra normalized by microscale λ and fluctuating velocity component u'_i showed good similarity and were not affected by location in the vessel, agitator speed and size of the mixing vessel. Then, effects on energy spectra by agitator speed and by the size were fairly represented by u'_i and by λ , respectively. The Kolmogoroff's universal similarity of spectrum was not observed except in the very high wave-number range.

Introduction

The main difficulty faced in application of the theories of turbulence to phenomena in mixing vessels comes from incomplete information about turbulence in mixing vessels, though some studies have been reported in literature^{3,7,8,11,13}.

In a previous paper¹⁰, behaviors of time-mean velocity and fluctuating velocity in baffled mixing vessels were reported. In this paper, structure of turbulence in baffled mixing vessels is discussed through measurement of turbulence energy spectra.

Theory of Turbulence^{2,6)}

One-dimensional energy spectrum E_i of a fluctuating velocity component u_i is given by the following equation.

$$u_i'^2 = \int_0^\infty E_i(k_1) dk_1 \quad (i=1, 2, 3) \quad (1)$$

where u'_i is root-mean-square value of u_i , k_1 is wave-number related to fluctuating frequency n by $k_1 = 2\pi n/\bar{U}_1$ and the suffix i means the direction of u_i ; u_1 is in the direction of time-mean velocity and u_2 and u_3 are in the directions normal to it.

For isotropic turbulence, Eqs. (2)–(4) are given.

$$E_2(k_1) = E_3(k_1) = \frac{1}{2} \left[E_1(k_1) - k_1 \frac{\partial}{\partial k_1} E_1(k_1) \right] \quad (2)$$

$$\lambda_f = \sqrt{2} \lambda_g \quad (3)$$

$$\varepsilon = 30\nu(u_1'^2/\lambda_f^3) = 15\nu(u_2'^2/\lambda_g^3) \quad (4)$$

$$\frac{1}{\lambda^2} = \frac{1}{2u_i'^2} \int_0^\infty k_1^2 E_i(k_1) dk_1 \quad (5)$$

(put $i=1$ for λ_f and $i=2$ or 3 for λ_g)

where λ is micro scale defined by Eq. (5), ε is the energy dissipation rate of turbulence and ν is the kinematic viscosity.

For actual turbulence which does not show perfect isotropy, the definitions of local isotropy and local homogeneity by Kolmogoroff are effective. Where isotropy is seriously deviated from in actual cases, Eq. (4) is not valid to calculate the dissipation rate, and the following equation is applied, based in the theory of axisymmetric turbulence by Batchelor.¹⁾

$$\varepsilon = \frac{\alpha}{\alpha + \beta} \left(30\nu \frac{u_1'^2}{\lambda_f^3} \right) + \frac{\beta}{\alpha + \beta} \left(15\nu \frac{u_2'^2}{\lambda_g^3} \right) \quad (6)$$

where the values of $\alpha=1$ and $\beta=2$ are used.

Kolmogoroff also proposed the following scales for the universal similarity of energy spectra:

$$\text{length scale} \quad \eta = (\nu^3/\varepsilon)^{1/4} \quad (7)$$

$$\text{velocity scale} \quad \nu = (\nu\varepsilon)^{1/4} \quad (8)$$

and the following spectrum law for the inertial sub-range.

$$E_i(k_1) = \text{const.} \varepsilon^{2/3} k_1^{-5/3} \quad (k_1 \ll 1/\eta) \quad (9)$$

Experimental

Three mixing vessels, 15 cm, 30 cm and 60 cm in diameters were used in this study. The three were geometrically similar and the vessel diameter D was taken as the representative dimension. The shape

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Fig. 1 Mixing vessel and measuring points

and dimensions of the vessel and impeller are shown in Fig. 1. The working liquid was water.

The measurement of energy spectra was performed using a constant temperature hot-film anemometer system. This system consisted of a DISA type 55D01 anemometer unit, type 55D10 linearizer, type 55D25 auxiliary unit with band filters, type 55D35 RMS voltmeter and type 55A83 hot-film probe. The probe sensor was a quartz-coated thin nickel film, 1 mm long and 0.2 mm wide. The probe was calibrated before and after each run using a calibration equipment which consisted of diverging and converging nozzles. The responsive frequency range of this anemometer system was from DC to about 30 kHz. The electronic noise was discarded by a low-pass filter of 20 kHz. Details on these electrical circuits and the probes are given elsewhere^{4,5}.

From the movement of micro particles suspended in the mixing vessel, it was confirmed that two-dimensional horizontal flow regions occupied fairly large space in the vessel. In the measurement of turbulence in these regions, the film was placed in a horizontal plane with a probe support inserted vertically through the top cover of the vessel. After the flow direction was determined by rotating the probe about the support axis, the turbulence signals were analyzed with the band filters to obtain longitudinal and lateral energy spectra, E_1 and E_2 . Here, the longitudinal spectrum means the spectrum of fluctuating velocity in the flow direction at the location, and the lateral means that in the direction normal to it in the horizontal plane.

As shown in Fig. 1, the measurement was made in the four vertical sections I–IV, and a measuring point was indicated with height from the vessel bottom, H , and radial distance from the center axis, r . In this figure, the five typical measuring points P_1 , P_2 , P_3 , P_1' and P_3' are also shown.

Fig. 2 Energy spectra of u_1 and u_2 ($D=30$ cm, $N=160$ r.p.m.)

Result and Discussion

1 Isotropy and homogeneity

In Figs. 2a–2c, measured E_2 -curves are compared with the curves of E_2^0 calculated from measured E_1 by Eq. (2) for isotropic turbulence. The difference between E_2 and E_2^0 means a degree of nonisotropy, and the two must overlap in the higher wave-number range for locally isotropic turbulence⁶.

As shown by the failure to agree of E_2 and E_2^0 in Fig. 2a, nonisotropy of the turbulence at P_1 in the impeller stream is very large in the lower wave-number range and remains as even at the high wave-number of 100 cm^{-1} . However, the fact that the nonisotropy becomes smaller in the higher wave-number range implies local isotropy. As can be seen from Figs. 2b and 2c, at P_2 and P_3 , in regions other than the impeller stream, local isotropy almost exists from the much lower wave-number of 10 cm^{-1} .

In Fig. 3, E_1 -curves for several measuring points are compared to examine the homogeneity of turbulence. As E_1 for P_1 is much larger than the others, especially in the lower wave-number range, nonhomogeneity is high for the low wave-number turbulence in the impeller stream. In the higher wave-number range, all the curves approach one another and imply the homogeneity in the higher wave-number range.

E_2 -curves also show the tendency to approach one another in the higher wave-number range. However, E_2 is more homogeneous than E_1 , because Figs. 2a–2c show that, at P_1 , E_2 is much smaller than E_1 while, at P_2 and P_3 , E_2 is a little smaller than E_1 .

These observations can be summarized as follows. The turbulence in the impeller stream deviates largely from both isotropy and homogeneity, especially in the lower wave-number range. In other regions, these deviations become smaller, so that the turbulence is regarded as isotropic in the higher wave-

Fig. 3 Energy spectra of u_1 at several points ($D=60$ cm, $N=80$ r.p.m.)

number range and homogeneous in the much higher wave-number range.

Accordingly, the nonhomogeneity and even the nonisotropy should be taken into account for more precise analyses of mixing vessels, although the assumption of local isotropy in the whole of the vessel is applicable as a first approximation.

2 Change of energy spectrum

2.1 Effect of agitator speed As shown in Fig. 4, the agitator speed has little effect on the curves of spectra $E_i/u_i'^2$. This means that in changing agitator speed, log-log plotted E_i -curves are little transformed, and translated parallel to the ordinate axis alone due to change of $u_i'^2$. For this reason, spectrum $E_1/u_1'^2$ is treated hereafter.

2.2 Micro scale, λ The log-log plotted $E_1/u_1'^2$ -curves measured at different locations or in the vessels different in size were not identical as those in Fig. 4, but were nearly of the same shape. This leads one to consider that the $E_i/u_i'^2$ -curves are also shifted depending on measuring conditions except for agitator speed.

From Eq. (1) any spectrum must satisfy the following condition.

$$\int_0^\infty E_i/u_1'^2 \cdot dk_1 = 1 \quad (10)$$

Therefore, a parallel translation of a $E_1/u_1'^2$ -curve is possible only along a line of slope -1 . Here, supposing a curve of $E_i(k_1)/u_1'^2$ is shifted along the abscissa axis by distance $\log x$ and along the ordinate axis by distance $-\log x$, then the change of micro scale becomes as follows, from Eq. (5).

$$[E_i(k_1)/u_1'^2]_{pt} = E_i(k_1/x)/xu_1'^2 \quad (11)$$

Fig. 4 Energy spectra of u_1 at several agitator speeds ($D=30$ cm, at P_1')

Fig. 5 Longitudinal micro scale and agitator speed ($D=30$ cm)

$$\begin{aligned} \frac{1}{(\lambda_f)_{pt}^2} &= \frac{1}{2} \int_0^\infty k_1^2 [E_i(k_1)/u_1'^2]_{pt} dk_1 \\ &= \frac{x^2}{2} \int_0^\infty (k_1/x)^2 E_i(k_1/x) / xu_1'^2 \cdot dk_1 \\ &= \frac{x^2}{\lambda^2} \end{aligned} \quad (12)$$

$$x = \lambda / (\lambda_f)_{pt} \quad (13)$$

where the suffix pt means the result of the parallel translation of $E_i/u_1'^2$. Eq. (13) shows that the translation is expressed by a change of λ .

The effect of agitator speed, stated above, shows that λ_f is affected little by agitator speed. The result is presented in Fig. 5. At P_1' near the impeller tip, λ_f is nearly constant and independent of agitator speed, though some deviation from constancy is seen in the regions away from the impeller, at P_3 for example. The tendency that λ_f was independent of agitator speed was observed in a nonbaffled mixing vessel⁹.

2.3 Effect of location Values of λ_f exhibit much dependence on location in the vessel, as shown in

Fig. 6 Longitudinal micro scale distribution ($D=60$ cm, $N=80$ r.p.m., section II)

Fig. 6, which shows that λ_f is quite similar to u'_1 in distribution. This is based on the following, supposing, that the E_1 -curve at the location where u'_1 is smaller can be obtained by shifting suitably the E_1 -curve at the location where u'_1 is larger; then, the latter curve would be shifted downwards because of the smaller value of u'_1 of the former. For homogeneity to exist in the very high wave-number range of the two curves, this shift should be accompanied by a shift to the higher wave-number side. As a result, from Eq. (13), λ_f is smaller where u'_1 is smaller.

2.4 Effect of apparatus size Generally, value of λ depends on a given size of the apparatus as well as on location¹²⁾. The values of λ_f in this study were 0.05–1.04 mm in the 15 cm vessel, 0.08–2.22 mm in the 30 cm vessel and 0.18–4.38 mm in the 60 cm vessel. Apparently, λ_f is proportional to the vessel size though it is not known which size of the mixing apparatus affects λ_f , for the three vessels are similar in shape. According to Eq. (13), this change of λ_f means that when a mixing vessel is scaled up the energy spectra are shifted to the lower wave-number side.

The difference between the effect on spectra by agitator speed and that by apparatus size implies that the scale-up method of keeping the energy spectra the same in all wave-number ranges is impossible. The spectrum shift by the scale-up effect can not be compensated by adjusting agitator speed, which is effective only for the shift along the ordinate axis. Hence, there can not be only one criterion that is applicable to all the turbulence phenomena in mixing vessels.

2.5 Lateral micro scale, λ_g λ_g , which characterizes a E_2 -curve, was similar to λ_f in behavior. Ratios of λ_f/λ_g were 1.5–2.5 in the impeller stream and 1.0–1.4 in other regions. Accordingly, Eq. (3), the condition of isotropy, is satisfied only as a rough approximation.

3 Similarity of energy spectra

According to Kolmogoroff, there exists a universal similarity of energy spectra normalized by η and ν for fully developed turbulence. On the other hand, the parallel translation of spectrum curves, stated above, is based on a tacit assumption of spectrum similarity by the scales λ and u'_1 .

The former is examined in Figs. 7a, 7b and 7c with parameters of location, agitator speed and apparatus size, respectively. Then, η and ν were evaluated by using Eqs. (6)–(8). The normalized E_1 -curves in Fig. 7b show good similarity, but those in Figs. 7a and 7c show similarity only in the higher wave-number range, say $k_1\eta > 0.2$. That is, the universal similarity can exist only in this range and it is largely deviated from in the lower wave-number range, due to particular effects of location and apparatus size. In Figs. 4 and 7b, with the same parameter of agitator speed, a range of slope $-5/3$ is observed over one decade of wave-number. But this does not mean the existence of an inertial subrange, for the universal similarity in this range is deviated from in Figs. 7a and 7b.

It is evident from the above that a turbulence phenomenon observed with changing agitator speed alone seems to be consistent with a theoretical analysis based on the Kolmogoroff's spectrum law, Eq. (9), but the experimental result leave unsettled the effects of location and apparatus size.

In Figs. 8a–8c, the other similarity is confirmed with the same data and parameters used in Figs. 7a–7c. Curves of E_1 normalized by λ_f and u'_1 show good similarity in each case of the three parameters. Hence, all the E_1 -curves in geometrically similar mixing vessels can be reduced to one curve independent of location, agitator speed and apparatus size. For the E_2 -curves, scales λ_g and u'_2 are used instead of λ_f and u'_1 .

From the present data, it can not be said which of the two methods of similarity is more effective for the turbulence of the very high wave-number that has not been measured in this study. Accordingly, scales λ and u'_1 will be useful, at least, in considering the turbulence phenomena in mixing vessels which are mainly determined by the turbulence of the lower wave-number than $1/\eta$.

Conclusion

1) The turbulence in the impeller stream was largely nonisotropic and nonhomogeneous, especially in the lower wave-number range. In regions other than the impeller stream, the turbulence was nearly or locally isotropic and roughly homogeneous, the ratios of λ_f/λ_g were 1.0–1.4.

2) Energy spectra normalized by the scales λ and u'_1 showed good similarity independent of location,

a) location as a parameter
($D=60$ cm, $N=160$ r.p.m.)

b) agitator speed as a parameter
($D=30$ cm)

c) apparatus size as a parameter

Fig. 7 Energy spectra of u_1 normalized by η and ν

a) location as a parameter
($D=30$ cm, $N=160$ r.p.m.)

b) agitator speed as a parameter
($D=30$ cm)

c) apparatus size as a parameter

Fig. 8 Energy spectra of u_1 normalized by λ_f and u_1'

agitator speed and apparatus size under geometrically similar condition of apparatus. Thus, the effect on the spectra by location were expressed by both λ and u_1' , and the effects by agitator speed and by apparatus size were expressed by u_1' and λ , respectively.

3) The Kolmogoroff's universal similarity of energy spectra was not observed except in the very high wave-number range, due to the effects of location and apparatus size.

Nomenclature

B_w = width of baffle plate [cm]

b = blade width of impeller [cm]
 c = blade length of impeller [cm]
 D = vessel diameter [cm]
 d = impeller diameter [cm]
 E_i = energy spectrum of $u_i'^2$ in wave-number space [cm³/sec²]
 H = height from vessel bottom [cm]
 H_v = vessel height [cm]
 k_1 = wave-number, $2\pi n/\bar{U}_1$ [cm⁻¹]
 N = agitator speed [r.p.m.]
 n = fluctuation frequency [Hz]
 n_B = number of baffles [-]
 n_p = number of impeller blades [-]
 r = radial distance from center axis [cm]

$\bar{U}_i$	= time-mean velocity component	[cm/sec]
u_i	= fluctuating velocity component	[cm/sec]
u_i'	= root-mean-square value of u_i	[cm/sec]
ϵ	= energy dissipation rate per unit mass of fluid	[cm ² /sec ³]
η	= Kolmogoroff's length scale	[cm]
λ	= micro scale	[cm]
λ_f	= longitudinal micro scale	[cm]
λ_g	= lateral micro scale	[cm]
ν	= kinematic viscosity	[cm ² /sec]
υ	= Kolmogoroff's velocity scale	[cm/sec]

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