THE EFFECT OF INTERFACIAL TURBULENCE ON MASS TRANSFER

— AN EXPRESSION OF THE ENHANCEMENT FACTOR —

NOBUYUKI IMAISHI, MITSUNORI HOZAWA*, AND KATSUHIKO FUJINAWA
Chemical Research Institute of Non-Aqueous Solutions, Tohoku University, Sendai, 980

When cellular convections are induced by the mass transfer through an interface between two fluids, the rate of the mass transfer usually increases as compared with the undisturbed state. The increase of the mass transfer rate owing to interfacial turbulence has been formulated, adopting the concept of the enhancement factor $R^{4-7)}$. This factor is defined as the ratio of the mass transfer coefficient of the disturbed system to that of the standard system; standard system means the state where no interfacial turbulence exists.

This paper presents a general expression of the enhancement factor derived based upon the surface renewal model with the unsteady state age distribution functions.

Theoretical

Expression of the enhancement factor

It will be useful to suppose a mass transfer system in which (a) the main resistance of the mass transfer is in one fluid side and (b) the concentration driving force Δc is constant because of the sufficient bulk volumes of both fluids. With the theory of unsteady state diffusion, the mass transfer rate N_τ of the fluid element with the age τ is expressed by Eq. (1).

$$N_\tau = \Delta c \sqrt{D/\pi\tau} \quad (1)$$

The mass transfer rate $N_o(t)$ of the standard system whose surface age distribution function is $\phi_o(t, \tau)$ is expressed by Eq. (2).

$$N_o(t) = \int_0^\infty N_\tau \phi_o(t, \tau) d\tau \quad (2)$$

It is assumed that the surface renewals induced by the interfacial turbulence are superinduced on the surface renewals owing to the bulk turbulence. The mass transfer rate $N_d(t)$ of the disturbed system is expressed by Eq. (3).

$$N_d(t) = \int_0^\infty N_o(\tau) \phi_d(t, \tau) d\tau \quad (3)$$

Here, $\phi_d(t, \tau)$ is the age distribution function of the

surface elements owing to the renewals induced by the interfacial turbulence.

The enhancement factor R is then expressed by the following equation.

$$R = k_d/k_o = N_d(t)/N_o(t) \quad (4)$$

Unsteady state age distribution function

Two cases of surface renewals can be considered. One is the case in which the renewals of surface elements occur independently of age with same probabilities. This type of surface renewals is called the Danckwerts type¹⁾. The other is the case in which the renewals of surface elements occur always to the elements with the oldest age. This is the Higbie type³⁾.

According to Tadaki and Maeda⁷⁾, the unsteady state age distribution function of the Danckwerts type is expressed by the following equation.

$$\phi(t, \tau) = e^{-st} \delta(\tau - t) + s e^{-s\tau} \{1 - u(\tau - t)\} \quad (5)$$

Here, s is the average rate of the renewals.

In the Higbie type, the age distribution function is given as follows, the time required for a renewal being θ .

$$t \leq \theta; \phi(t, \tau) = (1/\theta) \{1 - u(\tau - t)\} + \delta(\tau - t)(1 - t/\theta) \quad (6-a)$$

$$t > \theta; \phi(t, \tau) = (1/\theta) \{1 - u(\tau - \theta)\} \quad (6-b)$$

Derivation of enhancement factor

One may consider the case in which the Danckwerts type surface renewals of the rate s_d induced by the interfacial turbulence are superinduced on the Danckwerts type surface renewals of the rate s_o owing to bulk turbulence. In this case, both $\phi_o(t, \tau)$ and $\phi_d(t, \tau)$ are expressed by Eq. (5).

Considering the effect of interfacial turbulence in the steady state, the enhancement factor R for this case is expressed by Eq. (7) according to Eqs. (1)~(4) and (5).

$$R = \sqrt{1+a} \quad (7)$$

a is the ratio of the surface renewal rate of the interfacial turbulence to that of the standard system.

$$a = s_d/s_o \quad (8)$$

With the same procedures as used in the derivation

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Table 1 Expressions of Enhancement Factor

Standard system	Interfacial turbulence	Definition of a	Correlation between R and a
Dankwerts type $s=s_o$ $\phi_o(t,\tau)$; Eq. (5)	Dankwerts type $s=s_d$ $\phi_d(t,\tau)$; Eq. (5)	$a=s_d/s_o$	$R=\sqrt{1+a}$ (I) (=Eq. (7))
	Higbie type $\theta=\theta_d$ $\phi_d(t,\tau)$; Eq. (6-b)	$a=1/s_o\theta_d$	$R=\sqrt{a/\pi} \exp(-1/a) + (1+a/2)\text{erf}(1/\sqrt{a})$ (II) ⁵⁾
Higbie type $\theta=\theta_o$ $\phi_o(t,\tau)$; Eqs. (6-a) and (6-b)	Dankwerts type $s=s_d$ $\phi_d(t,\tau)$; Eq. (5)	$a=s_d\theta_o$	$R=(\sqrt{\pi a}/2)\{e^{-a/\pi} + (1+1/2a)\text{erf}(\sqrt{a})\}$ (III) ^{6,7)}
	Higbie type $\theta=\theta_d$ $\phi_d(t,\tau)$; Eq. (6-b)	$a=\theta_o/\theta_d$	$R=1+a/3, a \leq 1$ $R=\sqrt{a} + (1/3)\sqrt{a}, a > 1$ (IV) ⁴⁾
Film type The thickness is B $\phi_o(t,\tau)$; Eq. (10)	Dankwerts type $s=s_d$ $\phi_d(t,\tau)$; Eq. (5)	$a=B^2s_d/D$	$R=1+2\sum_{n=1}^{\infty} 1/(1+n^2\pi^2/a)$ $=\sqrt{a} \coth(\sqrt{a})$ (V) ^{2,8)}
	Higbie type $\theta=\theta_d$ $\phi_d(t,\tau)$; Eq. (6-b)	$a=B^2/D\theta_d$	$R=1+a/3 - (2a/\pi^2) \sum_{n=1}^{\infty} (1/n^2) \exp(-n^2\pi^2/a)$ $=(2\sqrt{a/\pi})\{1+2\sqrt{\pi} \sum_{n=1}^{\infty} \text{ierfc}(n\sqrt{a})\}$ (VI) ⁸⁾

Fig. 1 Correlations between R and a

of Eq. (7), one can readily derive the expressions of R for the other three cases. The equations are shown in Table 1 together with Eq. (7).

Application to the standard system represented by the film theory

It can be considered that there is a diffusion film of thickness B on the interface of the standard system and that no surface element is renewed. When interfacial turbulence is induced in this case, the elements of the surface film of thickness B may be renewed according to either the Dankwerts type or the Higbie type.

Instead of Eq. (1), N_τ is now expressed by Eq. (9-a) or (9-b)^{2,8)}.

$$N_\tau = \Delta c(D/B) \left\{ 1 + 2 \sum_{n=1}^{\infty} \exp(-n^2\pi^2 D\tau/B^2) \right\} \quad (9-a)$$

$$N_\tau = \Delta c\sqrt{D/\pi\tau} \left\{ 1 + 2 \sum_{n=1}^{\infty} \exp(-n^2B^2/D\tau) \right\} \quad (9-b)$$

In this case, $\phi_o(t, \tau)$ is expressed by Eq. (10).

$$\phi_o(t, \tau) = \delta(\tau - t) \quad (10)$$

The expressions of R for the film type standard system can be obtained from Eqs. (2)~(6), (9) and (10). The equations are included in Table 1, though

the expressions were previously solved mathematically^{2,8)}.

Discussion

It will be noted from the equations in Table 1 that the expressions composed of Eqs. (1)~(4), include the formulas for R reported previously⁴⁻⁷⁾. Eq. (7) is the first expression for the case in which the Dankwerts type surface renewals by the interfacial turbulence are superinduced on the Dankwerts type surface renewals of the standard system. The correlations between R and a are shown in Fig. 1.

The application of these expressions to an actual apparatus requires knowledge of both the fluid dynamics of the standard system and the characteristic of the interfacial turbulence. The selection from the expressions would have a little effect on the final result, however, since all correlations are closely adjacent to one another in Fig. 1.

Conclusion

A general expression for the enhancement factor has been proposed on the basis of the surface renewal model, adopting the unsteady state age distribution

functions. A new expression of R has been derived for the case in which the Danckwerts type surface renewals by the interfacial turbulence are superinduced on the Danckwerts type surface renewals of the standard system. R for the film type standard system are also given.

Nomenclature

a	= modulus defined by Eq. (8) or in Table 1	[—]
B	= thickness of diffusion film	[m]
Δc	= concentration driving force	[mol·m ⁻³]
D	= diffusion coefficient	[m ² ·s ⁻¹]
k	= mass transfer coefficient	[m·s ⁻¹]
$N(t)$	= average mass transfer rate	[mol·m ⁻² ·s ⁻¹]
N_τ	= mass transfer rate of element with age τ	[mol·m ⁻² ·s ⁻¹]
R	= enhancement factor	[—]
s	= average surface renewal rate	[s ⁻¹]
t	= time	[s]

$u(t)$	= unit step function	[s ⁻¹]
$\delta(t)$	= unit impulse function	[s ⁻¹]
θ	= time required for a renewal	[s]
τ	= age of element	[s]
$\phi(t, \tau)$	= age distribution function	[s ⁻¹]

<Subscripts>

d	= disturbed system
o	= standard system

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GAS-SIDE MASS TRANSFER COEFFICIENT IN ABSORPTION OF CARBON DIOXIDE INTO WATER JET — EFFECT OF DESORPTION OF AIR FROM WATER —

HAKARU MITSUTAKE* AND MASASHI SAKAI

Department of Industrial Chemistry, Kyushu Sangyo University, Fukuoka, 813

Several experiments¹⁻⁹⁾ have been reported on gas-liquid mass transfer in water jet and cocurrent flow of liquid. Among them, Onda *et al.*^{4,5)} have reported on the gas-side resistance in the absorption of gas, without taking into account the desorption of air from liquid.

This experiment has been performed to investigate the effect of the accumulated air in gas phase by desorption of air from water during absorption, on the gas-phase mass transfer coefficient, k_G by setting a ratio N_B/N_A in terms of p_{BM}/P .

Theoretical

The following assumptions are made in this work.

1) The concentration of X_{AS} at the interface and that of X_{AO} at the main flow of gas are taken as constant in order to solve the problem simply, in spite of the fact however, that they vary very slightly in the direction of flow of liquid.

2) Instantaneous equilibrium exists at the interface. Therefore the following equation follows.

$$(P/RT)v_A^* - \partial n_A / \partial \theta = N_A A \quad (1)$$

$$(P/RT)v_B^* - \partial n_B / \partial \theta = -N_B A, \quad v_A^* - v_B^* = v^*, \quad v_B^* = 0 \quad (2)$$

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where

$$\left. \begin{aligned} n_A &= (PV/RT)X_{AO} \\ n_B &= (PV/RT)X_{BO} = (PV/RT)(1 - X_{AO}) \\ P, V &= \text{constant} \end{aligned} \right\} \quad (3)$$

From Eqs. (1), (2) and (3)

$$N_A = \frac{P}{RTA} \left(v^* - V \frac{\partial X_{AO}}{\partial \theta} \right) \quad (4)$$

$$N_B = -\frac{PV}{RTA} \frac{\partial X_{BO}}{\partial \theta} = \frac{PV}{RTA} \frac{\partial X_{AO}}{\partial \theta} \quad (5)$$

Let

$$\alpha = \frac{N_B}{N_A} = -1 + v^* \left/ \left(v^* - V \frac{\partial X_{AO}}{\partial \theta} \right) \right. \quad (6)$$

Then, from Eqs. (4) and (6)

$$N_A = \frac{Pv^*}{RTA} \frac{1}{1 + \alpha} \quad (7)$$

On the other hand, N_A in liquid film is

$$N_A = \bar{C} \cdot Q_L / A = K_L \cdot \Delta C_{lm} \quad (8)$$

Where

$$\Delta C_{lm} = 2\bar{C} / \ln \frac{C^* - C_o}{C^* - C_2} \quad (9)$$

K_L can be obtained by using the experimental values of N_A and ΔC_{lm} . Besides, k_G in different