A Study of Proof Testing: Unloading Time Expressed in the Term of an "Equivalent Service Time"

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保証試験の研究：“等価使用時間”で表された除荷時間

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Proof test is useful for rejecting ceramics of undesirable short life time. But the crack in the proof test survivors propagates in the unloading period of the proof stress. So the shortest life time of the survivors may be shorter than the needed life time. In this paper, the influence of unloading time on the shortening of the life time of the proof test survivors was studied on the basis of fracture mechanics of ceramics. Some formulae are derived for expressing the shortened life time of the survivor during unloading, the ratio of the shortened life time to the needed one, and the longest unloading time allowable to make the shortest life time of the survivors longer than the needed one. According to these formulae, the unloading time gives substantial influence on the life time of the survivors from the point of view of the shortening effect for the life time. The formulae expressing the ratio of shortened life time to the needed one tells that the unloading time must be of the order of $10\ \mu\text{s}$ to make the shortest life time negligibly small compared with the needed one, when the proof stress is selected so as to be the value which corresponds to the needed life time. In an actual application of proof test to ceramics, some safety factor is multiplied to the proof stress, so the safety factor is studied in correlation with the unloading time. According to the study, the unloading time can be increased in proportion to $\Delta p/(1+\Delta p)^2$, where $(1+\Delta p)$ is the safety factor. Then, the unloading time reaches the maximum at the value of 2, in an approximation. But even when 2 is selected as the safety factor, the unloading time must be of the order of $5\text{--}10\ \mu\text{s}$ to make the shortest life time of the survivors equal to or longer than the needed value. Such a short unloading time seems to be inapplicable to the test. Therefore, it can be concluded that the complete elimination of the ceramics having undesirable short life time needs another complementary method such as AE monitoring for proof test.

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1. Introduction

The importance of proof testing in the structural ceramic industries is now well understood, and the test is adopted in several ceramic makers as well as consumers to prove the life time of a ceramics.

To assure the reliability of the test, some attention must be added to the testing conditions, such as temperature, humidity, load, contact stress caused by testing load, unloading time, and so on. Effect of temperature, humidity and load on the test are studied experimentally and theoretically by some authors.^{1),2)} About the unloading time, Evans and Wiederhorn³⁾ studied the influence of the time on the strength degradation of the ceramics submitted to the test. On the basis of this work, the distribution of the failure stress of the proof test survivors is given, and the life time distribution is given also. The procedure to evaluate the latter, however, is a little tedious. And some simpler methods are needed to evaluate the influence of the time on the life time degrada-

tion. A method for that is to express the unloading time in the term of the "equivalent service time," which corresponds to the time shortened during the unloading period.

In this work, the equivalent time was studied on the basis of the slow crack growth of ceramics.

2. Theory

2.1 The equivalent service time to the unloading time

The substantial object of the proof test is to select the ceramics which have the initial crack length shorter than some fixed value, α_p . The ceramics which survived the test are desired to have the life time, L_s , longer than some needed minimum value which depends on the value of α_p . Then, the criterion on the unloading time of the proof stress will be given by studying the propagation of the crack of the length of α_p .

The proof stress, σ_p , is selected so as to break the ceramics having a crack longer than α_p , or it is determined from the following equation,

$$Y\sigma_p\alpha_p^{1/2}=K_{IC} \quad (1)$$

where Y stands for the geometrical factor for the stress intensity factor, and K_{IC} for the critical stress intensity factor of the ceramics.

The length of the largest crack in the ceramics tested is α_p at the beginning of unloading period of the test stress. In the period of unloading, the crack grows gradually resulting the service time shorter than the expected one, L_s . The value of the growth is given from the following equation, which is an integral expression for the slow crack growth⁽¹⁾ based on the fracture mechanics of ceramics.

$$\alpha_p^{(2-n)/2}\alpha_{pe}^{(2-n)/2}=H\int_0^{t_{pu}}\sigma^n(t)dt \quad (2)$$

$$H=\frac{n-2}{2}AY^n \quad (3)$$

where α_{pe} , $\sigma(t)$ and t_{pu} represent respectively, the crack length at the final moment of unloading, the stress in the unloading period, and the unloading time, and A and n stand for material constants expressing the crack velocity, V , as a function of stress intensity factor, K_I , in the form of $V=AK_I^n$ in the region of K_I just below K_{IC} .

In most cases, $\sigma(t)$ is expressed as a linear function of time, t , or that of square of t , as follows;

$$\sigma(t)=\sigma_p\left(1-\frac{t}{t_{pu}}\right) \quad (4)$$

or

$$\sigma(t)=C\omega_p^2\left(1-\frac{t}{t_{pu}}\right)^2 \quad (5)$$

$$\sigma_p=C\omega_p^2 \quad (6)$$

where C and ω_p are constants. Unloading expressed by Eq. (4), occurs in the proof test in which the decreasing rate of the proof stress in the unloading period is fixed constant. That expressed by Eq. (5), occurs frequently in the proof test in which the test stress is applied with centrifugal force and the decreasing rate of the rotational speed in the unloading period is fixed constant. And in the proof test, ω_p represents the top speed of the rotation, and the proof stress, σ_p , is expressed by Eq. (6).

In the unloading period, the crack of the length of α_p grows and it reaches finally the length α_{pe} . α_{pe} is given by combining Eqs. (2), with (4) or (5), as follows;

$$\alpha_p^{(2-n)/2}-\alpha_{pe}^{(2-n)/2}=\frac{H}{n+1}\sigma_p^n t_{pu} \quad (7)$$

for σ_p expressed by Eq. (4)

or

$$\alpha_p^{(2-n)/2}-\alpha_{pe}^{(2-n)/2}=\frac{H}{2n+1}\sigma_p^n t_{pu} \quad (8)$$

for σ_p expressed by Eq. (5)

The equivalent service time, t_{equ} , would be

defined as the service time in which the crack of the length of α_p grows to the value of α_{pe} under the service environment and under the service stress of σ_s .

The service stress is assumed to be constant for simplicity.

Then, from Eq. (2)

$$\alpha_p^{(2-n')/2}\alpha_{pe}^{(2-n')/2}=H'\sigma_s^{n'}t_{equ} \quad (9)$$

$$H'=\frac{(n'-2)}{2}A'Y'^{n'} \quad (10)$$

where n' and H' stand for the constants, corresponding respectively to n and H , in the region of the stress intensity factor, K_{IS} , defined by the crack length, α_p , under the service stress, σ_s , or $K_{IS}=Y\sigma_s\alpha_p^{1/2}$. n' as well as H' is not always the same as n or H from two reasons. One is that the service environment is different from that of the proof test in general. The other is that, because of the nonlinearity of the $\ln V$ - $\ln K_I$ curve the value of n as well as H varies with the value of K_I or with the deviation of K_{IS} from K_{IC} (Fig. 1).

In Eqs. (7), (8) and (9), the approximation, $\alpha_p \sim \alpha_{pe}$, can be assumed, because the primary object of a proof test is to prove the length of the largest crack not to be larger than some needed value (α_p), and consequently the proof test condition must be selected not to make the crack length so large in comparison with α_p . For an example, the value of α_{pe} which is two

Fig. 1. $\ln V$ - $\ln K_I$ curves for proof test, practical service and failure stress measurement.

K_{IS} : Stress intensity factor for the crack of the length, α_p , under the service stress ($K_{IS}=Y\sigma_s\alpha_p^{1/2}$).

K_{IC} : Critical stress intensity factor.

curve (1): For the proof test,

curve (2): For the practical service,

curve (3): For the failure stress measurement.

times larger than α_p results in the life time of 2^{-10} times shorter than the expected one on the assumption of $n=20$, which can not be allowed for the survivors in proof testing.

In addition, the stress, σ , in the rotating component is a function of radial and tangential position, the dimension of the component, rotating speed, etc. Then the exact integration of Eq. (2), must be made regarding position. As mentioned above, however, the variation of the crack tip position is the order of the crack length or less, and the variation is thought to be very small usually in comparison with the dimension of the component as well as the absolute value of the position from the center. Therefore, so long as the crack growth of the proof test survivors is concerned, the stress at the crack tip is assumed to be constant. On this assumption, Eq. (7), holds.

Therefore, on the approximation, Eqs. (7), (8) and (9), can be rewritten in the following forms;

$$(n-2) \alpha_p^{(2-n)/2} \left(\frac{\Delta \alpha_{pe}}{\alpha_p} \right) = \frac{2H}{n+1} \sigma_p^n t_{pu} \quad (11)$$

for σ_p expressed by Eq. (4),

$$(n-2) \alpha_p^{(2-n)/2} \left(\frac{\Delta \alpha_{pe}}{\alpha_p} \right) = \frac{2H}{2n+1} \sigma_p^n t_{pu} \quad (12)$$

for σ_p expressed by Eq. (5),

$$(n'-2) \alpha_p^{(2-n')/2} \left(\frac{\Delta \alpha_{pe}}{\alpha_p} \right) = 2H' \sigma_s^{n'} t_{equ} \quad (13)$$

where $\Delta \alpha_{pe} = \alpha_{pe} - \alpha_p$.

Dividing Eq. (11), by Eq. (13), gives the following relation;

$$\frac{(n-2)}{(n'-2)} \alpha_p^{n'-n} = \left(\frac{H'}{H} \right) \frac{\sigma_p^n}{\sigma_s^{n'}} \left(\frac{t_{pu}}{t_{equ}} \right) \frac{1}{(n+1)} \quad (14)$$

Substitution of Eqs. (1), (3), (10) for (14) gives the following expression for t_{equ} ,

$$t_{equ} = \frac{A}{(n+1)A'} \left(\frac{Y}{Y'} \right)^{n'} K_{ic}^{n-n'} \left(\frac{\sigma_p}{\sigma_s} \right)^{n'} t_{pu} \quad (15)$$

for σ_p expressed by Eq. (4).

In a similar way, dividing Eq. (10) by Eq. (13) and the same substitution made for derivation of Eq. (15) give the following expression for t_{equ} ,

$$t_{equ} = \frac{A}{(2n+1)A'} \left(\frac{Y}{Y'} \right)^{n'} K_{ic}^{n-n'} \left(\frac{\sigma_p}{\sigma_s} \right)^{n'} t_{pu} \quad (16)$$

for σ_p expressed by Eq. (5).

Moreover, Eqs. (15) and (16), can be reduced to the following, when the unloading condition is common to the proof test and the practical service and the value of n is the same as that of n' ,

$$t_{equ} \div \frac{1}{n+1} \left(\frac{\sigma_p}{\sigma_s} \right)^n t_{pu} \quad (17)$$

(from Eq. (15))

Fig. 2. Equivalent service time plotted as a function of the ratio of proof stress to service stress, for various values of n .

$$t_{equ} \div \frac{1}{2n+1} \left(\frac{\sigma_p}{\sigma_s} \right)^n t_{pu} \quad (18)$$

(from Eq. (16))

t_{equ}/t_{pu} given from Eqs. (17) and (18), is plotted as a function of (σ_p/σ_s) for various value of n in Fig. 2.

2.2 The ratio of the equivalent service time to the needed life time

Eqs. (17) and (18), give the absolute value of t_{equ} . But special interest is put on the ratio of t_{equ} to the needed life time, L_s . L_s depends on α_p and service conditions. In this study where σ_s is assumed to be constant, it is given by the integration of Eq. (2), as follows,

$$L_s = \alpha_p^{(2-n')/2} H'^{-1} \sigma_s^{-n'} \quad (19)$$

Calculation of L_s or α_p from Eq. (19), needs the full knowledge on the actual value of A' as well as Y' . For a ceramic component of complicated profile, however, these value can not easily be given. Then another convenient variable must be used. The failure stress seems to be useful for that purpose. The stress is measured normally by applying stress, which increases linearly with time, to a ceramic. The failure stress of the ceramics with the crack of the length of α_p , σ_f , can be given from the following equation,⁵⁾ which is resulted from Eq. (2),

$$\alpha_p^{(2-n'')/2} = \frac{H''}{n''+1} \sigma_f^{n''} t_{fp} \quad (20)$$

$$H'' = \frac{n''-2}{2} A'' (Y'')^{n''}$$

where A'' and n'' stand respectively for the

material constants in the region of the stress intensity factor corresponding to the fracture stress measurement condition. Y'' stands for the geometrical factor for the stressing condition and the shape of the ceramics, and t_{fp} stands for the failure time. The ratio of t_{equ} to L_s can be given by dividing Eq. (15) by Eq. (19), and the substitution of Eq. (20) for the quotient as follows,

$$\frac{t_{equ}}{L_s} = \frac{(n'-2)H\sigma_p^n t_{pu}}{(n-2)(n+1)} \left\{ \frac{n''+1}{H''\sigma_t^{n''} t_{fp}} \right\}^{(n-2)/(n''-2)} \quad (21)$$

for the unloading condition expressed by Eq. (4).

In the similar way, dividing Eq. (16), by Eq. (19), and the substitution give the following expression for t_{equ}/L_s as follows,

$$\frac{t_{equ}}{L_s} = \frac{(n'-2)H\sigma_p^n t_{pu}}{(n-2)(2n+1)} \left\{ \frac{n''+1}{H''\sigma_t^{n''} t_{fp}} \right\}^{(n-2)/(n''-2)} \quad (22)$$

for the unloading condition expressed by Eq. (5).

For ceramics such as silicon nitride and silicon carbide, which have no plateau region in $\ln V - \ln K_I$ curve, Eqs. (21) and (22), can be reduced to the following simpler forms, on the assumption that the failure stress is measured under the same environment as that in the proof test ($A=A''$, $n=n''$),

$$\frac{t_{equ}}{L_s} = \frac{n'-2}{n-2} \left(\frac{Y}{Y''} \right)^n \left(\frac{\sigma_p}{\sigma_t} \right)^n \left(\frac{t_{pu}}{t_{fp}} \right) \quad (23)$$

for the condition expressed by Eq. (4), and

$$\frac{t_{equ}}{L_s} = \frac{(n'-2)(n+1)}{(n-2)(2n+1)} \left(\frac{Y}{Y''} \right)^n \left(\frac{\sigma_p}{\sigma_t} \right)^n \left(\frac{t_{pu}}{t_{fp}} \right) \quad (24)$$

for the condition expressed by Eq. (5).

2.3 Unloading time from the point of view of safety factor for proof stress

In proof test, the proof stress is selected often so as that the survived ceramics all might have longer life than that needed in practical use. In this proof test, the crack length of all survived ceramics which propagated during the unloading period must be shorter than the length which corresponds to the needed life time. For this purpose, the proof test stress, σ_p , is selected as slightly larger value than the needed stress, σ_1 . The safety factor is assumed to be $(1+\Delta p)$. Then

$$\sigma_p = \sigma_1(1+\Delta p) \quad (25)$$

$$Y\sigma_p\alpha_p^{1/2} = K_{Ic} \quad (1')$$

where α_p stands for the crack length corresponding to the short time failure stress, σ_p .

The increment of α_p during the unloading period, $\Delta\alpha_p$, is given by applying Eq. (9), or

$$(n-2)\alpha_p^{(2-n)/2}(\Delta\alpha_p/\alpha_p) = \frac{2H\sigma_p^n t_{pu}}{n+1} \quad (11')$$

The crack length, $(\alpha_p + \Delta\alpha_p)$, must be equal to or smaller than α_1 , corresponding to σ_1 , then

$$\alpha_p + \Delta\alpha_p \leq \alpha_1 \quad (26)$$

The crack length, α , is related to the failure stress, σ , as follows on the basis of fracture mechanics of ceramics,

$$\alpha \propto \sigma^{-2} \quad (27)$$

Eqs. (25) to (27) result in the following equation,

$$\Delta\alpha_p/\alpha_p \leq 2\Delta p \quad (28)$$

Substitution of Eq. (28) to Eq. (11') gives the unloading time, t_{pu} , as follows,

$$t_{pu} \leq \left\{ \frac{2(n+1)}{AK_{Ic}^n} \right\} \left(\frac{K_{Ic}}{Y\sigma_1} \right)^2 \left\{ \frac{\Delta p}{(1+\Delta p)^2} \right\} \quad (29)$$

for the unloading condition expressed by Eq. (4).

In a similar way, the needed value for t_{pu} is given as follows in the unloading condition, expressed by Eq. (15).

$$t_{pu} \leq \left\{ \frac{2(2n+1)}{AK_{Ic}^n} \right\} \left(\frac{K_{Ic}}{Y\sigma_1} \right)^2 \left\{ \frac{\Delta p}{(1+\Delta p)^2} \right\} \quad (30)$$

AK_{Ic}^n in Eqs. (29) and (30), is the crack propagation velocity at the value of stress intensity factor of K_{Ic} which is given by extrapolating $\ln V - \ln K_I$ curve. Then assuming the velocity as V_c , Eqs. (29) and (30), can be written in the following form,

$$t_{pu} \leq 2 \left(\frac{n+1}{V_c} \right) \left(\frac{K_{Ic}}{Y\sigma_1} \right)^2 \left\{ \frac{\Delta p}{(1+\Delta p)^2} \right\} \quad (31)$$

$$t_{pu} \leq 2 \left(\frac{2n+1}{V_c} \right) \left(\frac{K_{Ic}}{Y\sigma_1} \right)^2 \left\{ \frac{\Delta p}{(1+\Delta p)^2} \right\} \quad (32)$$

3. Discussion

The influence of the unloading time, t_{pu} , on the shortening of the needed life time, t_{equ} , is given by Eqs. (15) to (18) and is shown in Fig. 2. As seen in these equations or the figure, t_{equ} is very large in comparison with t_{pu} , because the service stress σ_s is very small in comparison with σ_p and n is very large. Therefore, to minimize the influence of the unloading time on the needed life time, t_{pu} must be made as small as possible.

The ratio of t_{equ} to L_s represents the influence well. According to Eqs. (21)-(24), t_{pu} must be selected so as to make the right hand side of the equations much less than unity. A rough estimation of the largest value allowable for that purpose is made by taking $Y=Y'$ on the assumption that the loading method is common to the proof test and the strength measurement. On this assumption, the following expression can be given from Eqs. (23) and (24),

$$\frac{t_{equ}}{L_s} < \left(\frac{\sigma_p}{\sigma_t} \right)^n \left(\frac{t_{pu}}{t_{fp}} \right) \quad (33)$$

because $n' < n$ in most ceramics as the stress intensity factor acting on the ceramics in practical

use is much less than K_{IC} , and there $\ln V - \ln K_I$ curve has small gradient. Moreover, the following equation holds on the basis of fracture mechanics of ceramics,⁶⁾

$$\sigma_t^n t_{tp} = \sigma_p^n t_{pp} \quad (34)$$

where t_{pp} is the failure time of the ceramics having the crack of the length of a_p . Therefore, t_{pu} must be much less than t_{pp} so that t_{equ} can be neglected in comparison with L_s . The value of t_{pp} is not exactly measured, but it is thought to be of the same order of the failure time in an impact experiment. In an impact experiment of Si_3N_4 ,⁷⁾ one example of the time during which stress acts on the specimen is measured as about 0.6 ms. Therefore, t_{pu} must be smaller than 60 μs for $t_{equ}/L_s < 0.1$.

Such small unloading time does not seem possible to apply. Then it will be concluded that some ceramics of the proof test survivors have not the needed life time, when the proof stress is taken so as to select the ceramics having just the needed life time, on the basis of the study of the ratio t_{equ}/L_s .

The possibility of the complete elimination of the ceramics having the life time less than the needed one is studied on the basis of Eqs. (31) and (32), which are derived for the proof test with some safety factor, $(1 + \Delta p)$. The unloading time, t_{pu} , must be less than the value given by Eqs. (31) and (32) for that purpose. In these equations, V_c is roughly estimated to be the order of 1000 m/s, from the experimental studies of the crack propagation velocity of Si_3N_4 .⁸⁾⁻¹⁰⁾ Moreover, n is about 100 and the value of σ_1 is about 500 MPa in normal application of ceramics to some mechanical components. Y is estimated about 1, and K_{IC} is about 5 MPa $\cdot m^{1/2}$ for structural ceramics such as Si_3N_4 and SiC. By substitution of these values to Eqs. (31) and (32), it is concluded that t_{pu} must be less than about $(10-20) \cdot \Delta p / (1 + \Delta p)^2$ (μs). The upper limits of t_{pu} increases with Δp until Δp reaches unity on the rough approximation. And it must be less than 5-10 μs , even for $\Delta p = 1$. This value (5-10 μs) of t_{pu} is too short to be applied in the actual proof stress. Therefore, the complete elimination of the ceramics which has undesirable life time can not be made only by the application of proof test. Then, another method such as AE monitoring should be used in combination with proof test for the complete elimination.

In a previous paper,¹¹⁾ the failure probability of

proof test survivors was studied, on the assumption of $t_{pu} = 0$. But this assumption is not right as mentioned above, and the true value of the probability will be larger than that given in the paper.

In addition, the approximated formulae, Eqs. (29) and (30), show that Δp or safety factor is desirable to be less than some value (1 in the present approximation), on the point of view of unloading time. This conclusion would be valid except the value, 1. Therefore the safety factor is recommended again to be small, as concluded in the previous paper¹¹⁾ from the point of view of the failure probability of the proof test survivors.

4. Conclusions

(1) The influence of the unloading time on the life time was studied, in a proof test. And some formulae giving the influence in term of the life time equivalent to the unloading time, which is the shortened life time during the unloading period, were given. Also the ratio of the equivalent service time to the needed life time and the allowable largest unloading time on the consideration of the safety factor for the proof stress were given.

(2) From these studies, it was concluded that the complete elimination of the ceramics having the life time less than the needed one can not be made only by the proof test, so long as the unloading time is not made shorter than about 5-10 μs .

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