Application Technique and Evaluation for Functional and Structural Design of Ceramics

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Strength properties of monolithic ceramics have been studied from the viewpoints of fundamental strength and practical application through the interdisciplinary research and development of ceramic applied equipment such as a gas turbine, an automobile engine and so on by researchers, engineers and material researchers. However for the design, manufacture and reliability estimation for various commercial equipment using ceramics, the ceramics based on a fundamental strength evaluation does not necessarily result in sufficient safety and reliability of actual ceramic components, because of foreign object damage (FOD), Hertz crack generation and chipping failure due to ceramic brittleness. Therefore, by extracting the basic concept and data of ceramic strength for functional and structural design from research for practical applications as well as interdisciplinary research, the author would like to emphasize in this paper significant points concerning the ceramic strength estimation method useful for the application of practical engineering international standard.

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1. Introduction

NON-OXIDE ceramics such as SiC and Si₃N₄ have been successful in some practical applications to machinery structural components making the best use of certain characteristics which in ceramics are superior to those of metal materials. Examples of typical applications with high performances are automobile engine components such as turbo-rotor and valve, and ceramic bearings for precise working machines and jet engines. Currently, ceramics such as Al₂O₃, AlN and so on are applied as structural materials as well as functional ones as in the past. In many of these applications, the ceramic components are hardly made as a simple and are composed as a complex product which joins with metal materials. For the design, manufacture and quality control of these practical ceramic components, it is very significant that ceramic fundamental properties are studied and a strength estimation method is established which considers the ceramic characteristics of both extremely low toughness and large strength variance.

The estimation methods of the fundamental strength properties of ceramics are conducted probabilistically with respect to the statistical properties and become the basis for the reliability design, quality control and proof test of practical ceramic components.

In this paper, it is explained from the viewpoint of the ceramic fracture mechanism, governed mainly by time dependent fracture under practical application, that our proposed unified strength estimation method can be applied to evaluate in unification all strengths against fast fracture, static fatigue, cyclic fatigue, and ring crack failure due to contact stress.

On the other hand, it is explained from the viewpoint of fracture mechanics that the strength properties against a stress distribution with a steep slope such that found in contact structural parts and jointing parts with metal were clarified with respect to the cause of fracture and could be estimated on the basis of a simple ceramic strength.

For the practical design and manufacturing techniques based on ceramic fundamental strength, the fracture criteria have been established that are significant to probabilistic design analysis and design criteria for ceramic components of automobile engines and small gas turbines and so on.

Through this general comment paper, the author would like to emphasize the basic concepts of ceramic strength estimation method useful for implementation of the international standard, extracting in a suitable manner just the property data and technology information actually available for practical functional and structural design in interdisciplinary studies which tend to make a phenomenal study just under easy severe experiment condition without notice of reality and practice.

2. Concepts and evaluation for fundamental strength

2.1 Common concept

2.1.1 Fracture origin and mechanism

It is well known generally that the origin of a fracture in ceramics is one of innumerable initial flaws. The fracture mechanism is that a crack generated from a initial flaw grows very slowly with the increase of applied stress or with the passage of stress hold time until a critical size, a₀, for burst fracture not more than 2 times of a initial size, a₀. The crack growth of ceramics is such that the growth rate, being extremely slow, depends on the stress intensity factor, K, at the tip of a crack propagating from a initial flaw and is significantly affected by conditions such as humidity and temperature. K is a peak value, K_{max} in the case of either monotonic fracture or delayed fracture at a constant stress, and an amplitude value, 4K_{π}, is used in the case of repeated fatigue fracture as shown in Fig. 1. In the crack growth process, the wake zone and process zone, where grain bridging and crack interlocking can occur as shown in Fig. 2, have been reported to influence greatly the crack growth rate. These phenomena such as grain bridging and crack interlocking occurs only during testing using a compact tension specimen with a large crack and so should not be related to the fracture strengths obtained by testing.
2.1.3 Difference in fracture mechanism between specimens tested at room temperature and those tested at high temperature

The fracture mechanism differs according to whether the ceramic fracture is characterized as brittle slow crack growth or as crack growth together with viscous elastic deformation of softened glass phase, occurrence of cavitations within the grain boundary phase and rotation of some grains. Thus, the fracture strength properties have a transition temperature region where the fracture mode changes.

2.1.4 Difference in ceramic response of thermal and mechanical shock

The ceramic response differs according to whether the applied stress is generated due to temperature gradient or due to mechanical force. If fracture strengths are estimated from the stress intensity factor with respect to the stress distribution, stress magnitude, shape and size of a flaw and so on, the difference in strength properties can not be attributed to the two kinds of applied stresses.

2.2 Unified evaluation method

2.2.1 Strength properties at room temperature in atmosphere

Ceramic fracture strengths are governed by the properties of crack growth from one of innumerable initial flaws in a test specimen, as expressed by the following formula

\[ \frac{da}{dt} = C K_{\text{max}}^n \left( \frac{\sigma}{\sigma_0} \right)^m, \]

(1)

where \( \sigma \) is applied stress, \( \phi \) is the shape factor and \( n \) is the crack growth rate dependence index.

By integrating Eq. (1) and rearranging, the fracture stress, \( \sigma_0 \), can be expressed as

\[ \sigma_0 = B \left( \frac{1}{\sqrt{a}} \right)^{n-2}, \]

(2)

where \( a \) is the size of the fracture origin flaw, \( B = 2/(\nu - 2) C (\phi \pi)^2 \) and \( t_{\text{eff}} \) is the effective hold time expressed as

\[ t_{\text{eff}} = \int_0^t \left( \frac{\sigma(t)}{\sigma_i} \right)^m dt. \]

(3)

On the other hand, let us suppose that the inert strength of the ceramics follows the two-parameter Weibull distribution, then the distribution function of \( a \) can be expressed in turn as

\[ F(a) = \exp \left\{ - \left( \frac{a}{\alpha_0} \right)^{-m/2} \right\}. \]

(4)

From Eqs. (3) and (4), the probability, \( R_i \), that \( \sigma_i t_{\text{eff}} \) becomes larger than a critical value, can now be given as

\[ R_i = \exp \left\{ - b_0 \left( \sigma_i t_{\text{eff}} \right)^{-m/2} \right\}, \]

(5)

where \( R_i \) is the non-fracture probability of a ceramic element, and \( b_0 \) is the material constant given by \( b_0 = 1/(\alpha_0 B^{-m/2}) \). The non-fracture probability can be expressed as

\[ R = \pi R_i = \exp \left\{ - b_0 \int_0^\infty \left( \sigma_i t_{\text{eff}} \right)^{-m/2} \frac{dV}{d\sigma} \right\}. \]

(6)

Now considering a concept of effective volume as

\[ V_{\text{eff}} = \int \frac{dV}{d\sigma} = \int \frac{\sigma_i}{\sigma_0} \frac{dV}{d\sigma_0} \]

(7)

the expression, from which all strength data obtained by various tests such as the tensile and bending tests and so on can be estimated unifiedly as the native ceramic strength, can be derived from Eqs. (6) and (7) and is given as

\[ \sigma_i = \sigma_0 \left( \frac{V_{\text{eff}}}{\sigma_0} \right)^{m/2}. \]

(8)

where \( \sigma_0 \) is a scale parameter, and \( m \) is the Weibull modulus. \( \sigma_0 t_{\text{eff}} V_{\text{eff}}^{1/n} \) in Eq. (8) is replaced to \( \sigma_i \) called a normalized strength, and the scatter of \( \sigma_i \) indicates the two-parameter Weibull distribution. Therefore, the normalized strength can be expressed as a probability variable \( \sigma_i \)

\[ \sigma_i = \sigma_0 t_{\text{eff}} V_{\text{eff}}^{1/m}. \]

(9)

It was proposed by the author that making a probabilistic estimation of \( \sigma_i \) was as the best method for Weibull statistical treatment.

2.2.2 Temperature dependence of strength properties

Regarding the super-high temperature properties of monotonic or fatigue fracture, the temperature dependence of the normalized strength, \( \sigma_i \), can be considered using the activation energy, \( Q \), and the cumulative effective hold time, \( t \), as follows

\[ \tilde{\sigma}_i = \sigma_0 t_{\text{eff}} V_{\text{eff}}^{1/m} \exp \left\{ \frac{Q}{k} \left( \frac{1}{T_{\text{c}}} + \frac{1}{T_D} \right) - \frac{1}{T_D} \right\}, \]

(10)

where \( k \) is the Boltzmann constant, \( T_{\text{tran}} \) is the transition temperature where the fracture mode changes as mentioned previously, and \( T_D \) is an arbitrary temperature. The activation energy, \( Q \), is the temperature dependence value corresponding to either the crack growth rate or the softening of the grain boundary phase.
3. Special strength properties under practical applications

3.1 New concept for estimation of monolithic ceramic strength

3.1.1 Fast fracture strength

In the strength estimation for the practical design of ceramic components, it is very important to consider that stresses generated in ceramic components have distributions with various slopes. The concept of effective volume is applicable to a stress distribution with a gentle slope, but it is not applicable to a stress distribution with a steep slope, because this will lead to overestimation of the ceramic strength to be very high.

Therefore, a new concept is required. The problematical stress distributions with a steep slope are those generated in the cases of contact stress, particularly jointing boundary stress and so forth.

Then, let the stress distribution from the surface to the inside be expressed as follows

$$\sigma(x) = f(x) \sigma_{\text{max}}$$

where \(f(x)\) expresses a distribution function of stress \(\sigma(x)\) normalized by \(\sigma_{\text{max}}\). The stress intensity factor, \(K_I\), at the tip of a surface defect is given by the following expression

$$K_I = 2 \sqrt{\frac{c}{\pi}} \int_0^c \frac{f(z)}{\sqrt{c^2 - z^2}} \, dz$$

where \(c\) represents a surface defect and is an equivalent radius of semicircular crack.

Now considering a distribution of stress which decreases gradually from the surface to the inside, the distribution of the stress intensity factor with increasing the magnitude of a surface crack can be calculated by Eq. (12), and changes to indicate the maximum value of \(K_I\) in the case of a stress distribution with an extremely steep slope as shown in Fig. 3.

In the case of a stress distribution with a gentle slope, the maximum value of \(K_{\text{max}}\) cannot be determined from the stress intensity factor \(K_I\) for defect sizes below 200 \(\mu\)m. On the other hand, in the case of a stress distribution where \(K_{\text{max}}\) can be determined from the \(K_I\), the magnitude of a defect relating to fracture are limited within the sizes correspond to the range of \(K_{\text{th}} < K_I < K_{\text{max}}\) where \(K_{\text{th}}\) is threshold of \(K_I\) for growing a crack.

In the case of a stress distribution where \(K_{\text{max}}\) cannot be determined, the fractures and determined from the defect distribution according to the strength distribution obtained by a bending test or tensile test. It is due to the properties of fracture under stress distribution with \(K_{\text{max}}\) that fracture strength increases and its scatter decreases as the stress distribution slope becomes increasingly steeper.

Therefore, the fracture strength under a steep stress distribution can be estimated as the upper limit value, \(K_{IC}\), of \(K_{\text{max}}\) by the following formula,

$$\sigma_1 = \frac{K_{\text{max}}}{\varphi / \sqrt{\pi c}}$$

where \(K_{IC}\) is the fracture toughness and the shape factor \(\varphi\) is given by an integral equation of \(z\) as follows

$$\varphi = \frac{2}{\pi} \int_0^c f(z) \sqrt{c^2 - z^2} \, dz$$

On the other hand, the fracture strength under a gentle stress distribution can be estimated from the mean length \(\bar{c}\) of surface cracks, by the following formula

$$\sigma_1 = \frac{K_{IC}}{\varphi / \sqrt{\pi c}}$$

where \(\bar{c}\) can be estimated from both the strength standardized for a volume of 1 mm³ and the stress distribution.

3.1.2 Delayed fracture strength

As for the delayed fracture strength, in the case of cyclic fatigue, strength can be estimated as shown in Eq. (16) by considering an effective stress hold time, \(\varepsilon_{\text{eff}}\) per cycle by using Eq. (3):

$$\sigma_1 = \frac{K_{IC}}{\varphi / \sqrt{\pi c} (N_f)^{1/6}}$$

where \(N_f\) is the number of cycles to failure.

3.2 Strength under Hertz's contact stress

3.2.1 Contact stress applied by a sphere

Figure 4 shows a schematic diagram of the stress distribution on the surface of a ceramic plate subjected to perpendicular compressive load by a ceramic ball. Surface compressive stress, \(\sigma_{\text{max}}\), indicates a maximum value at the center of the contact region but rises steeply as a parabolic curve...
until it reaches the contact boundary, where the contact stress indicates a maximum tensile stress, $\sigma_{t\text{max}}$. This tensile stress under Hertz\'s contact stress, decreases gently up to zero with leaving outside the contact boundary, inside which a ball contacts with plate.

The stress distribution toward the direction of plate thickness, which governs fracture strength, shows a stress distribution with the steepest slope at the contact boundary as shown in Fig. 5. For such a stress distribution, $K_I$ becomes $K_{I\text{max}}$ at $r=1.5a$.

When $K_I$ becomes $K_{I\text{max}}$, with the increase of Hertz\'s contact stress, a ring crack initiates. The first ring crack remains at a certain length after a slight growth with the increase of the contact compressive load applied by a ceramic ball. However, with the increase of the contact load above a critical value, the crack again starts to grow as a cone crack as shown in Fig. 6. Consequently, a cone fracture occurs. During the growth of a cone crack, $K_I$ at the crack tip is given by the following equation

$$K_I = 2\sqrt{\pi \int_{r_C}^{r} \frac{e^{\pi n} \sigma \left[ (1 + f(y) \right]}{\sqrt{r^2 - a^2}} \, db, \quad (17)$$

Judging from the fracture process mentioned above, a contact fracture strength can be reasonably estimated by using Hertz\'s tensile stress at the contact boundary of $r=a$ just at the moment when the cone fracture occurs.

On the other hand, a perpendicular directional component of the contact compressive force decreases with the increase of inclination $\theta_0$ against the normal direction of the plate.

3.2.2 Contract stress applied by a cylindrical bar

Figure 7 shows the distribution of a width directional stress in the case of a ceramic rectangular plane subjected to contact load by a ceramic rod. According to the results of stress analysis by Finite element method (FEM), contact tensile stress is maximum at both outside edges and decreases with moving to the center of the width of the plane. As shown in Fig. 8, the maximum tensile stress decreases steeply with leaving the contact boundary and then becomes compressive stress. Under such a stress field, the fracture strength, $\sigma$, should also be estimated by using Eqs. (13) and (14).

3.3 Strength under jointing residual stress

3.3.1 DBC semiconductor ceramic plate

As shown in Fig. 9, the thermal stress generated during the bonding process due to the difference between the thermal expansion coefficients of ceramics and metals remains as residual stress within the bonding layer of Al$_2$O$_3$ which...
exists between the copper sheet and ceramic plate in a semiconductor substrate. This residual stress governs the fracture strength under exfoliation or crack penetration into the semiconductor substrate in the direct bonding copper (DBC) method. As shown in Fig. 10, $K_I$ at the tip of a crack existing in the bonding boundary phase changes in distribution with the increase of the crack length. Therefore, it is found to be necessary that fracture mechanics should be considered as a fracture criterion for the strength estimation of the semiconductor substrate.

### 3.3.2 Strength of copper/cone jointing structure for a neutron detector

Residual stress with a steeply sloping stress distribution generated within $\text{Si}_3\text{N}_4$ in the ceramics/metal joints used for neutron detector as shown in Fig. 11. Being subjected to neutron irradiation over long period, $\text{Si}_3\text{N}_4$ swells and the copper for releasing thermal stress gradually becomes brittle which decreases its stress releasing ability.

Consequently, as shown in Fig. 12, the changing of the residual stress distribution due to this phenomenon increases the stress intensity factor at the tip of a defect, which decreases the fracture life by causing slow crack growth. The estimation method mentioned previously is indispensable in predicting the fracture life of ceramics/metal joints constituting a neutron detector.

### 3.4 Strength under collision of a foreign object

When the ceramic parts are subjected to collision of a small ball as shown in Fig. 13, the fracture strength depends on the thickness of the ceramic part as well as the size of the colliding ball and so the fracture modes differ respectively according to the dependence. This dependence is closely related to both the stress distribution on the contact surface and the stress distribution toward the thickness direction outside the contact circle region as shown in Figs. 14, 15 and 16. Each maximum contact stress has a relationship with plate thickness as shown by a curve in...
Calculating $K_I$ from Eq. (12) at the tip of a surface crack, assumed to be one of innumerable defects existing latently under the distribution of each stress $\sigma(x)$, three modes of...
relations between $K_I$ and the plane thickness, $h$, can be observed as shown in Fig. 18 and the fracture mode is suggested according to the range of plane thickness.

On the other hand, contact load, $F$, against collision velocity, $V$, of a small ball was clarified from the experimental relation between an impulse and a collision velocity. Thus each maximum stress, $\sigma_{\text{max}}$, can be calculated from $F$ and so the maximum value of $K_I$ can be found against the stress distribution with maximum stress $\sigma_{\text{max}}$. Figure 19 shows three diagrams of the relationships between $K_I$ and $V$ and the critical collision velocity, $V_C$, can be suggested according to plane thickness in each case of Si$_3$N$_4$ and SiC.

3.5 Strength under thermal stress

For example, ceramic yarn guide parts, used as heat-resistant material in the heat treatment furnace with temperature 200°C of a textile machine, generate a steeply sloping thermal stress distribution near the guide ditch, because heat in the ceramic part transfers to a yarn moving in high speed and a steep temperature gradient is generated. All of the ceramic guide parts which had been fractured in the actual service, fracture from the bottom of the guide ditch as shown in Fig. 20. This ceramic fracture, also, depends on not only a peak value of thermal stress but also on the stress distribution, as shown in Fig. 21. The estimation of fracture life and structural strength design should be conducted using Eq. (13) to Eq. (16).

4. Concept for advanced unified evaluation method

Generally the stresses generated in the products applied practically have various distributions, ranging from gentle to steep slopes. Therefore, in the case of estimating fracture strength with respect to effective volume, it should be considered that the size of cracks involved in each fracture differs according to whether the stress slope is gentle or steep. In order to improve the unified evaluation method proposed by the author in the past, the range of defect sizes as shown in Fig. 22 should be introduced into Eq. (4). As a result the unified evaluation method becomes applicable to the unified analysis of various strengths under contact stress, collision, residual stress and so on.

5. Conclusion

Many engineers, in pursuit of the development of ceramic equipment, have conducted research into practical applications by means such as evaluation of FOD, thermal shock fracture, strength of jointing ceramic/metal and so on. However, these studies have been carried out individually without concept for a unified estimation method. On the other hand, interdisciplinary studies, such as the estimation of fast/delayed fracture strength properties, crack growth properties, fracture toughness and so on under various standard applied stresses, have been carried out without sufficient consideration of actual fractures in practical applications.

In conclusion it cannot be said that a consistent estimation method has been developed from a combination of the two study approaches until now. Some interdisciplinary research may contribute to an estimation method for initial structural strength design and strength comparison in material development, but it cannot clarify the strength under contact stress and jointing residual stress required for practical ceramic products, related to the fundamental standard strength.

In this paper, from the viewpoint of standardization as well as practical application, the significant methodologies for evaluating ceramic strength properties can be summarized as follows.

(1) Test/estimation method for fundamental strength should be applied to clarify the applicable range of effective volume and stress hold time. Otherwise, overestimation occurs for the strength under a steeply sloping stress.

(2) In the estimation of ceramic strength under a steeply sloping stress distribution, the estimation should be conducted based on stress intensity factor $K_I$.

(3) In the estimation of strength and fracture life for
various jointing components, the total stress added the applied stress to the residual stress and its distribution should be considered in the calculation of the stress intensity factor.

(4) Though the impulse force calculated from the collision of a foreign object, the strength under the collision stress can be estimated as a dynamic contact strength problem. Then the influence of plate thickness and size of the colliding object on the strength should be considered.

(5) Considering the range of crack lengths involving in fracture, the unified evaluation method can be improved to enable it to estimate various strengths under contact stress, collision, residual stress and so on, as an advanced unified evaluation method.

References
Nagatoshi Okabe was born in 1941, Fukuoka Japan. He received the B.S. degree from Kumamoto University in 1965. He served as an assistant in Kurume National College of Technology from 1965 to 1968. He worked as a researcher for the development of energy equipment, applying new materials and reliability analysis/estimation technology on structural/functional design at Heavy Apparatus Laboratory of Toshiba Corporation from 1968 to 1995. He received from Kyoto University in 1983. He has served as Professor of Ehime University in Department of Mechanical Engineering, Ehime University since 1995.