A Model for Two-Dimensional Compaction of Cylinders

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円柱の2次元圧密モデル

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A model for two-dimensional compaction of cylinders is proposed. The equation $\ln(P_2/P_1) = k_1 k_s N_t (1/Z_{s2} - 1/Z_{s1})$ based on the model well described an experimental relation between the applied pressure P_a and the number of propagation sources of rearrangement, Z_s , where k_1 is a constant, k_s is a coefficient of friction, N_t is the total number of contacts among cylinders, Z_{s1} and P_1 are the initial value of Z_s and that of P_a , and Z_{s2} and P_2 are the final ones, respectively. The equation indicates that $\ln(P_2/P_1)$ is roughly proportional to $1/Z_{s2}$ under the condition $P_1 \ll P_2$ because of $Z_{s2} \ll Z_{s1}$, which means that a feature in the initial packing structure vanishes on compaction. The k_s value, on the other hand, has appreciable influence on the Z_{s2} value, i. e., the final packing structure.

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1. Introduction

There is an essential difference between a regular array of particles in a sintering model and a random one in an actual powder compact. This causes severe questions⁽¹⁾⁻⁽³⁾ for application of theoretical equations of sintering to practical data of a usual powder. If one can elucidate packing structure of particles in a usual powder compact, a sintering theory must be effectively developed.

Successive studies⁽⁴⁾⁻⁽¹⁰⁾ about compaction have been made in various fields such as ceramics,⁽⁴⁾ powder metallurgy,⁽⁵⁾ soil,^{(6),(7)} pharmacy⁽⁸⁾ and applied physics.^{(9),(10)} Compaction phenomena of a powder are so complicated that most of the studies have derived only empirical equations relating an applied pressure to the volume of a powder compact. Mathematical studies,^{(9),(10)} on the other hand, have been made under an assumption that a powder compact can be treated as a continuum medium. This assumption is reasonable for macroscopic behaviors⁽¹¹⁾ of a powder. It is, however, well-known that properties of ceramics critically subject not only to macroscopic inhomogeneity⁽¹²⁾ in packing of particles but also to packing defects⁽¹³⁾ of particles in microscopic regions. The present study was made to obtain information about microscopic packing structure of particles. Since it is very difficult to analyze and describe a three-dimensional packing structure of particles,

a two-dimensional one was examined through analysis of compaction process of cylinders located in the flat box of a compaction apparatus.⁽¹⁴⁾

2. A compaction model

2.1 Typical arrays of cylinders

Figure 1 shows two typical examples of packing of cylinders, from which one can find some statistical relation between a number of cylinders C_p and the shape of a polygon linked with their centers. Figure 2 shows four examples of the polygons, in which polygons with $C_p \leq 5$ are inevitably convex. If a concave polygon is defined as the polygon having one concave site at least, a number fraction, R_c , of the total number of concave polygons for a given C_p abruptly increases with increasing the C_p value from 6. Most of polygons of $C_p \geq 8$ are accordingly concave. Figure 2 shows that when cylinder a or b at a convex site shrinks toward the center of the polygon, cylinders c, d, ... near it inevitably move toward outer side. The applied pressure restricts such movement in general. The polygons, therefore, are considered to be hard in the sense that the restriction force prevents them from breaking down. It is, on the other hand, probable that cylinder e or f at a concave site in Fig. 2 shrinks toward the center of the polygon together with cylinders g, h, ... near it. The concurrent

Fig. 1. Two typical arrays of adhesive cylinders. (A) random, (B) granules

Fig. 2. Four examples of polygons linked with C_p cylinders.

shrinkages indicate no operation of the restriction force stated above, and hence the polygon is considered to be soft in the sense that its collapse can easily occur. Furthermore, most concave sites in a polygon can play the role of propagation sources, PS's, at the rearrangement of cylinders.

2.2 Rearrangement of cylinders in a sub-region

We consider a slide at a contact between two cylinders. Such a slide occurs when an applied force, F_a , increases by Δf_a . Physics^{(6), (7), (9)} lead an equation,

$$\Delta f_a = k_s F_a \quad (1)$$

where k_s is a coefficient of friction. A sub-region is defined as a region where the rearrangement of cylinders is directly induced by that at one or several PS's. If simultaneous slides occur at Z_r contacts in the sub-region, $Z_r \Delta f_a$ is equal to the least increment of F_a which just causes the rearrangement. There is a relation,

$$P_a = n_a F_a \quad (2)$$

among F_a , an applied pressure, P_a , and a number of cylinders per a unit length for the planar packing, n_a . n_a is statistically equal to $1/(\sqrt{\rho_r} \times d_c)$, where ρ_r is an area fraction of occupation of cylinders to the total area of the compaction field and d_c is the radius of a cylinder. The change of P_a extends over several orders in general. The previous study,⁽¹⁴⁾ on the other hand, showed the practically slight change of the ρ_r value from 60%

to 80%. This inappreciable change as compared with the former one roughly means that n_a is constant during compaction. Consequently, Eq. (2) indicates that F_a is proportional to P_a . It is natural that $Z_r \Delta f_a$ is induced by increase, ΔP_r , of P_a . From these relations and Eq. (1),

$$\Delta P_r = k_s Z_r P_a \quad (3)$$

is obtained.

2.3 Decrease of PS in number

To derive an equation which relates a number of sub-regions to an applied pressure, the following are assumed:

(1) An applied pressure, P_a , results in some balanced array of cylinders. At this time, force F_a , which is proportional to P_a , acts at a contact between cylinders.

(2) The rearrangement of cylinders at PS has directly influence on directions of movement of other cylinders in the same sub-region, but has indirectly influence on those in the other sub-regions.

(3) A number, Z_r , of sliding contacts in a sub-region is proportional to the total number of contacts Z_m in it.

Compaction induces translations or rotations of cylinders and their clusters with slides at contacts among cylinders as well as those without such slides. Z_r , therefore, is smaller than Z_m . It is very difficult exactly to estimate the Z_r value. If, however, rearrangement of cylinders is fully random, a constant ratio is statistically evaluated between sliding cylinders and the other ones. This situation is in accord with assumption (3).

From the definition, every sub-region has a PS or more PS's. PS's which exist closely must belong to the same sub-region, and co-operatively induce the rearrangement of cylinders in it. The existence of such sub-regions means that the total number, Z_s , of sub-regions is not more than that, Z_p , of PS's. Equation (3), however, indicates that the ΔP_r value decreases with decreasing the Z_r value. This tendency means that rearrange-

ment of cylinders easily occurs in a compaction field with small (many) sub-regions in comparison with that with larger (fewer) ones. This condition suggests that sub-regions with one PS are much more than the other with several PS's in general, and Z_s is roughly estimated as Z_p . The compaction field is, thus, divided into Z_p sub-regions from PS to PS.

Assumption (1) indicates that the increase of P_a to $P_a + \Delta P_r$ results in a new balanced array of cylinders with PS's of $Z_s - 1$ from the old one with Z_s . Rearrangement from $(Z_s - 1)$ to $(Z_s - 2)$ occurs if the applied pressure increases from $P_a + \Delta P_r$ to $P_a + 2\Delta P_r + \Delta_1$, further one from $(Z_s - 2)$ to $(Z_s - 3)$, also, proceeds after increase of the applied pressure from $P_a + 2\Delta P_r + \Delta_1$ to $P_a + 3\Delta P_r + \Delta_2$, and so on. If $dZ_s - 1 \ll Z_s$, $\Delta_q \ll \Delta P_r$ ($q = 1, 2, \dots$). dP_a is defined as the increment of the applied pressure during which the value of Z_s decreases by dZ_s . $dP_a = \Delta P_r dZ_s$. Very difficult estimation of the ΔP_r value results in that of the dP_a value. A relation that ΔP_r is proportional to ΔP_r , however, may hold because of slight structural change of array of cylinders from Z_s to $Z_s - 1$. Both the values of Z_s and dZ_s are proportional to the area of the compaction field, which, on the other hand, has inconsiderable influence on the value of dZ_s/Z_s . dP_a is hence related to dZ_s/Z_s to eliminate the effect of the area of the compaction field on the relation, and dP_a is found to be proportional to the product of ΔP_r and dZ_s/Z_s . N_t denotes the total number of contacts in the compaction field, and Z_s is equal to N_t/Z_m . From the aforesaid relations, $dP_a \propto \Delta P_r \times dZ_s/Z_s$, $Z_r \propto Z_m$ and $Z_s = N_t/Z_m$

$$dP_a = -k_1 k_s \frac{N_t}{Z_s} P_a \frac{dZ_s}{Z_s} \quad (4)$$

is derived from Eq. (3), where k_1 is a constant. If N_t is a constant, integrating Eq. (4) results in,

$$\ln(P_2/P_1) = k_1 k_s N_t \left(\frac{1}{Z_{s2}} - \frac{1}{Z_{s1}} \right) \quad (5)$$

where P_1 and Z_{s1} are the initial values of P_a and Z_s , and P_2 and Z_{s2} are the final ones, respectively.

3. Experimental

The apparatus, CA,¹⁴⁾ was fabricated to observe compacting process of something like 500 acrylic resin cylinders with 10 mm in diameter by 16 mm in high. The cylinders were compacted with the four movable walls which slide on the flat plate of the apparatus. The purchased cylinders are slippery, and their sides were covered with a pressure sensitive adhere double coated tape to give adhesiveness to the cylinders. Analysis was carried both on a packing process of the randomly arranged cylinders and on that of granules consisting with about 25 cylinders. Packing structure was taken with a camera located above CA. An applied pressure was measured with a strain gauge. ρ_r was estimated as the ratio of the total basal area of cylinders to the area of the square encircling them.

4. Results and discussion

Table 1 shows ρ_r , P_a and distributions of numbers of polygons. From the Table, increase of P_a occurred slightly from $\rho_r = 68\%$, and appreciably from $\rho_r = 78\%$. It was, however, very difficult to obtain a larger ρ_r value than 81%.

If there is a direct interdependence among all the slides at contacts in the compaction field (this corresponds with the compaction field having one sub-region), it is very difficult to explain the reason why densification at a relatively low ρ_r (< 68%) easily occurs even at an insignificantly applied pressure. This perplexed situation induces the conception, that is, "sub-regions".

Table 1. Applied pressures, P_a , ratios of occupation, ρ_r , and numbers of polygons linked with C_p cylinders for a random array.

P_a	ρ_r	C_p											
		3	4	5	6	7	8	9	10	11	12	13	
9	0.674	81	38	14	24	11	12	6	7	3	0	1	
2.5	0.698	89	39	17	27	14	15	5	4	2			
5.8	0.730	104	45	24	32	17	8	3	3	1			
7.7	0.739	112	62	26	32	18	5	2	2				
14.2	0.760	131	70	34	32	12	3	3	1				
20.4	0.770	153	71	36	24	15	2	3					
22.6	0.780	177	66	38	21	14	2	2					
24.0	0.793	189	74	35	21	14	1	2					
37.2	0.796	189	78	40	23	11	1	1					
55.0	0.807	196	64	49	23	10	1	1					
87.0	0.807	206	71	47	20	9	1	1					
122.0	0.808	215	82	42	19	7	1	1					
135.0	0.809	204	84	46	17	7	1	1					
156.0	0.810	209	84	46	17	6	1	1

Fig. 3. A relation between P_2/P_1 and $1/Z_{s2} - 1/Z_{s1}$.

This conception, also, well explains one of origins for the abrupt increase of P_a from $78\% < \rho_r$ because of appreciable decrease of PS in number.

Numbers of polygons of $C_p=3$ to 5 notably increased, but those of $C_p \geq 8$ decreased on compaction. These different tendencies in change of the number may be attributed to the shapes of polygons. That is, the polygons of $C_p=3$ to 5 are inevitably convex, but R_c for a given C_p value increased with increasing C_p , and most of polygons of $C_p \geq 8$ are concave.

The value of Z_s has to be evaluated to test the reasonability of Eq. (5), but not only this evaluation but also that of Z_p is very difficult. The major part of concave site, on the other hand, was identified as propagation sources of rearrangement of cylinders, and Z_s may be substituted with a number of all the concave sites in the compaction field. We, then, enumerated such sites to calculate Eq. (5). The solid line in Fig. 3 shows the result obtained from Eq. (5) under an assumption that the k_1 value is constant. The close fit of the theoretical values to the experimental data supports the reasonability of this assumption.

Equation (5) indicates that the value of $1/Z_{s2} - 1/Z_{s1}$ approaches to that of $1/Z_{s2}$ with increasing P_2 . This means that if an applied pressure is appreciably high, Z_{s1} , an initial array of cylinders, has slight influence on the Z_{s2} value, the final array of cylinders. Our previous paper¹⁴⁾ has shown that the initial array of cylinders has strong influence on packing structure in the early stage

of compaction. This feature vanished on compaction as suggested from Eq. (5). The data, also, roughly support that the k_1 value is constant. It was, however, very difficult completely to eliminate the trace of the packing feature in the initial array in spite of applying a very high pressure. Further development of the present compaction model is required to specify the trace.

It is well-known that a coefficient of friction¹¹⁾ has appreciable influence on packing structure. Equation (5), also, theoretically confirms this influence irrespective of a degree of compaction. This result suggests that it is very desirable to fabricate a powder with a very small k_s for achievement of a very small Z_{s2} value, effective elimination of relatively large pores.

5. Conclusions

The presently derived equation describes a two-dimensional compacting process, where the initial feature in a packing array vanishes on compaction. The equation, also, indicates that a final structure by compaction statistically depends both on a friction coefficient among particles and a final compression pressure.

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