Cyclic Fatigue of Ceramics: A Fracture Mechanics Approach to Subcritical Crack Growth and Life Prediction

Robert O. RITCHIE and Reinhold H. DAUSKARDT
Center for Advanced Materials, Materials Science Division, Lawrence Berkeley Laboratory, and Department of Materials Science and Engineering, University of California, Berkeley, CA 94720, U.S.A.

Cyclic fatigue, and specifically fatigue-crack propagation, in ceramic materials is reviewed both for monolithic and composites systems. In particular, stress/life (S/N) and crack-propagation data are presented for a range of ceramics, including zirconia, alumina, silicon nitride, SiC-whisker-reinforced alumina and a pyrolytic-carbon/graphite laminate. S/N data derived from unnotched specimens often indicate markedly lower lives under tension-compression compared to tension-tension loading; similar to metals, 10⁸-cycle fatigue limits generally approach ~50% of the tensile strength. Crack-growth results, based on studies on "long" (>3 mm) cracks, show fatigue-crack propagation rates to be markedly power-law dependent on the applied stress-intensity range, \(\Delta K\), with a threshold, \(K_{TH}\), of the order of ~50% of \(K_c\). Conversely, for "small" (<250 \(\mu m\)) surface cracks, fatigue-crack growth is seen to occur at \(\Delta K\) levels some 2 to 3 times smaller than \(K_{TH}\), and to show a negative dependency on applied stress intensity. At ambient temperatures, lifetimes are shortened and crack-growth rates are significantly accelerated by cyclic, compared to quasi-static loading, although limited data suggest the reverse to be true at very high temperatures in the creep regime. Such results are discussed in terms of the primary crack-tip shielding (toughening) mechanisms and potential mechanisms of cyclic crack advance. Finally, implications are discussed of long and small crack cyclic fatigue data to life prediction and safety-critical design of ceramic components.

Key-words: Cyclic fatigue, Crack growth, Small cracks, Fatigue threshold, Ceramics, Ceramic-matrix composites, MgPSZ, Alumina, Silicon nitride, SiC-reinforced alumina, Pyrolytic-carbon/graphite laminate, Damage-tolerant design, Life prediction

1. Introduction

The importance of mechanical degradation under cyclic fatigue loading in the design of metallic structures and components is characterized by either a classical stress/life (S/N) approach or using fracture mechanics concepts which incorporate the sub-critical growth behavior of both "long" and "small" cracks (Fig. 1). Over the past few years, several studies have provided persuasive evidence of a similar susceptibility of ceramic and ceramic-matrix composite materials to cyclic fatigue. In fact, extensive data have now been reported indicating reduced lifetimes during S/N testing under cyclic, compared to...
sustained (quasi-static) loads, and accelerated cyclic crack-propagation rates at stress intensities less than required for environmentally-enhanced crack growth (static fatigue) during fracture-mechanics testing; results exist for zirconia, graphite, alumina, silicon nitride and silica glass ceramics and LAS-SiCf, Al2O3-SiC, and laminated graphite-pyrolytic carbon composites.

While the precise micro-mechanisms for such cyclic fatigue are still unclear, behavioral characteristics are in general qualitatively similar to those in metallic materials. For example, typical ambient-temperature stress/life data for alumina, TZP, silicon nitride and silicon carbide, taken from the four-point bend (zero-tension) data of Kawakubo et al., are shown in Figure 2. It is clear that for lives in excess of $10^3$ cycles, the time to failure under cyclic loads is less than that under quasi-static loads at the same maximum stress, and further, with the exception of SiC, the cyclic fatigue life is less than that predicted using the static data by integrating over the fatigue cycle. However, the inverse slope, $n$, of these curves is far higher than that found in metals; values range from 11 in TZP to 132 in SiC.

Alternatively, typical crack-propagation data (long crack), taken from the authors' own ambient-temperature studies on compact-tension specimens, are shown in Fig. 3; these results illustrate the dependency of cyclic fatigue-crack growth rates, $da/dN$, on the applied (far-field) stress-intensi-
ty range ($\Delta K = K_{\text{max}} - K_{\text{min}}$), for three zirconia ceramics, graphite, a pyrolytic-carbon-coated graphite laminate and a SiC-whisker-reinforced alumina ($\text{Al}_2\text{O}_3-\text{SiCw}$) composite, and are compared with similar data for steel and aluminum high-strength metallic alloys.$^{35,36}$ Similar to metals, the ceramic fatigue data appear to follow a simple Paris power-law relationship$^{37}$ of the form:

$$\frac{da}{dN} = C(\Delta K)^m,$$

where $C$ and $m$ are scaling constants; the exponent, $m$, however, varies between 12 to over 40, which is far higher than the typical values of between 2 and 4 generally found for metals in this regime.

In addition to such crack-propagation data for "long" cracks (typically longer than $\sim 3\ \text{mm}$), more recent results on "small" surface cracks have been observed to far exceed those of "long" cracks at equivalent applied stress-intensity levels, and more importantly to occur at stress intensities less than the fatigue threshold, $\Delta K_{\text{TH}}$, below which (long) cracks are presumed to be dormant.$^{7-9}$ Accordingly, the objective of this review is to survey current understanding of the cyclic fatigue behavior of monolithic and reinforced ceramics, with respect to stress/life and crack-propagation data under both constant and variable-amplitude loading. Long and microstructurally-small crack-growth behavior will be reviewed and possible micro-mechanisms advanced. Finally, the implications for fatigue life prediction and design criteria for ceramic components are discussed.

2. Testing methods

2.1 Stress/life testing

In general, stress/life ($S/N$) cyclic fatigue tests on ceramic materials have been performed in similar fashion to the standard techniques used for metallic materials, although much greater care has been required in the design of the loading grips. Techniques include zero-tension and tension-tension testing in cantilever bend and three- and four-point bending,$^{6,9,25}$ tension-compression testing in rotary bending,$^{14,18}$ and uniaxial push-pull, and tension-torsion testing.$^{27}$ Typical stress/life results are shown in Fig. 2.

2.2 Constant-amplitude crack-propagation testing

Cyclic fatigue-crack propagation studies on through-thickness long cracks have been conducted on a variety of pre-cracked fracture mechanics samples; results have been reported for single-edge-notched specimens in three- and four-point bending, single- and double-anvil configured specimens in bending, tapered double-cantilever beam specimens and compact-tension $C(T)$ specimens. The system used by the current authors, illustrated in Fig. 4(a), utilizes $\sim 1$ to 6 mm thick $C(T)$ specimens which are cycled at various positive load ratios ($R = \text{ratio of minimum to maximum loads}$) at frequencies up to 50 Hz.$^{1,4}$ Following crack initiation from a wedge-shaped starter notch,$^{1}$ crack-growth rates are determined over the range $\sim 10^{-12}$ to $10^{-4}$ m/cycle using computer-controlled $K$ decreasing and $K$ increasing conditions; applied loads are varied such that the instantaneous values of the crack length, $a$, and stress-intensity range, $\Delta K$, vary according to the equation:

$$\Delta K = \Delta K_0 \exp[C^*(a-a_0)],$$

where $a_0$ and $\Delta K_0$ are the initial values of $a$ and $\Delta K$, and $C^*$ is the normalized $K$ gradient $[(1/K)(dK/da)]$ set typically to $\pm 0.08$ mm$^{-1}$. Linear elastic stress-intensity solutions for the various specimen geometries are given in standard handbooks.$^{39}$

As an alternative to optical crack-length measurements, electrical-resistance techniques, using $\sim 0.1\ \mu\text{m}$ NiCr films evaporated onto the specimen surface, can be employed to continuously monitor in situ the crack length to a resolution of within $\pm 5\ \mu\text{m}$. In addition, unloading compliance measure-
ments using back-face strain gauges have been utilized to assess the extent of fatigue crack closure caused by premature contact of the crack surfaces during the loading cycle. This is achieved by defining a far-field stress intensity, $K_{cl}$, at first contact of the crack surfaces during the unloading cycle, i.e., at the highest load where the elastic unloading compliance deviates from linearity. In metal fatigue, the value of the closure stress intensity is often used to compute an effective (near-tip) crack-driving force given by the difference of the maximum stress intensity and $K_{cl}$, i.e., $\Delta K_{eff}=K_{max}-K_{cl}$ (where $K_{cl}>K_{min}$).

Corresponding fatigue-crack propagation data for small ($<250\ \mu m$) surface cracks have generally been determined through replication or optical examination of the top surface of cantilever-beam specimens loaded in tension-compression (Fig. 4 (b)); depending upon the ease of crack initiation, such specimens are either unnotched or contain stress concentrators such as micro-hardness indents or a notch. Ideally tests are interrupted after ~100 cycles to ascertain damage accumulation during initial loading and then subsequently at $10^2$ to $10^3$ cycle intervals until failure. Small-crack lengths can be readily quantified, with a resolution better than ±0.5 μm, by digitizing optical micrographs of the specimen surface using an image analyzer, in order to compute growth rates between $10^{-12}$ to $10^{-6}$ m/cycle.

Stress-intensity factors for such surface cracks can be obtained from linear elastic solutions for three-dimensional semi-elliptical surface cracks in bending (and/or tension) (Fig. 4 (b)) in terms of crack depth $a$, crack length $c$, elliptical parametric angle $\phi$, shape factor $Q$, specimen width $t$, specimen thickness $B$, and remote (outer surface) bending stress, $\sigma$;

$$K = H_{\sigma} \sigma (\pi a/Q)^{1/2} F(a/c, a/t, c/B, \phi)$$

where $H_{\sigma}$ is the bending multiplier and $F$ is a boundary correction factor.

2.3 Variable-amplitude crack-propagation testing

Limited results have also been reported by the authors for cyclic fatigue-crack propagation behavior in ceramic materials during variable-amplitude loading sequences. Such tests have been performed on long cracks, specifically in transformation-toughened Mg-PSZ and a SiC-whisker-reinforced alumina, by applying various load excursions during steady-state fatigue-crack growth, and monitoring the transient crack-growth response as a function of crack extension until steady-state growth is re-established. The principal loading spectra that have been used include single tensile overloads and block-loading sequences (both low-high and high-low), comprising selected constant stress-intensity ranges. In addition, growth rates have been examined under constant $K_{max}$/variable-$R$ conditions, where the maximum stress intensity is held constant as $K_{min}$ is increased; such loading spectra have been used in metal fatigue studies to minimize the influence of crack closure on growth-rate behavior.

3. Stress-life behavior

As noted above, cyclic stress/life ($S/N$) data for most ceramic materials at ambient temperatures show lifetimes to be less than that to cause failure at the same maximum stress under quasi-static loads, and to be less than that predicted from the static stress/life data assuming only time-dependent (and not cycle-dependent) processes. In metals, the fatigue strength decreases with number of cycles, although the dependence of life on applied stress, i.e., the inverse slope, $n$, of the $S/N$ curve, is far higher; typical values are listed in Table 1. In addition, $S/N$ curves appear to be much more sensitive to the load ratio than with metallic materials.

$S/N$ curves on unnotched cantilever-beam samples of mid-toughness (MS grade) and overaged (pre-transformed) Mg-PSZ, under both zero-tension ($R=0$) and tension-compression ($R=-1$) loading, are shown in Fig. 5; corresponding data from Swain et al. for MS-grade Mg-PSZ, tested in four-point bend ($R=0$) and rotating bend ($R=-1$), are included for comparison. It is apparent that, similar to behavior in steels, the MS-grade material shows evidence of a “fatigue limit” of roughly 50% of the single-cycle tensile (or bending) strength, at $10^6$ cycles ($R=-1$). However, not all ceramics have such a distinct knee in the $S/N$ curve above $10^6$ cycles; sintered alumina, for example, can fail at lives in excess of $10^8$ cycles, with a $10^8$ cycle endurance strength some 25 to 40% of the single-cycle strength.

Similar to results reported for Si$_3$N$_4$, the Mg-PSZ data in Fig. 5 also indicate that cycling in tension-compression is significantly more damaging than in tension. This is apparent from optical examination of specimen surfaces in MS-grade material at both $R$-ratios, where quantitative analysis of the crack-size distributions reveals an increased distribution of larger microcracks at $R=-1$ (Fig. 6).

Table 1. Values of parameters for $S/N$ and crack-propagation fatigue tests for ceramic materials at ambient temperatures.

<table>
<thead>
<tr>
<th>Material</th>
<th>$S/N$ slope $m$</th>
<th>Crack Propagation ($R = 0$) slope $m$</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alumina</td>
<td>4.0</td>
<td>2.7-33</td>
<td></td>
</tr>
<tr>
<td>Mg-PSZ</td>
<td>16.0</td>
<td>4.4-4.7</td>
<td>18,20</td>
</tr>
<tr>
<td>(MS-grade)</td>
<td>11.5</td>
<td>2.0-2.5</td>
<td></td>
</tr>
<tr>
<td>(overaged)</td>
<td>3.9</td>
<td>0.9-1.2</td>
<td></td>
</tr>
<tr>
<td>Silicon nitride</td>
<td>6.0</td>
<td>2.0-4.3</td>
<td>21</td>
</tr>
<tr>
<td>3Y-TZP</td>
<td>5.3</td>
<td>2.0-4.3</td>
<td>21</td>
</tr>
<tr>
<td>Al$_2$O$_3$/SiC$_n$</td>
<td>4.5</td>
<td>2.0-4.3</td>
<td>21</td>
</tr>
<tr>
<td>Graphite/epoxy C</td>
<td>1.6</td>
<td>0.9-1.2</td>
<td>12</td>
</tr>
</tbody>
</table>

*TS = 7 hr at 1000°C, MS = 3 hr at 1100°C, AF = as fired, overaged = 24 hr at 1100°C.
Microcracks appear in regions of surface uplift from transformation and their alignment is primarily orthogonal to the stress axis, although some cracks form at angles of \(-45^\circ\) to the stress axis at \(R=-1\).

This effect, however, is not general to ceramic fatigue. Unlike results on smooth samples,\(^{7,8,26}\) \(S/N\) data derived from results on micro-indent cantilever-bend specimens, of a SiC-whisker-reinforced alumina,\(^{33}\) show no such difference between tension-compression and tension-tension cycling (Table 2). This may be because fully reversed cyclic loading has a greater effect on initiating fatigue damage in the form of microcracks, which in unnotched samples can account for a significant proportion of the life. In fact, it is known that where defects pre-exist (as micro-indents), lifetimes in tension and tension-compression are comparable as crack-growth rates are similar (Fig. 7).\(^{33}\)

<table>
<thead>
<tr>
<th>Load Ratio</th>
<th>(S_{\text{max}}/S_{\text{min}})</th>
<th>Life, (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>245</td>
<td>(6 \times 10^{5})</td>
</tr>
<tr>
<td>0.03</td>
<td>247</td>
<td>(10^{6})</td>
</tr>
<tr>
<td>0.03</td>
<td>245</td>
<td>(10^{6})</td>
</tr>
<tr>
<td>-1.0</td>
<td>245</td>
<td>(6 \times 10^{5})</td>
</tr>
<tr>
<td>-1.0</td>
<td>245</td>
<td>(6 \times 10^{5})</td>
</tr>
</tbody>
</table>

Fig. 7. Surface crack length vs. number of cycles for small-crack growth at similar applied stress levels in unnotched cantilever-bend specimens of MS-grade Mg:PSZ tested at \(R=0\) and \(R=-1\).\(^{8}\)

4. Fatigue-crack propagation behavior

4.1 Long-crack behavior

The variation in cyclic fatigue-crack propagation rates with the applied stress-intensity range, shown in Fig. 1 for long cracks in a range of ceramic materials, illustrates the extremely high dependency of \(d\alpha/dN\) on \(\Delta K\). Similar results are apparent for several transformation-toughened (sub-eutectoid aged) and pre-transformed (overaged) microstructures in Mg:PSZ\(^4\) (Fig. 8 (a)), SiC-whisker-reinforced alumina (Fig. 9),\(^{33}\) silicon nitride,\(^{29}\) silica glass,\(^{30}\) and a pyrolytic-carbon coated graphite laminate\(^{12}\) (Fig. 10). As noted above, these data in general conform to a power-law relationship (Eq. 1) over the mid-range of growth rates, with very high exponents (i.e., the slope \(m\) of the \(d\alpha/dN\) vs. \(\Delta K\) curve). Values of \(m\), listed for several ceramics in Table 1, can vary between \(-10\) and over \(100\); the constant \(C\), on the other hand, appears to scale inversely with the fracture toughness \(K_c\). Fatigue threshold \(\Delta K_{\text{TH}}\) values may be defined at a maximum growth rate of \(\sim 10^{-10}\) m/cycle since the \(d\alpha/dN/\Delta K\) curves do not always show a sigmoidal shape; these values are typically of the order of 50% of \(K_c\).

In view of past skepticism over the fatigue of ceramics, experiments have been performed to demonstrate unequivocally that crack growth in these instances is cyclically induced and not simply a conse-
Cyclic Fatigue of Ceramics: A Fracture Mechanics Approach to Subcritical Crack Growth and Life Prediction

To achieve this, crack extension has been monitored at constant $K_{\text{max}}$ with varying $K_{\text{min}}$, as shown for a pyrolytic-carbon/graphite laminate in Fig. 11. It is apparent that, whereas crack extension proceeds readily under cyclic loading conditions (region a), upon removal of the cyclic component by holding at the same $K_{\text{max}}$ (region b), crack-growth rates are markedly reduced. Aside from establishing that the crack-growth behavior is a true cyclic fatigue phenomenon, this result illustrates that subcritical crack-growth rates due to cyclic fatigue are far in excess of those due to static fatigue at the same $K_{\text{max}}$.

In certain experiments, crack closure has been observed during fatigue-crack growth in ceramics; results are shown in Fig. 12 for four microstructures in Mg-PSZ with increasing degrees of transformation toughening. Data are plotted in terms of the far-field closure stress intensity, $K_{\text{cl}}$, normalized by $K_{\text{max}}$, as a function of the applied $\Delta K$. $K_{\text{cl}}$ values are measured globally at first contact of the fracture surfaces during the unloading cycle; specifically, values of $K_{\text{cl}}$ are calculated from the highest load at which the elastic unloading compliance line deviates from linearity. In metals, such closure primarily results from premature contact between the crack surfaces by fracture-surface asperities, and can have a potent influence in locally affecting the “crack-driving force”. The degree of closure in ceramics, however, appears to be smaller, and in materials like Mg-PSZ is dwarfed by other mechanisms of crack-tip shielding such as transformation toughening. In fact, resistance to cyclic fatigue-crack extension in Mg-PSZ is observed to increase directly with the
degree of crack-tip shielding resulting from increased transformation toughening (Fig. 8(a)). Such crack-growth data can therefore be normalized to a single curve (Fig. 8(b)) by characterizing in terms of the near-tip stress-intensity range, $\Delta K_{tip}$, which allows for the extent of shielding, as outlined below.4)

In Mg-PSZ, the tetragonal phase, which typically consists of approximately 40 vol% of lens-shaped precipitates of maximum size $\sim 300$ nm, can undergo a stress-induced martensitic transformation to a monoclinic phase in the high stress field near the crack tip. The resultant dilatant transformation zone in the wake of the crack exerts compressive trac-

$$K_{tip} = K - K_s,$$

$$K_s = A \left( E/(1 - \nu^2) \right) \tau_{trans} f w^{1/2}. $$

Under cyclic loading, shielding from transformation far exceeds that due to crack closure; accordingly, the local near-tip stress-intensity range can be expressed by:4)
\[ \Delta K_{\text{tip}} = K_{\text{max}} - K_s. \]  

By computing \( K_s \) using the integrated form of Eq. (5) and Raman spectroscopy measurements of the transformation-zone size, growth rates can be characterized in terms of \( \Delta K_{\text{tip}} \), thereby accounting for the differing fatigue resistance of the four microstructures (Fig. 8(b)). An equivalent result can be achieved by normalizing the crack-growth data in terms of \( \Delta K/K_c \). 

FRACTOGRAPHICALLY, little discernible difference is generally apparent between monotonic and fatigue fracture surfaces in ceramics; this is in marked contrast to metals where clear distinction is generally observed between striation growth under cyclic loads and microvoid coalescence or cleavage, for instance, under quasi-static loads. Fatigue and fracture surfaces in silicon carbide, graphite, pyrolytic carbon, and Mg-PSZ are essentially identical; in Mg-PSZ, for example, surfaces are primarily transgranular with crack paths showing evidence of crack deflection, branching, and uncracked-ligament bridging behind the crack tip. In certain ceramics, however, the fractography of the resulting cyclic and monotonic fracture surfaces do show some distinction. An example of this is Al\(_2\)O\(_3\)-SiC\(_w\). Fatigue fracture surfaces appear to be more textured and rougher with a higher degree of SiC whisker pull out than monotonic fracture surfaces. At higher magnification, crack paths formed under monotonic loading can be seen to be predominantly transgranular, with a high incidence of cleavage steps (indicated by the letter C on the micrograph) and a flatter appearance (Fig. 13(a), (b)). The increased roughness of the fatigue surfaces, conversely, appears to be associated with an increasing degree of intergranular fracture (Fig. 13(c), (d)).

4.2 Small-crack behavior

The question of small cracks is undoubtedly of special importance to ceramics simply because the majority of ceramic components in service will not be able to tolerate the presence of physically long cracks. In metal fatigue, it is known that where crack sizes approach the dimensions of the microstructure or the extent of local crack-tip plasticity, or where cracks are simply physically small (typically \( \leq 250 \mu m \)), the similitude of crack-tip field characterization can break down (see, for example Ref. 38). Specifically, small-crack growth rates are often far in excess of those of corresponding long cracks at the same applied \( \Delta K \) levels; moreover, small-crack growth can occur at stress intensities well below the (long-crack) threshold \( \Delta K_{\text{TH}} \). Although such apparently anomalous behavior can be attributed to a number of factors, the primary reason is that the extent of crack-tip shielding (in metals, principally from crack-closure phenomena) is generally diminished in small flaws by virtue of their limited wake. In fact, a more precise definition of a small crack is one whose length is small relative to the size of the shielding zone. Since crack-tip shielding, from phase transformation, microcracking, crack bridging etc., plays a pivotal role in the toughening of many monolithic and composite ceramics, it is to be expected that such small-crack effects may also be prevalent in ceramic fatigue behavior.

Results on small fatigue cracks in ceramics, however, are very limited. Typical crack-growth data for Mg-PSZ, monitored from unnotched cantilever-beam S/N samples and characterized in terms of the applied \( K_{\text{max}} \), are compared to corresponding long-cracks data in Fig. 14(a). In sharp contrast to long-crack results, the small cracks grow at progressively decreasing growth rates with increase in size, until finally linking together as the density of cracks across the specimen surface increases; the specimen then fails. Small-crack propagation rates display a negative, dependency on stress intensity and occur at applied stress-intensity levels well below \( \Delta K_{\text{TH}} \) (specifically at \( K_{\text{max}} \) levels of 1.6 MPa \( \sqrt{m} \), a factor of seven less than \( K_c \)). Moreover, growth rates are clearly not a unique function of \( K_{\text{max}} \) and appear to be sensitive to the level of applied stress.

Similar behavior has been reported for ceramic composites. The crack lengths of selected microcracks at similar maximum applied stress lev-
els, emanating from micro-indent in cantilever-beam samples of SiC-whisker-reinforced alumina, are compared as a function of number of cycles at $R=0.05$ and $-1$ in Fig. 15. As in monolithic ceramics, the small cracks grow at progressively decreasing growth rates, i.e., when plotted as propagation rates as a function of the applied $K_{\text{max}}$ (Fig. 9), the crack-growth rate data display a characteristic negative dependency on applied stress intensity.

Such sub-threshold small-crack growth behavior can be explained by analogy to small-crack behavior in metals (e.g., Refs. 36, 38). Although the nominal (far-field) stress intensity $K$ increases with increase in crack size, the shielding stress intensity $K_s$ is also enhanced as a shielding zone is developed in the crack wake. The near-tip stress intensity (Eq. (6)) and hence the crack-growth behavior is thus a result of the mutual competition between these two factors; initially $K_{\text{tip}}$ is diminished with crack extension until a steady-state shielding zone is established, whereupon behavior approaches that of a long crack. In Mg-PSZ, the primary shielding is due to phase transformation, whereas in the $\text{Al}_2\text{O}_3/\text{SiC}_w$ composite, it is presumed to result from crack bridging by unbroken whiskers in the crack wake, and from the residual stress field surrounding the micro-indent (Fig. 9 (b)).

With a detailed knowledge and quantitative modeling of the salient shielding mechanisms, it is possible to normalize long and small crack data by characterizing in terms of the near-tip stress intensity, $K_{\text{tip}}$, i.e., after subtracting out the stress intensity due to shielding. Computations of $K_s$ and hence $K_{\text{tip}}$ have been achieved for small fatigue cracks in a SiC-fiber-reinforced LAS glass-ceramic, where the predominant shielding resulted from fiber bridging, and in $\text{Al}_2\text{O}_3$, $\text{Si}_3\text{N}_4$, and 3Y-TZP where the effect of the residual stress field associated with the crack-initiating micro-indent was taken into account. This is illustrated in Fig. 9 (b), and in Fig. 16 where small and long crack data in 3Y-TZP are similarly normalized by defining $K_{\text{tip}}$ in terms of a superposition of the applied and residual stress fields. In addition, long-crack data in Mg-PSZ has been characterized in terms of $K_{\text{tip}}$ after correcting for transformation shielding; it is apparent that the $da/dN$ ($K_{\text{tip},\text{max}}$) curve corresponds closely to that associated with the initial growth of the small cracks. However, whether such analytical procedures to normalize small and long crack data are practical on a regular engineering basis for all ceramic materials in service is clearly questionable.

4.3 Variable-amplitude loading

In addition to the cyclic fatigue of ceramics under constant-amplitude loading described above, limited studies have been performed to examine crack-propagation behavior under variable-amplitude loading. These studies have been focused on the transient
crack-growth response to single tensile overload, block loading and constant-$K_{\text{max}}$/variable-$R$ (increasing $K_{\text{min}}$) sequences during steady-state fatigue-crack growth, specifically in MS-grade Mg-PSZ and a SiC-whisker-reinforced $\alpha$-Al$_2$O$_3$ composite.

Behavior in Mg-PSZ, shown for MS- and TS-grades in Fig. 17, is similar to that widely reported in metals (e.g., Ref. 52). In Fig. 17(a), a high-low block overload, from $\Delta K = 5.48$ to $5.30$ MPa m$^{-1}$, is first applied to an MS-grade sample; on reducing the cyclic loads, an immediate transient retardation is seen followed by a gradual increase in growth rates until the (new) steady-state velocity is achieved. Similarly, by subsequently increasing the cyclic loads so that $\Delta K = 5.60$ MPa m$^{-1}$ (low-high block overload), growth rates show a transient acceleration before decaying to the steady-state velocity. The increment of crack growth over which the transient behavior occurs is $\sim 700 \mu$m, approximately five times the measured transformed-zone width of $\sim 150 \mu$m. This is consistent with zone-shielding calculations which compute the maximum steady-state shielding to be achieved after crack extensions of approximately five times the zone width. In fact, relatively accurate predictions of the post load-change transient crack-growth behavior have been obtained (dotted line in Fig. 17(a)) by computing the extent of crack-tip shielding in the changing transformation zone following the load changes. The changing size of the crack-tip transformation zone can be readily mapped using spatially-resolved Raman spectroscopy techniques. For example, zone sizes for the block-load-
The precise role of variable-amplitude loading on cyclic crack growth in the various ceramic systems is still uncertain. It is known that in metals and phase-transforming ceramics, an overload cycle creates an enlarged crack-tip plastic or transformation zone which then promotes crack-growth retardation by reducing the near-tip stress intensity by residual compressive stress and resulting crack-closure effects in metals or by transformation shielding in ceramics. However, in non-transforming ceramics, such as Al₂O₃/SiCw which may rely on shielding by crack bridging, the overload cycle may enhance shielding by increasing the extent of the bridging zone in the crack wake, or conversely diminish shielding by causing the failure of more bridges. Resolution of this effect must await more detailed bridging calculations and experimental measurements of the bridging-zone sizes following various loading sequences.

### 4.4 High temperature fatigue

Despite the anticipated high-temperature use of ceramics in service applications, studies on the cyclic fatigue of monolithic and composite ceramics at elevated temperatures are extremely limited. However, studies on the S/N behavior of MS- and TS-grade Mg-PSZ using 100 Hz rotational flexure (R = -1) tests at moderately high temperatures up to 400°C in air clearly show a progressive decrease in fatigue resistance with increase in temperature, concurrent with a general increase in the inverse slope; e.g., the exponent, n, varies from 34 at 20°C to 116 at 200°C in TS-grade material. Resistance to cyclic fatigue-crack propagation rates is similarly diminished in Mg-PSZ at these temperatures; in TS-grade material, ΔKTH thresholds have been reported to drop from 3.5 MPa √m at 20°C to 3.0 MPa √m at 650°C, with the exponent, n, increasing from 25 to 70 over the same temperature range.

At high homologous temperatures in the creep regime, however, cyclic fatigue mechanisms in ceramics do not appear to be as damaging as static fatigue or creep mechanisms. For example, S/N uniaxial tensile fatigue studies on alumina at 1200°C reveal the lives of samples loaded in cyclic fatigue (R = 0.1) in general to exceed those loaded quasi-statically at the same maximum stress (Fig. 20). Cyclic fatigue lifetimes were found to be dependent upon the frequency and cyclic waveform, specifically to the duration of the application of the maximum stress. In
fact, assuming only time- (and not cycle-) dependent processes, cyclic lifetimes predicted from the static fatigue data were in general less than measured life times, in sharp contrast to behavior at ambient temperatures (Fig. 2(a)). Similar results have been reported for hot pressed Si$_3$N$_4$ at 1200°F. Corroborating evidence from crack-propagation studies in alumina at 1050°C and SiC-whisker-reinforced alumina at 1400°C (frequency=2Hz, $R$=0.1) show the cyclic crack-growth rates to be less than those measured under static loading at the same applied maximum stress intensity (Fig. 21). In these cases, the improved cyclic fatigue resistance has been attributed, at least in part, to the bridging of crack surfaces by the glassy phase, present in grain boundaries or formed in situ by the oxidation of the SiC whiskers.

5. Mechanisms of ceramic fatigue

In metals, mechanisms of cyclic fatigue have been based primarily on crack-tip dislocation activity leading to crack advance via such processes as alternating blunting and resharpening of the crack tip. Accordingly, the refuted existence of a true fatigue effect in ceramics has been based in the past on the very limited crack-tip plasticity in these materials. However, it is now apparent that other inelastic deformation mechanisms may prevail; these mechanisms include microcracking and transformation "plasticity" in monolithic materials, the frictional sliding or controlled debonding between the reinforcement phase and ceramic matrix in brittle fiber or whisker reinforced composites, and the plastic deformation of a metallic or intermetallic phase itself in ductile-particle toughened composites. In fact, these factors are the prime reason for marked resistance-curve ($R$-curve) fracture-toughness properties of ceramic composites, as such crack-tip shielding mechanisms develop progressively (until steady state) with increasing initial crack extension. Such deformation processes, like dislocation activity in metals, can similarly contribute to fatigue damage due to their nonlinear irreversible nature, although the precise micromechanisms of crack advance are presently unknown.

Despite the uncertainty in the mechanistic nature of ceramic fatigue, two general classes of mechanisms can be identified (where failure is associated with a dominant crack), specifically involving either intrinsic or extrinsic mechanisms; these are illustrated schematically in Fig. 22. Intrinsic mechanisms involve the actual creation of a fatigue-damaged microstructure ahead of the crack tip, and thus result in a crack-advance mechanism unique to cyclic fatigue. This could entail crack extension via alternating crack-tip blunting and re-sharpening, as in metals where the consequent striation fracture morphology is distinct from monotonic fracture modes, or through the formation of shear or tensile cracking at the tip due to contact between the mating fracture surfaces on unloading. In fact, fatigue striations have been reported for cyclic failure in zirconia ceramics. Another example is the high tempera-
ture fatigue of SiC-whisker-reinforced alumina: cyclic fractures show clear evidence of whisker breakage, whereas under quasi-static loads the majority of whiskers oxidize to form glass pockets.32)

Extrinsic mechanisms, conversely, do not necessarily involve a change in crack-advance mechanism compared to monotonic fracture; here, the role of the unloading cycle is to affect the magnitude of the near-tip stress intensity by diminishing the effect of crack-tip shielding. Several mechanisms are feasible. These include reduced shielding through an enhanced forward transformation zone in transforming ceramics, although careful Raman spectroscopy measurements have not revealed any change in zone size in Mg-PSZ for cyclic and quasi-static conditions at the same $K_{\text{max}}$.4,50) An alternative mechanism is the reduction in crack-tip bridging via the cyclic induced failure of the bridges in the crack wake, where several possible processes exist. In ductile-particle toughened composites, cyclic loading may simply cause fatigue cracking in the ductile phase, which would otherwise remain intact under quasi-static loads and act as a crack bridge. An example of this is in γ-TiAl intermetallics reinforced with TiNb particles.56) Under monotonic loading, bridging zones of uncracked TiNb particles in excess of 3 mm are formed in the crack wake leading to an elevation in the fracture toughness $K_c$ from 8 MPa m to over 25 MPa m in the composite; under cyclic loading, conversely, the TiNb particles suffer fatigue failure to within ~150 µm of the crack tip, such that the ductile-particle bridging mechanism essentially is obliterated. Also it is feasible that brittle fibers or whiskers in specific composites may similarly fracture and/or buckle due to the unloading cycle, again diminishing shielding by crack bridging under cyclic loads. Finally, in certain monotonic ceramics, such as coarse-grained alumina, where toughening is achieved (primarily for intergranular fracture) by grain bridging or the interlocking of fracture-surface asperities between the crack faces,57,58) in cyclic fatigue, the unloading cycle may cause cracking and/or crushing of the asperities, or may degrade the grain-bridging mechanism through progressive wear of the sliding interfaces. In fact, evidence for the latter mechanism has recently been obtained in alumina using in situ scanning electron microscopy.59)

Indirect evidence for the extrinsic mechanism for cyclic fatigue-crack advance by the suppression of shielding can also be found from studies on small cracks in alumina and silicon nitride.19,29) In a comparison of cyclic and quasi-static behavior, small surface cracks, which because of their limited wake would have developed only minimal shielding, were found to show no acceleration under cyclic loading; long cracks, conversely, were found to be significantly accelerated by the cyclic loads, consistent with their higher shielding levels.29)

6. Design considerations

For structural design where failure results from the propagation of a single dominant crack, cyclic fatigue in ceramic materials presents unique problems. In safety-critical applications involving metallic structures, damage-tolerant design and life-prediction procedures generally rely on the integration of crack velocity/stress intensity $(v/K)$ curves (e.g., Eq. (1)) to estimate the time or number of cycles for a presumed initial defect to grow to critical size. Although such data are now available for many ceramic materials, the approach may prove difficult to utilize in practice because of the large power-law dependence of growth rates on stress intensity, which implies that the projected life will be proportional to the reciprocal of the applied stress raised to a large power.8)

For example, for a cracked structure subjected to an alternating stress $\Delta \sigma$, where the applied (far-field) stress-intensity factor $K$ can be defined in terms of an applied stress $\sigma$ and a geometry factor $Q$ as:

$$K = Q \sigma \sqrt{\pi a}. \quad (7)$$

the projected number of cycles $N_t$ to grow a crack from some initial size $a_0$ to a critical size $a_c$ is given by:

$$N_t = \frac{2}{(m-2)C(Q\Delta\sigma)^{m/2}} \left[ a_0^{(m-2)/2} - a_c^{(m-2)/2} \right], \quad (8)$$

assuming a crack-growth relationship of the form of Eq. (1) (for $m \neq 2$). For metallic structures where the exponent $m$ is of the order of 2 to 4, a factor of two increase in the applied stress reduces the projected life by roughly an order of magnitude; in ceramic structures, conversely, where $m$ values generally exceed 20 (and can be as high as 50 to 100), this same factor of two increase in stress reduces the projected life by some six to thirty orders of magnitude!

Thus, conventional damage-tolerant criteria for ceramics imply that marginal differences in the assumption of the component in-service stresses will lead to significant variations in projected life. Furthermore, any fluctuation of the applied stress and possible overloads frequently encountered in service may also result in highly non-conservative design lives. Two additional features of the approach provide further complication. First, in many practical applications, acceptable projected component life may only be guaranteed by restricting the initial defect size to extremely small sizes, often below the resolution of non-destructive evaluation techniques. Expensive and time consuming on-line scanning electron microscopy or proof testing at elevated loads of individual components are therefore required; in fact, these procedures are currently in use in quality control procedures for pyrolytic-carbon prosthetic heart-valve devices. Second, and perhaps most im-
portant, the approach generally does not consider, and in this case cannot easily be modified to include, small-crack effects such as those described above.

An alternative procedure is to redefine the critical crack size in terms of the fatigue threshold, $\Delta K_{TH}$, below which crack growth is presumed dormant; this in essence is a crack-initiation criterion where $\Delta K_{TH}$ is taken as the effective toughness, rather than the fracture toughness $K_c$. This procedure, in addition to sharing similar problems associated with defining a minimum detectable defect size, also does not address small-crack effects which may arise at loads considerably below those required for the growth of long cracks. As a result, although data on the small-crack effect in ceramic fatigue are very limited, the observed sub-threshold crack-growth behavior implies that conventional damage-tolerant design criteria may be again highly non-conservative for ceramics.

Like many high strength metallic materials, however, crack-initiation effects may involve a very significant portion of lifetime under alternating loads. Having referred to a number of potential limitations of a fracture-mechanics approach to ensure component reliability, it is important to note that these approaches are typically highly conservative in the sense that they assume crack growth from the first loading cycle. Although the degree of this conservatism is difficult to quantify, due in part to the paucity of data from crack-initiation studies, the disadvantages of the approach may in fact be considerably outweighed by neglecting the cycles required for initiation. Selection of a damage-tolerant approach is, therefore, at present undertaken on a case by case basis, often affected by the requirements of appropriate regulatory agencies.

The importance of fatigue-crack initiation does, however, suggest an additional design criterion for ceramic components; this more traditional approach is invariably based on stress/life $S/N$ data. In addition to simple consideration of the fatigue limit, a more sophisticated approach might include a damage-mechanics analysis utilizing detailed finite-element stress analyses for components of complex shape and a statistical evaluation of the pre-existing defect population. While this approach has serious limitations in some applications, particularly in large structures where numerous defects may be present and therefore fatigue-crack initiation periods diminished, it might be quite appropriate for small ceramic components. Clearly, more indepth studies of both fatigue-crack initiation and crack-growth behavior, together with improved design criterion are required to enhance the in-service reliability and life prediction of ceramic and ceramic-matrix components subjected to alternating loads.

7. Conclusions

(1) Results on a wide range of monolithic and composite ceramics clearly indicate a strong susceptibility to mechanical degradation under cyclic loading. Ceramic fatigue is promoted by inelastic deformation mechanisms, such as microcracking and transformation "plasticity" in monolithic ceramics and irreversible deformation in the reinforcement phase or its interface with the matrix in composites.

(2) Stress/life ($S/N$) data show lifetimes to be shorter under cyclic loading compared to corresponding lifetimes under quasi-static (static fatigue) loading at the same maximum stress. In many ceramics, "fatigue limits" at $10^6$ cycles ($R = -1$) approach $\sim 50\%$ of the single-cycle tensile (or bending) strength. Data on unnotched samples indicate that cycling in tension-compression ($R = -1$) is generally more damaging that in tension-tension ($R > 0$).

(3) With the exception of SiC, cyclic fatigue lifetimes in $S/N$ tests at ambient temperatures are less than those predicted from static fatigue data assuming only time-dependent (and not cycle-dependent) processes; conversely, at high homologous temperatures, results for alumina at 1200°C suggest that the reverse is true.

(4) Cyclic fatigue-crack growth rates for long ($> 3 \text{ mm}$) cracks are found to be power-law dependent on the applied stress intensity ($\Delta K, K_{max}$), with an exponent $m$ much larger than typically observed for metals (i.e., in the range $\sim 10$ to over 100). Crack-growth behavior is sensitive to several factors including microstructure, load ratio, environment and crack size. Fatigue threshold stress intensities ($\Delta K_{TH}$), below which long cracks are presumed dormant, approach $\sim 50\%$ of the fracture toughness ($K_c$).

(5) At ambient temperatures, crack-growth rates under cyclic loading generally exceed those by static fatigue (at the same $K_{max}$) by many orders of magnitude. Conversely, at high homologous temperatures (1000°C to 1400°C) in the creep regime, cyclic fatigue-crack growth rates in monolithic and SiC-reinforced alumina have been observed to be less than those measured under quasi-static loads.

(6) Cyclic fatigue-crack growth rates for small ($< 250 \mu\text{m}$) surface cracks are found to be in excess of corresponding long cracks at the same applied stress intensity, and furthermore to occur at stress intensities significantly smaller than the nominal long-crack threshold, $\Delta K_{TH}$. Similar to metallic materials, small-crack growth rates show a negative dependency on applied stress intensity and appear sensitive to the applied stress level. Such behavior is attributed to diminished crack-tip shielding (e.g., by transformation toughening in Mg-PSZ or crack bridging in whisker- or fiber-reinforced composites) with cracks of limited wake.

(7) The fractography of failures by cyclic fatigue is frequently similar and often indistinguishable from corresponding failures under quasi-static loading. Such similarity complicates the post-fracture failure analysis of ceramic components.
Although precise micro-mechanisms of cyclic fatigue are currently uncertain, two classes are identified. Intrinsic mechanisms involve the creation of an inherent fatigue-damaged microstructure ahead of the crack tip and hence result in a crack-advance mechanism unique to cyclic fatigue (as in metals). Extrinsic mechanisms involve no change in crack-advance mechanism compared to that under quasi-static loading; here the unloading cycle acts to enhance the near-tip stress intensity by diminishing the role of crack-tip shielding.

Cyclic fatigue-crack growth-rate data, particularly pertaining to small cracks, have significant implications for damage-tolerant lifetime prediction and design criteria in ceramic materials. The observed marked dependency of growth rates on the stress intensity implies an excessive dependency of projected lifetime on applied stress; this leads to difficulties in applying conventional damage-tolerant procedures. An indepth understanding of the initiation and growth of microstructurally-small cracks is required to provide more reliable design criteria for safety-critical applications.

Acknowledgments The work on monolithic ceramics was supported by the Director, Office of Energy Research, Office of Basic Energy Sciences, Materials Sciences Division of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098; research on composite ceramics was funded by the Office of Naval Research under Grant No. N00014-89-J-1094.

References

24) T. Fett and D. Munz, Proc. 7th World Ceramics Congress (CIMTEC), Montecatini Therme, Italy, 1990, in press.
1062 Cyclic Fatigue of Ceramics: A Fracture Mechanics Approach to Subcritical Crack Growth and Life Prediction


Robert O. RITCHIE has been Professor of Materials Science at the University of California in Berkeley for ten years and is presently Director of the Center for Advanced Materials and Deputy Director of the Materials Sciences Division in the Lawrence Berkeley Laboratory. He holds M. A., Ph. D. and Sc. D. degrees in Physics and Metallurgy/Materials Science, all from Cambridge University in England. His research is focused on the mechanistic aspects of fatigue and fracture mechanics in engineering materials including advanced metals, intermetallics and ceramics and their respective composites, which recently earned him (with Reiner Dauskardt) the U.S. Department of Energy Most Outstanding Scientific Accomplishment Award in Metallurgy and Ceramics.

Reiner H. DAUSKARDT is currently a Research Scientist in the Department of Materials Science at the University of California in Berkeley and in the Center for Advanced Materials at the Lawrence Berkeley Laboratory. He holds Bachelors and Masters degrees in Physics, Mathematics and Materials Science from the University of the Witwatersrand in Johannesburg, and completed his Ph. D. degree at the University of California in Berkeley in 1987. His current research interests lie in the study of the fracture and fatigue behavior in advanced materials and ceramic/metal interface systems. Dauskardt is well known for his research on cyclic fatigue in ceramics for which he was a recipient of the U.S. Department of Energy Most Outstanding Scientific Accomplishment Award in Ceramics and Metallurgy in 1989.