Fundamental theory of void fraction of cohesive spheres with logarithmic normal size distribution

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The fundamental theory of void fractions with an arbitrary size distribution was applied to a system with a lognormal distribution. The theoretical void fraction was in good agreement with the experimental values for a glass-sphere system. We also considered the particle-shape effect under extreme conditions for spherical particles and crushed particles in a system having a lognormal distribution.

Key-words: Void fraction, Size distribution, Logarithmic normal distribution, Standard deviation, Glass spheres, Crushed glass, Shape effect

1. Introduction

The fundamental theory of void fraction of cohesive spheres with size distribution is unique to the calculation of the void fraction of a system in which the size distribution is represented by a pure mathematical function. One of the authors (Y. Aikawa) formulated the fundamental theory of void fraction with an arbitrary size distribution based on a statistical method by using a modified version of the method of author M. Suzuki. This report addresses the problem of the void fraction of a system in the case where the size distribution is represented by a lognormal distribution function.

It is problematic to analyze theoretically the void fraction of cohesive spheres with a lognormal distribution function. Moreover, it is known empirically that an increase in the standard deviation of a lognormal distribution is accompanied by a decrease in the void fraction. One author (M. Suzuki) has shown experimentally the correlation between the standard deviation and the void fraction by using glass spheres, and has been confirmed the expected relationship between the standard deviation and the void fraction notated above.

However, the experimental results from a lognormal distribution system have not yet been supported theoretically. In this report, the void fraction of the lognormal distribution was calculated from the fundamental theory, and the theoretical result was compared with the experimental result. This is an application of the fundamental theory to a lognormal distribution function, a typical arbitrary distribution function.

In addition, the particle-shape effect of the void fraction is very important concern when calculating the void fraction of a non-spherical particle system. To consider the shape effect, we addressed the problem with extremes of spherical and crushed particles. As a result, it emerged that the particle-shape effect on the void fraction is represented by a numerical factor, which is multiplied by the void fraction of the spheres in the system.

2. Theory

According to Y. Aikawa et al., the void fraction, \( \delta \), in an arbitrary distribution system can be determined using the following equation:

\[
\delta = 1 - p.
\]

where

\[
p = \sum_i \frac{r_i^2 f(r_i)}{\langle r_i^2 \rangle} p_{ji}(\text{max}).
\]

Where, \( r_i \) is the radius of the \( i \)-th particle, \( f(r_i) \) is the distribution function of the system of spherical particles, and the notation \( \langle \cdot \rangle \) denotes the average:

\[
\langle r^p \rangle = \sum_i r_i^p f(r_i),
\]

and \( p_{ji}(\text{max}) \) is defined as follows:

\[
p_{ji}(\text{max}) = 1 - \delta_{ji}(\text{min}),
\]

where \( \delta_{ji}(\text{min}) \) is the void fraction around the \( j \)-th particle within the radius of the hypothetical sphere \( \sqrt{(r_j + r_i)^2 - r_i^2} \) densely enclosed by \( i \)-particles of equal radius \( r_i \). We can calculate the void fraction given by Eq. (1) with any arbitrary size distribution \( f(r) \) by using the \( \delta_{ji}(\text{min}) \) value which is calculated by a geometrical analysis or numerical calculation.

3. Experiment

Spherical glass beads and crushed glass with a density of \( 2.49 \times 10^3 \text{ kg/m}^3 \) were used for the experiments. The samples of crushed glass are prepared by using a type of mill. The test particles were classified using \( 2^{1/4} \) series JIS standard sieves and were mixed to adjust to a lognormal distribution. Before the packing tests, the glass particles were dried in vacuum desiccators at 120°C for more than 2 h. The packing tests were carried out using a high-speed high-pressure air flow.
out using a Hosokawa Micron Powder Tester in an air-conditioned room at 50–60% relative humidity and 20–25°C room temperature. A cylindrical stainless steel container of 50.5 mm inner diameter and 50 mm depth was used for our packing tests. The packing volume of the container is 100 cm³. Two packing methods were used for our experiments.

1) Tapping: Particles were packed in the container to a volume of 130–150 cm³ and a height of 40 mm, the plastic cover and the container includes the particles and the plastic cover was tapped 180 times from a height of 20 mm.

2) Prodding: 40–50 cm³ particles were packed in the container with the plastic cover and prodded 25 times by a 7 mm diameter glass rod. 40–50 cm³ particles were added and prodded as the same process and another 40–50 cm³ particle were added and prodded as same process again.

After the packing operation, the plastic cover was removed and the particle over the container was cut by a stainless steel blade. The total weight of particles in the container was measured to determine the void fraction of the particle packed bed.

4. Comparison between experimental and theoretical results

The lognormal distribution function is given by

\[ f(r_i) = \frac{1}{\sqrt{2\pi} \sigma \ln \sigma} \exp \left[ -\frac{1}{2} \left( \frac{\ln r_i - \ln \langle r_i \rangle}{\ln \sigma} \right)^2 \right], \quad (4) \]

where \( \sigma \) is the standard deviation. The function \( f(r_i) \) is shown by Fig. 1 at various \( \sigma \) values. By applying Eq. (4) to Eq. (1), the void fraction as a function of \( \sigma \) is calculated as shown in Fig. 2. The \( \sigma \)-dependency of the theoretical curve is consistent with the experimental values both qualitatively and quantitatively. In the range between the minimum and maximum radii of the glass spheres is too large, there arise a segregation because that the fine particles drop to bottom by threading their way through the large particles. Therefore, it is very difficult to construct a uniformed random packing state at \( \sigma \geq 2.5 \) because of the segregation, and accordingly the value of the void fraction in this region is not confirmed from experiments.

To analyze the particle-shape effect, an extreme example, we prepared crushed glass samples, shown in Fig. 3. The experimental void fraction of these samples is shown in Fig. 4, and the solid curve is 1.25 times the theoretical void fraction of spheres. The main factor influencing the void fraction for crushed glass is represented by a numerical coefficient \( n = 1.25 \), which is characterized by the grain-shape effect, as shown in Fig. 3. It is considered that the grain-shape effect, \( n \), reflects the statistical grain shape in a particle system. The value of the coefficient \( n \) corresponding to each statistical grain shape must be determined experimentally.

5. Conclusion

The theory of the void fraction of cohesive spheres with an arbitrary size distribution has been applied to a lognormal size distribution of spheres. As a result, the theoretical calculation is almost consistent qualitatively and quantitatively with the experimental result. Moreover, the experimental data of the void fraction of another lognormal distribution system, consisting of crushed glass, is accordance with \( n = 1.25 \) times the theoretical void fraction for spheres. If the statistical relation
between $n$ and the corresponding grain shape should be shown empirically, it would very useful for calculating the void fraction of various grain-shape systems in the field of rheology.

References