FULL PAPER

Estimation of the stress σ_{CR} generated at propagating crack front in zone-tempered glass

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High-speed photo-elastic color photography was introduced to a study of dynamic crack propagation in zone-tempered glass using recent photo-processing technologies. The study had a special focus on the value of stress σ_{CR} generated at the propagating crack front. It was recognized that the crack connection phenomenon could be explained using the stress σ_{CR} . The values of stress σ_{CR} were estimated at about 5 MPa in zone 1 and 3–4 MPa in zone 2 utilizing the color change of the stress field when propagating at terminal velocity. A slant-propagating crack was observed, which could be explained by a generation mechanism involving the stress σ_{CR} and the stress release, and propagation in the same way as other general crack propagation based on the maximum normal stress criterion.

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1. Introduction

Tempered glass has been widely used in the windows of buildings, automobiles, trains etc., because of its high strength as compared with ordinary float glass and its fragmentation when fractured. The fragmentation phenomenon is extremely important, because it gives tempered glass the characteristics of safety glass. As crack propagation in tempered glass is a very high-speed phenomenon, analysis of the fragmentation is not easy.

Residual inner tensile stress is known to affect the fragmentation phenomenon; a relationship between fragment density and tensile stress in tempered glass were reported, for example.^{1),2)} The authors reported that fragment density and spline crack generation were dependent on the local residual stress pattern,³⁾ the crack velocity in tempered glass was identified as about 1.50×10^3 m/sec,⁴⁾ and the difference in propagating energy before and after the crack divergence⁵⁾ was measured utilizing a Cranz-Scharadin high-speed camera.⁶⁾ The crack velocities and transverse and longitudinal velocities of shock waves were measured at the same time,⁷⁾ and moreover, it was observed that collisions of reflective transverse waves with a propagating crack front affected the fragmentation.⁸⁾

The maximum normal stress criterion for propagation of a crack in glass in the normal direction with the maximum principal stress is widely known.⁹⁾ As ordinary tempered glass has been produced to possess the most uniform stress

Fig. 1. A typical fracture pattern in the central area of zone-tempered glass.

distribution possible, the principal stress difference ($\sigma_1 - \sigma_2$) should ultimately be small. Zone-tempered glass with coarse fragment zones in central area and fine fragment zones in the outer area was proposed. An example of the fracture pattern in the central area of zone-tempered glass is shown in Fig. 1. The coarse fragmentation in the central area of zone-tempered glass is produced by arranging the two zones alternately (with mainly vertical crack propagation in zone 1 and mainly transvers crack propagation in zone 2) by changing the principal stress distribution. From this standpoint, zone-tempered glass has characteristics not of “partly tempered glass” but of “partly non-uniform tempered glass”. Analysis of crack propagation seemed to become easier when using zone-tempered glass. Few reports have been published concerning crack propagation using zone-tempered glass specimens, how-

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ever, due to prohibition of the use of zone-tempered glass for automobile windshields.

The authors^{10),11)} reported a crack connection phenomenon and a neutral line movement in zone-tempered glass by simultaneous observation of the crack propagation and change in the stress field, using high-speed photo-elastic color photography. The existence of a stress σ_{CR} at the propagating crack front was proposed to explain this crack connection phenomenon. This stress was also confirmed from the observation results.¹²⁾ That is, crack propagation should be induced by stress comprising the sum of the stress σ_{CR} and residual stress in zone-tempered glass, and the value of the stress σ_{CR} should not be neglected.

However, no study concerning the stress σ_{CR} generated at propagating crack front, including its existence and values, has been reported. Cracks which seem to be propagated in different directions based on the maximum normal stress criterion in zone-tempered glass can be observed. And the seemingly strange generation and propagating mechanism of these cracks has not yet been fully explained.

Photo-processing technology has advanced recently, and the brightness of photographs can be adjusted to desirable levels. As a result, new information has become obtainable from photographs that could not be utilized former by due to illegibility.

Crack propagation in zone-tempered glass was investigated utilizing new photographs whose brightness could be improved. This study focused especially on estimation of the stress σ_{CR} at the propagating crack front and observation of seemingly strange crack propagation in the slant direction.

2. Experimental procedure

2.1 Specimens

Zone-tempered glass specimen plates made by Central Glass Co., Ltd. were used in the study. The size was 600 mm long, 400 mm wide and nominal 5 mm thick. The zone-tempered glass specimens were produced using the ordinary tempering method in which heated glass is quenched by air through multiple nozzles. Tempering was conducted by two-step quenching in the central area so that the principal stress values and directions of two zones were different. While the tempering was conducted using ordinary quenching in the outer area, uniform quenching was possible even in zone-tempered glass. The chemical composition of the glass specimens is shown in **Table 1**.^{10)–12)}

2.2 Observation method

An in-laboratory constructed Cranz-Schardin-type camera⁶⁾ was used for high-speed photography. As this camera

had 30 sets of light sources and lenses, a maximum of 30 photographs could be acquired. **Figure 2** shows the experimental set-up. Light from the light source was transmitted to the camera through the specimens, a reflective mirror, two polarizing plates (polarizer and analyzer) in the crossed Nicol state, a tint plate (a sensitive color plate) and a focusing lens. An air gun propelled a small bullet of cement carbide with a conical head of velocity of about 25 m/sec, such that the bullet impacted the glass specimen. To ensure that specimen deformation upon collision with the impactor was as small as possible, a very light weight bullet of 0.2 g was used.

2.3 Principles of the observation method

High-speed photo-elastic color photography was used to observe crack propagation and changes in stress fields simultaneously. The birefringence phenomenon is generated when the stress occurs in a glass specimen. The phase difference δ generated by the birefringence phenomenon is acquired from Brewster's law by the following equation:

$$\delta = (2\pi d/\lambda)C(\sigma_1 - \sigma_2) \quad (1),$$

where d is the thickness of the glass specimen, λ the wavelength of the incident light, C the light elastic modulus, and σ_1 and σ_2 the principal stresses. As the thickness d of the glass specimens was 5 mm and the stress optical coefficient C was constant for the float glass composition, the phase difference δ is a function of the wavelength of incident light and the difference in the two principal stresses. Phase difference δ in Eq. (1) can be introduced to the equation of retardation R as below:

$$R = dC(\sigma_1 - \sigma_2) \quad (2)$$

With white light used as the light source and a tint plate between the two polarizing plates, retardation R can be observed as the change of color.

Figure 3 shows the stress fields before fracture by photo-elastic color photography. Under these experimental conditions, blue, white-blue and brown colors were observed in zone 1, deep-blue, light-blue, yellow and red-violet in zone 2, and black in the boundary area between zone 1 and zone 2. The blue color indicates the area with the highest stress difference ($\sigma_1 - \sigma_2$) in zone 1, where the strongest quenching was conducted through a quenching nozzle when the zone-tempered glass was produced, and

Fig. 2. An experimental set-up for high-speed photo-elastic color photography.

Table 1. Chemical composition of a glass specimen

SiO ₂	Al ₂ O ₃	CaO	MgO	Na ₂ O	K ₂ O	Others
72	2	8	3	13	1	1
Mass %

Fig. 3. Stress fields before fracture by photo-elastic color photography.

Table 2. Measurement results for the residual surface compressive stress at the center of each zone

Zone	σ_1 (MPa)	σ_2 (MPa)
1	112 ± 3.4	103 ± 2.2
2	70 ± 4.0	88 ± 5.8

the red-violet color in zone 2 indicates the area with the highest stress ($\sigma_1 - \sigma_2$) in zone 2, respectively. Black indicates a neutral line where ($\sigma_1 - \sigma_2$) was zero.

2.4 Surface stress measurement

The presence of compressive stress in the surface area and tensile stress in the inner area and of crack propagation based on the inner tensile stress are well known. Measurement of inner tensile stress is extremely difficult and unreliable. There is no doubt that the value of inner tensile stress is approximately half⁽⁴⁾ the value of surface compressive stress. Since measurement of surface compression is highly reliable compared with that of inner tensile stress, the value of surface compressive stress was measured instead of that of inner tensile stress.

Surface stress was measured with an FSM-10 surface refractometer made by Toshiba Glass Co., Ltd. The measurement results for surface stress are shown in Table 2.^{(10)–(12)} The surface stress was measured at the center of each zone. Thus, the values in Table 2 are the results of measurements at the center of the blue area in zone 1 and at the center of the red-violet area in zone 2.

3. Results and discussion

3.1 Vivified photograph and crack connection

The crack connection phenomenon resulting from upward and downward crack propagation in zone 1 seem a series of photographs taken at interval of $10\mu\text{sec}$ has been reported.^{(10),(11)} Figure 4 shows a series of photographs taken at intervals of $5\mu\text{sec}$ of the same phenomenon with the addition of the photographs in Figs. 4(a) and 4(c). Figures 4(b) and 4(d) have already been reported.^{(10),(11)} The brightness of Figs. 4(a) and 4(c) has been improved by the new photo-processing technology.

Fig. 4. A series of photographs taken at intervals of $5\mu\text{sec}$.

Fig. 5. The model of the stress σ_{CR} generated by a propagating crack front.

A model⁽¹²⁾ of the stress σ_{CR} generated at the propagating crack front was proposed to explain the crack connection phenomenon as shown in Fig. 5. The existence of stress σ_{CR} has been revealed by observation of the propagating crack front, and the stress σ_{CR} is not small enough to be ignored. It gives a different stress field compared with residual stress only. Two cracks propagated from upwards and downwards directions in zone 1 were induced by the addition of stress σ_{CR} to the maximum principal stress σ_1 . That is to say, upward crack and downward crack can be understood based on the collision of specify outgoing information for the stress σ_{CR} . The connection phenomenon of the two cracks in zone 1 can thus be explained.

As the intervals between observations could be shortened, more precise analysis was possible. While a color change caused by the principal stress difference ($\sigma_1 - \sigma_2$) was observed at the nozzle marks, which had the largest value of surface compressive stress (inner tensile stress) in the glass specimens, a color change caused by the stress σ_{CR} was observed at the propagating crack front shown in Fig. 4(a). Color changes caused by stress fields at the propagating crack fronts were affected by the sum of the residual stress σ_1 and the stress σ_{CR} shown in Fig. 4(b), while the residual stress was observably smaller. This tendency seemed to be emphasized as shown in Fig. 4(c), and the stress fields in front of propagating cracks also seemed to be connected with the residual stress σ_1 and the stress σ_{CR} . The crack connection phenomenon could be observed as shown in Fig. 4(d). It was difficult to judge the stress field changes from the photographs of Figs. 4(b) and 4(d) alone.

The crack connection phenomenon actually occurred in other location, too, where similar crack connection and stress field change could be observed. This suggests that the connection phenomenon is not particular but is made possible by the coincidence of the stress field and the crack propagating conditions.

3.2 Estimation of stress σ_{CR}

Value of σ_{CR} can be estimated approximately from color changes in the stress field observed in photo-elastic color photography. As mentioned, there is a close relationship between the principal stress difference and the color.

Table 3 shows the colors and principal stress difference values conjectured from surface compressive stresses with linear changes assumed according to the distance.¹²⁾ The values of the surface compressive stress in Table 2 were measured by surface refractometer at the center of each zone. For example, the value 112 ± 3.4 (MPa) of σ_1 was the result measured in the blue area of zone 1, and the value 103 ± 2.2 (MPa) of σ_2 was measured at a normal angle for the σ_1 direction of the same area in zone 1. Thus, the principal stress difference ($\sigma_1 - \sigma_2$) in the blue area was calculated as 9 MPa compressive stress. In the same way, the principal stress difference ($\sigma_1 - \sigma_2$) in the red-violet area of zone 2 area was calculated as 18 MPa compressive stress. The minus symbol indicates a crack propagated in the horizontal direction. The relationship between the change in the principal stress difference and the distance is thought of linear, and the principal stress difference in areas with other colors can be guessed. The principal stress difference at the center of the blue area in zone 1 was 9 MPa and that at the neutral line was 0 MPa, and the distance was about 15 mm. The principal stress difference at the center of the white-blue area which was 6.5 mm away from the neutral line, for example, was calculated as about 4 MPa compressive stress.

Three main methods can be considered for conducting the estimations. One analysis method is to use the neutral line movement from the original position before fracture to another position. The neutral line was observed as black, and it moved from the position before the fracture shown in **Fig. 6**. As the neutral line indicating ($\sigma_1 - \sigma_2$) is nearly to equal zero MPa, the value of the stress σ_{CR} is estimated from the color change from the original position of the neutral line or original color before fracture to the color after neutral line movement. As the color changed from

black to white-blue, the value of stress σ_{CR} was estimated about 4 MPa of surface compressive stress and 2 MPa of inner tensile stress.

The second method is to use the color changes of the crack propagating front in zone 2 shown in **Fig. 7**. The color of the crack front in zone 2 changed from deep-blue to light-blue. This color change corresponded to about 3 MPa of surface compressive stress and 1.5 MPa of inner tensile stress.

Figure 8 shows the third method, which is to use the changes in color at the propagating crack front in zone 1. The color of the crack front in zone 1 changed from white-blue to blue. This color change corresponded to about 5 MPa of surface compressive stress and 2.5 MPa of inner tensile stress.

The above values for the stress σ_{CR} seemed too small compared with the surface stress, but the color of a stress field can be changed to a different color with a small amount of stress. The color is changed by stress release with crack propagation from white-blue to brown, for example, with only 2 MPa of surface compressive stress in

Fig. 6. Movement of the neutral line in zone 2: (a) Movement of the neutral line in the stress field; (b) stress field before fracture.

Fig. 7. Color change in the propagating crack front in zone 2.

Table 3. Relationship between colors and the principal stress difference in the surface compression in zone 1 and zone 2

Zone	Color	$\sigma_1 - \sigma_2$ (MPa)
1	Blue	9
1	White-blue	4
1	Brown	2
Neutral	Black	0
2	Deep-blue	-3
2	Light-blue	-6
2	Yellow	-9
2	Red-violet	-18

Fig. 8. Color change in the propagating crack front in zone 1.

zone 1. Also, the color in zone 2 is changed from light-blue to deep-blue with only 3 MPa of surface compressive stress. The stress value σ_{CR} , which was estimated from the color change of the stress field in zone 1 and zone 2 can be assumed not to be too small.

The value of stress σ_{CR} should be expected to be dependent on the crack velocity. The crack velocity, in turn, is dependent on the surface stress.⁴⁾ Since surface stress in zone 1 is larger than that in zone 2, as shown in Table 2, the crack velocity in zone 1 is considered to be faster than that in zone 2. Therefore, the stress σ_{CR} in zone 1 has the possibility of being large compared with the stress to be σ_{CR} in zone 2. Thus, the results showing the stress σ_{CR} in zone 1 to be about 5 MPa and the stress σ_{CR} in zone 2 to be 3–4 MPa of surface compressive stress can be considered likely to be valid.

The values for stress σ_{CR} do, however, lack some degree of reliability due to possible errors in color change judgments and surface stress measurement. Due to lack of a quantitative approach, the stress values should be shown for reference purposes only at the present time.

3.3 Crack propagation for slant direction with maximum principal stress

Generally, a crack is propagated at a normal angle with maximum principal stress in glass. This phenomenon is widely known as “the maximum normal stress criterion”,⁹⁾ except when propagated immediately at crack divergence with the principal stress difference being extremely small, etc. **Figure 9** shows crack propagation in two areas of the zone, and many cracks in both areas can be observed being propagated in the vertical direction in this photograph. These cracks are considered to be propagated based on the maximum normal stress criterion. One crack to the extreme right in the right area of zone 1, indicated by the yellow arrow, is observed to be propagated in the slant direction rather than vertically, despite being propagated in middle area of zone 1. Although the principal stress σ_1 is definitely large compared with σ_2 , the above crack seemed to be propagated according to the maximum normal stress criterion. This one crack will be treated as a “slant-propagating crack” in this study.

Fig. 9. A slant-propagating crack observed in zone 1, which is indicated by the yellow arrow.

Fig. 10. The slant-propagating crack: (a) after about 10 μ sec from the divergence; (b) after about 30 μ sec from the divergence.

The slant-propagating crack seemed to be unusually independent of the maximum normal stress criterion. In this sense, it is a very rare case of the crack-propagating phenomenon. Few papers have reported it, however, including its existence and propagating mechanism, although this slant-propagating crack should be pointed out as an unusual crack.

Figure 10 shows photographs of slant-propagating cracks. Figure 10(a) shows a photograph after about 10 μ sec from crack divergence, and Fig. 10(b) after about 30 μ sec from crack divergence.

The symbols A_a and A_b indicate the slant-propagating cracks, B_a and B_b the cracks adjoining to the slant-propagating cracks A_a and A_b , and C_a the crack adjoining to B_a . There was no relationship between crack C and the slant-propagating crack generation, however, judging from the observation results. Crack D could be omitted from the analysis of the slant-propagating crack generation, because the propagation and connection of this crack were much later. And the yellow circle is the area including the start of the slant-propagating crack, i.e., the generation of crack divergence.

Figure 11 shows the slant-propagating crack generation phenomenon observed in a different specimen, which was produced by the same method and under the same conditions. In this case, three slant-propagating cracks A_{11} ,

Fig. 11. Three slant-propagating cracks generated in zone 1.

Fig. 12. Slant-propagating cracks observed in zone 2.

A_{12} and A_{13} were observed. Cracks B_{11} and D_1 corresponded to the adjoining cracks with characteristics similar to cracks B_a , B_b and D shown in Fig. 10.

Figure 12 shows crack propagation in the case of slant-propagating cracks in zone 2. Three slant-propagating cracks A_{21} , A_{22} and A_{23} were observed in this photograph. In this case, the adjoining cracks should be considered B_{21} and B_{22} representing two-crack symmetry. As described above, the number of slant-propagating cracks is not always one but can be two, three or more. The brightness of the photographs shown in Figs. 10–12 was effectively improved as compared with that of the original photos using current photo-processing technology.^{10),11)}

When considering the mechanism of the slant-propagating crack, the start of the crack, or generation of the crack divergence, should be investigated first. Two cracks A and B generated after the divergence were propagated at an adequate distance as if separated by a repulsive force as shown in Fig. 11. While cracks are not always propagated according to the maximum normal stress criterion immediately after divergence, a certain distance is needed for crack propagation according to the maximum normal stress criterion.

Two cracks generated by the crack divergence were propagated in their own respective ways, and two propagation types with a preceding crack and succeeding crack were observed, as shown in Fig. 10. The preceding crack corresponded to the adjoining crack B, the succeeding crack was the slant-propagating crack A, and the propaga-

tion relationship of the preceding crack and succeeding crack was maintained in this area. Most of the residual stress was maintained by the preceding crack propagation, and the stress of σ_1 and/or $(\sigma_1 - \sigma_2)$ decreased after propagation of crack B. Thus, the succeeding crack was propagated under conditions of decreasing residual stress. Since the stress field at the slant-propagating crack front placed a priority on stress σ_{CR} over the residual stress σ_1 , crack A was propagated in a straight line based on the stress σ_{CR} . As a result, crack A, i.e., the slant-propagating crack propagated in a different direction under the influence of the maximum principal residual stress σ_1 .

The mechanism of slant-propagating crack generation is considered to be summarized by the steps shown as below:

- 1) Generation of the crack divergence.
- 2) Generation of two adjoining cracks by the crack divergence.
- 3) Propagation of two cracks in their own respective ways under the influence of the maximum principal stress, the principal stress difference and the stress σ_{CR} .
- 4) Existence of two propagation types, the preceding crack and the succeeding crack.
- 5) Continuation of the relationship between the preceding crack and succeeding crack.
- 6) Decrease in the maximum principal stress and/or principal stress difference.
- 7) Priority of σ_{CR} over the maximum principal residual stress.
- 8) Straight-line propagation based on the stress σ_{CR} .
- 9) Generation of a “slant-propagating crack” that is propagated in a different direction with the maximum principal residual stress.

In the case of multiple slant-propagating crack generation, as shown in Fig. 11, a similar phenomenon is considered to be the cause. Thus, the preceding crack B_{11} was observed in front of crack A_{11} , and crack B_{11} was propagated in a straight line by the priority of σ_{CR} to the maximum principal residual stress σ_1 . Propagation of crack A_{12} was observed to proceed in a way similar to crack A_{11} which corresponded to the preceding crack with respect to crack A_{12} . Crack A_{13} was to be thought similar crack A_{12} in its correspondence to the preceding crack. Propagation of crack B_{11} seemed to proceed in a similar way the slant-propagating crack in the beginning of its entry into zone 1. The angle of incidence to zone 1 is thought to be an important factor for the crack propagation, which is affected by the principal stress difference.

The propagating phenomenon in zone 2 was similar, as shown in Fig. 12. In this case, two preceding cracks B_{21} and B_{22} were generated by the crack divergence and propagated in symmetry. Cracks B_{21} and B_{22} seemed to be propagated a slight angle to the normal direction with maximum principal stress σ_2 in zone 2, and were considered to be immediately affected by the divergence. Other conditions were to be considered closely similar as crack A_{21} , for example, was propagated in a straight line due to the priority of σ_{CR} generated by the preceding crack

propagation of crack B_{21} on the principal stress σ_2 . The slant propagation of cracks A_{22} and A_{23} can be explained by the above mechanism.

The relationship between the crack length from a divergence to the next divergence and the load stress was reported.¹³⁾ While the slant-propagating crack was propagated without divergence, the other cracks were propagated with divergence for some distance, as shown in Fig. 9. Crack divergence is thought to be caused when the propagating energy exceeds a certain constant value.¹⁴⁾ And slant-propagating cracks are propagated by the terminal velocity, as are other propagating cracks. This means that slant-propagating cracks are propagated by energy generated between the propagation by terminal velocity and the divergence. Propagation of the slant-propagating crack is inferred to be affected mainly by the crack divergence, the stress release and the stress σ_{CR} . Introducing the concept of stress σ_{CR} , enabled the slant-propagating crack to be explained. The slant-propagating crack is considered to be propagated in the same way as other general cracks based on the maximum normal stress criterion.

4. Conclusions

High-speed photo-elastic color photography was introduced for a study of dynamic crack propagation in zone-tempered glass utilizing recent photo-processing technology. The study focused specially on the stress value σ_{CR} generated at the propagating crack front. The results obtained were as follows:

- 1) It is recognized that the crack connection phenomenon can be explained using the stress σ_{CR} value in analyzing photographs using recent photo-processing technology.
- 2) The values of the compressive stress σ_{CR} were estimated at about 5 MPa in zone 1 and 3–4 MPa in zone 2 using the color changes under these experimental condition.
- 3) A slant-propagating crack was observed, which could be explained by the generation mechanism using the stress σ_{CR} . This slant-propagating crack is considered to be propagated in the same way as

other general cracks based on the maximum normal stress criterion.

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