To gain a better insight into the failure mechanism of reinforced sand, a FEM simulation of plane strain compression tests of dense Toyoura sand reinforced with planar reinforcement having a wide range of stiffness was performed. A new elasto-plastic constitutive model for sand, having a stress path-independent work-hardening parameter based on modified plastic strain energy, is applied. The constitutive model of sand is capable of simulating the effects on the deformation characteristics of stress history, stress path and pressure level, taking into account inherent anisotropy in the elastic and plastic properties and work-softening associated with strain localization into a shear band. The FEM code incorporating the above-mentioned new constitutive model is validated by a direct comparison between results from the physical experiments and the numerical simulation for sand specimens reinforced by using relatively flexible and rigid reinforcement as well as an unreinforced sand specimen. The comparison was made in terms of global stress-strain relationships and local strain fields showing the generation and development of shear band. It was found that the FEM code incorporating the proposed model work-hardening model simulates the experimental results much better than one with the previous shear strain-hardening model.

Keywords: Reinforced sand, Planar reinforcement, Plane strain compression, FEM, Work-hardening, elasto-plastic model, stress-strain relation, shear band
FEM Simulation of Failure of Reinforced Sand
Based on a New Work-hardening Constitutive Model

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1. INTRODUCTION

In order to predict the failure of full-scale reinforced soil structures under both operational and limit conditions, it is first required to understand their deformation and strength characteristics as a composite consisting of soil and reinforcement. Yamauchi (1985) and Tatsuoka and Yamauchi (1986) conducted a series of PSC tests on dense Toyoura sand that was either unreinforced or reinforced by using different planar-reinforcement materials. A simplified theoretical consideration was also made to investigate the interaction among reinforcement properties, soil properties, reinforcement spacing and initial confining pressure. They also observed precisely the strain fields of the specimens. Kotake (1998) and Kotake et al. (1999) simulated the results of plane strain compression (PSC) tests on small specimens of dense Toyoura sand that were either unreinforced or tensile-reinforced with various types of planar-reinforcements placed horizontal (Tatsuoka and Yamauchi, 1986) by using nonlinear elasto-plastic FEM, incorporating the plastic strain-hardening proposed by Tatsuoka et al (1993) and Siddiquee (1994). The FEM analysis method proposed for failure analysis of reinforced sand could successfully simulate both global and local behaviors observed in the physical experimental tests with respect to peak load and shear banding. It was noted, however, that for the two reinforced specimens, the maximum stress ratio $R^* = \sigma_1 / \sigma_y$ and pre-peak stiffness are somewhat larger in the FEM analysis than in the experimental PSC tests. This discrepancy can be attributed, at least partly, to that the assumption as to the plastic strain-hardening model for sand was inadequate, and for that reason, shear strains for stress paths traced in the sand were underestimated.

The objective of this study is, therefore, to numerically simulate the results from a series of plane strain compression (PSC) tests on reinforced dense Toyoura sand specimen conducted by Tatsuoka and Yamauchi (1986), based on the newly proposed work-hardening constitutive model for sand. The proposed elasto-plastic constitutive model for sand is based on a stress path-independent work hardening parameter. The model is capable of simulating the effects on the deformation characteristics of stress history and stress path, pressure level, and inherent anisotropy, while strain softening associated with strain localization into a shear band is taken into account. The FEM code incorporating the above new model is validated by a direct comparison between the numerical and experimental results. Results from tests on sand specimens reinforced by using relatively flexible and rigid reinforcement as well as an unreinforced sand specimen were analyzed. Comparisons between the results of the FEM analysis and those from experimental tests were made in global stress-strain relationships and generation and development of shear band. In addition, it was found that the FEM code incorporating the proposed work-hardening model could much better simulate the experimental results than the one with the previous strain-hardening model (Tatsuoka et al,
2. PSC TESTS ON REINFORCED SAND

In the PSC tests on reinforced dense air-dried Toyoura sand specimen conducted by Yamauchi (1985), a planar tensile reinforcing member was placed horizontally at the mid-height as shown in Fig. 1 (Tatsuoka and Yamauchi, 1986). The specimens were 7.5, 8.0 and 4.0 cm in the directions of \( \sigma_1 \), \( \sigma_2 \) and \( \sigma_3 \), respectively, and had an initial void ratio \( e_0 \) of 0.66. The surfaces of \( \sigma_1 \) and \( \sigma_2 \) were well-lubricated, and the measured boundary stresses were corrected for membrane forces and vertical friction working on the confining platens. The details are described in Appendix-I (Tatsuoka and Yamauchi, 1986).

3. FEM MODEL FOR PSC TESTS

3.1 Modeling of PSC Specimen

Typical tests on sand specimens reinforced by using relatively flexible and rigid reinforcement as well as an unreinforced sand specimen were analyzed. One test on unreinforced sand specimen and two tests on specimens reinforced with a urethane sheet and a rough brass plate having surfaces roughened by gluing sand particles, as flexible and rigid reinforcements respectively, were selected for simulation. The PSC specimen was discretised basically into 1.0 cm x 1.0 cm square plane strain elements (Fig. 2). The number of total nodal points was 81. The reinforcing members were modeled by 8 linear-elastic truss elements having only axial stiffness \( EA \), which was \( 4.51 \times 10^6 \) kN for the urethane sheet and \( 5.05 \times 10^4 \) kN for the rough brass plate, where \( E \) and \( A \) are the Young’s modulus and cross-section area per unit length, respectively. No
particular interface elements are used, since no pronounced slippage between reinforcement and sand was observed in the PSC tests. The initial isotropic effective confining pressure of $\sigma_c (=49 \text{kPa})$ was first given to all the plane strain elements. A displacement increment of $0.0005 \times 2 \text{ cm/step}$, i.e., an average axial strain increment $d \varepsilon_y$ of 0.0132 $\%$/step, was employed, which has been found to be small enough to keep accuracy and numerical stability with an equilibrium iteration tolerance of a force norm as well as an energy norm $\varepsilon_\varepsilon$ of $10^{-3}$. A set of non-linear equations was solved by the Dynamic Relaxation technique (Tanaka et al., 1988), which has a reputation in solving highly non-linear equations, especially for high friction angle materials as in the present case. The integration of the elasto-plastic equation was done by the Return Mapping scheme (Ortiz and Simo, 1986), which is a first order approximated Euler backward integration. The detail is described in Siddiquee et al. (1995, 1999).

3.2 Proposed Work-hardening Constitutive Model for Sand

The proposed material model is coupled with an isotropically work-hardening and softening, non-associated, elasto-plastic material description. The yield function and the plastic potential function are of, respectively, Mohr-Coulomb and Drucker-Prager type. As the details of the sand model have been reported in several previous papers by the authors (Peng et al. 2000 (a), (b) and 2001), only the essence will be described below.

The work-hardening parameter, $W_{\mu}^*$, was defined in the plane strain condition as follow:

$$W_{\mu}^* = \int \left[ t \cdot d\gamma^p + S \cdot d\varepsilon_{vol}^p \right] / (s / p_a')^{n} \right]$$

where $t = (\sigma_v' - \sigma_h')/2$, $s = (\sigma_v' + \sigma_h')/2$, $p_a' = 98 \text{kPa}$ and $n$ is a material constant. The hardening function for the pre-peak regime is assumed as follows (see Fig. 3):

$$X(W_{\mu}^*) = a \cdot \ln\left(W_{\mu}^*/b + h\right)$$

where $X$ is the following specific of stress level, given as:

$$X = t / s + r \cdot \ln(s / p_a')$$

where $a$, $b$, $h$ and $r$ are the material constants ($a=0.1288$, $b=3.4505 \times 10^{-5}$, $h=0.4310$, $n=0.9$ and $r=0.09$), which were obtained by fitting Eq. 2 to the data from the PSC tests of Toyoura sand shown in Fig. 3 (Yasin and Tatsuoka, 2000).

Fig. 4 shows the stress-dilatancy ($t / s \sim d\varepsilon_{vol}^p / d\gamma^p$) relations from the PSC tests. The average relationship between $t / s \sim d\varepsilon_{vol}^p / d\gamma^p$ was fitted as follows:
\[ t / s = m \cdot \left( -d \varepsilon_{vol}^P / d \gamma^P \right) + c \]  
(4)

where \( m \) and \( c \) are the material constants, which are, respectively, 0.596 and 0.639 for Toyoura sand in this present study.

In the present study, it was assumed that the deformation of a given sand element under uniform boundary stress conditions be homogeneous in the pre-peak regime, and that strain localization into a shear band starts suddenly at the peak stress state (Tanaka and Sakai, 1993). The smear method used to incorporate strain localization, is similar to the one proposed by Pietruszczak and Mroz (1981). Unlike their method, however, no direction of shear banding was specified in the present study.

Most granular materials deposited naturally or artificially, or compacted vertically, exhibit cross-anisotropic deformation properties. So in the present study, cross-anisotropic elasticity is considered, which is based on some physical tests with respect to anisotropic elastic deformation properties of sand (Hoque, 1996; Hoque & Tatsuoka, 1996, 1998).

4. RESULTS OF FEM SIMULATION

4.1 Global Stress-Strain Relationship

The global relationships between the boundary principal stress ratio \( R^* = (\sigma_1 / \sigma_c) \) and the average axial strain \( \varepsilon_a \) from the FEM analysis and the experimental PSC tests are compared in Fig. 5. For the unreinforced sand specimen and the sand specimens reinforced with urethane, it is seen that the result obtained from the FEM analysis is in a very good agreement with the experimental PSC test result, and it can be concluded that the FEM code with the proposed work-hardening model could simulate very well not only at pre-peak but also at post peak regime. For the sand specimen reinforced with a rough brass plate, the FEM results is similar to the PSC test results with respect to the effect of reinforcement on the global behaviors when using rigid reinforcement. It should be pointed, however, that the maximum stress ratio is somewhat larger in the FEM analysis than in the PSC tests. This discrepancy may be attributed...
partly to the effects of bedding error and slippage between reinforcement and sand in the PSC test.

4.2 Local Characteristics and Shear Band:

The distributions of incremental local maximum shear strains $\gamma_{\text{max}} (=\epsilon_1 - \epsilon_3)$ observed between two successive loading stages in the experimental PSC tests are shown in Fig. 6. The representative loading stages selected were “before peak (A)”, “around peak (B)”, and “after peak to residual (C)” as denoted in Fig. 5. The corresponding contours of $\gamma_{\text{max}}$ obtained from the FEM analyses are shown in the Fig. 6. The following trends can be noted from the comparison between the FEM simulation and the experimental PSC tests.

For the unreinforced specimen in the experimental PSC test, a V-shape shear band, reflecting at the bottom of the specimen appeared around the peak state, as seen from the incremental $\gamma_{\text{max}}$ field (A→B). Then, strain was localized more intensely into the shear band in the post-peak regime (B→C). Also in the FEM analysis, a diagonal shear band appeared around the peak state (A'→B'), which developed further, reflecting clearly at the specimen top and marginally at the bottom, in the post-peak regime (B'→C'). It can be said that the generation and development of shear band, affecting the global stress-strain behavior, observed in the experimental PSC test was well simulated by FEM analysis.

For the specimen reinforced with urethane, the shear band banding observed in the PSC tests was well simulated by FEM analysis. That is, the shear band pattern that appeared between the stages A and B was similar to the one in the unreinforced case, but the degree of strain localization was much less due to the restraint by reinforcement. However, the shear band penetrated through the reinforcement, perhaps due to the flexible nature of the reinforcement. Apparently, although it was relatively flexible, the reinforcement prevented the strain localization only into the initial shear band penetrating the

Fig. 6 Contours of incremental local maximum shear strain

For the specimen reinforced with urethane, the shear band banding observed in the PSC tests was well simulated by FEM analysis.
reinforcement. By this interaction, relatively large tensile strains, hence tensile stresses, were induced in the reinforcement. This interaction increased the global strength of the reinforced sand even after the sand reached its peak stress conditions locally (after stage A' in Fig. 5b), showing clear post-yield strain-hardening behavior (A'→B').

For the specimen reinforced with rough brass, the shear banding observed in the PSC tests was simulated very well by FEM analysis also in this case. That is, up to the peak state (A→B and A'→B'), very small shear strains were induced particularly in the vicinity of the rough brass due to its high confining effect while strain localization appeared to some extent around the both ends of the reinforcement. After peak (B→C and B'→C'), a greatly intense strain localization into a V-shape shear band developed in the top half of the specimen with sharp reflection at the top end. Afterwards, strain localized into the shear band while the reminder behaved like a rigid body. Due to a high rigidity of the brass reinforcement, shear band could not penetrate the reinforcement, and shear banding developed in the zones away from the reinforcement, controlling the global post-peak behavior. Therefore, despite that the global peak strength became very large by the large restraining effects of the reinforcement on the development of strains in sand, the residual strength was very small compared with the peak strength (see Fig. 5c).

4.3 Reinforcement Tensile Behavior

Tensile force distributions in the reinforcement of urethane at different loading stages are shown in Fig. 7(a), which are obtained from the FEM analysis. The tensile force in the urethane has the maximum at the center, but the distribution is somewhat skewed in the left half after the global peak state. This is due to that large local strains were induced to the left half of the specimen as the strain localization exhibited diffusion over the whole specimen, as seen from Fig. 6(b). The increase in the tensile force continues at a lower rate after the global peak state. This is due to the fact that shear banding diffused within the specimen and the lateral strains induced in the multiple shear bands were prevented from increasing by the reinforcement. This is the major factor for the post-peak ductile global behavior of the urethane-reinforced sand.

Tensile force distributions in the reinforcement of rough brass at different loading stages are shown in Fig. 7(b), which are also obtained the FEM analysis. The tensile force in the rough brass has the maximum at the center, but the distribution is somewhat skewed in the left half after the global peak state, which was similar to that of urethane-reinforced specimen. This is due to
that large local strains were induced to the left half of the specimen as the sand exhibited strain localization, as seen from Fig. 6(c). In the rough brass-reinforced specimen, the tensile force decreases noticeably after the global peak state. The decrease is due to the reduction in the lateral tensile strains of sand in the zones adjacent to the reinforcement. Such strain localization in a well defined shear band as described above is the major factor for the post-peak brittle global behavior of the brass-reinforced sand.

4.4. Comparison with previous strain-hardening model

The global relationships between the boundary principal stress ratio $R^\star (= \sigma_1 / \sigma_c)$ and the average axial strain $\varepsilon_a$ from the FEM analysis incorporating the proposed work-hardening model and the previous shear strain-hardening model (Kotake, 1998) and from the physical PSC test (Tatsuoka and Yamauchi, 1986) are compared in Fig 8. It is seen that the result obtained from the FEM analysis with the proposed work-hardening model is in a better agreement with the physical PSC test than that from FEM analysis with the previous shear strain-hardening model. This is due to that the proposed work-hardening model is capable of simulating the effects of stress history and stress path on the deformation of reinforced sand, as discussed above.

5. CONCLUSIONS

From the results presented above, the following conclusions can be derived:

(1) The relevant FEM analysis could simulate very well not only the global stress-strain behavior but also the local deformation of both unreinforced sand specimen and sand specimens reinforced with planar reinforcement, flexible and rigid, which are brought to failure in plane strain compression tests.

(2) The FEM analyses could well simulate the failure mechanism of sand specimens reinforced with flexible and rigid reinforcement. By an insight into the strain fields, the reinforcing mechanism by tensile reinforcement was clearly understood in relation to the global stress-strain relations.

Fig. 8 Comparison of results between proposed work-hardening model and previous strain-hardening model
The comparison of FEM results between the newly proposed work-hardening model and the previous shear strain-hardening model was conducted. It was found that the FEM code incorporating the proposed work-hardening model could much better simulate the results from the physical experiments than that with the previous shear strain-hardening model. The problem of FEM analysis with the previous strain-hardening model, which exhibited higher stiffness at pre-peak and smaller strains in the stress-strain relationship in the plane strain compression tests, has been improved.

REFERENCES


APPENDIX -I:

The physical PSC tests were conducted a) one unreinforced specimen; b) one specimen reinforced with urethane; and c) one specimen with rough brass planar. A constant confining pressure $\sigma_c$ equal to 49kPa was applied by a partial vacuum. The displacement fields on the $\sigma_2$ plane were obtained from the movements of targets that had been printed on the outer surface of latex rubber membrane at a 0.5 cm interval. The coordinates at the grid nodes were read from photographs that were taken through the transparent confining platen at different loading stages in each test. From the displacement vectors obtained at each node, strain fields were calculated, assuming uniform deformation within a 0.5cm times 0.5cm square element consisting of four nodes. The physical properties of reinforcement used for the PSC tests are listed in Tab. 1-1.

<table>
<thead>
<tr>
<th>Reinforcement</th>
<th>Young’s modulus $E$ (kN/m$^2$)</th>
<th>Poisson’s ratio $\nu$</th>
<th>Thickness $(mm)$</th>
<th>Friction angle Against Toyoura sand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Urethane</td>
<td>$4.51 \times 10^4$</td>
<td>0.5</td>
<td>1.0</td>
<td>$&gt; \phi^1$</td>
</tr>
<tr>
<td>Rough brass plate</td>
<td>$1.01 \times 10^8$</td>
<td>--</td>
<td>0.5</td>
<td>$&gt; \phi^1$</td>
</tr>
</tbody>
</table>

a) $\phi$ is the angle of internal friction of Toyoura sand;

b) Roughened by gluing Toyoura sand particles to its surfaces.