Application of Adaptive Sliding Mode Control with an Ellipsoidal Sliding Surface for Vehicle Distance Control

Taichi Mizoshiri * and Yasuchika Mori **

Abstract: This paper proposes a sliding mode control with an ellipsoidal sliding surface for vehicle distance control. The performance of two different sliding surfaces, namely ones that are ellipsoidal and linear, is evaluated under the same conditions. Each controller, regardless of sliding surface, is designed to achieve a similar level of control performance. It is shown through simulation that the sliding mode control with the ellipsoidal sliding surface proposed by the authors has advantages over conventional sliding mode control with a linear sliding surface, in that it is smoother and has lower energy consumption. Furthermore, a boundary layer width adaptation law is applied to prevent chattering.

Key Words: Adaptive Cruise Control system, sliding mode control, ellipsoidal sliding surface, boundary layer adaptation law, time-varying sliding surface.

1. Introduction

Sliding mode control (SMC) is a well-known nonlinear control method that has a high robustness against uncertainties, such as extraneous disturbances, modeling error, or parameter variation [1], especially if the state is constrained to the switching hyperplane (sliding mode). Since SMC has high robustness against unknown uncertainties, in recent years, it was adopted in Adaptive Cruise Control (ACC) [2]–[6]. It is generally said that human error is accounting for over 90 percent of car accidents. Especially, automating the inter-vehicle distance control task is effective to reduce car accidents while long driving. Furthermore, if every vehicle adopt an ACC system, traffic congestion can be relieved. ACC is a longitudinal vehicle control system for driving load reduction. In this system, the control objective is to maintain a desired inter-vehicle distance between one vehicle and its preceding vehicle. A sliding mode controller design with two different sliding surfaces [2] was proposed for ACC and Cooperative Adaptive Cruise Control (CACC) that added inter-vehicle communication to ACC. By using inter-vehicle distance, speed, and acceleration through inter-vehicle communication, CACC can reduce time delay and improve tracking performance than ACC. A sliding mode observer to estimate acceleration data and a second-order sliding mode control (CACC) that added inter-vehicle communication to ACC. The methods mentioned above have adopted second-order SMC with a linear sliding surface. Hence, in the presence of a sliding mode and a reaching mode, it is difficult to maintain both smooth tracking and a quick response. Moreover, there is no guarantee for robustness during the reaching mode and the energy consumption is not considered. In previous studies, tracking performance and robustness were well achieved. However, excessive jerks caused by chattering or rapid changes of control input occurred. This consequently sacrificed ride comfort.

In this paper, an ellipsoidal sliding surface that was proposed by the authors [7] is applied to an ACC system to maintain tracking performance without sacrificing ride comfort. This method is more robust than a conventional linear sliding surface because it has no reaching mode. Moreover, since the ellipsoidal sliding surface is a smooth curve without a cusp, chattering is reduced compared with a conventional linear sliding surface. The energy consumption is also reduced. Since the ellipsoid is a closed loop curve, the state is perturbed by chattering that occurs near the equilibrium point and does not remain at the equilibrium point. To ensure that the state remains at the equilibrium point, we define an auxiliary linear sliding surface because it has no reaching mode. Moreover, since the ellipsoidal sliding surface is a smooth curve without a cusp, chattering is reduced compared with a conventional linear sliding surface.

In this paper, an ellipsoidal sliding surface that was proposed by the authors [7] is applied to an ACC system to maintain tracking performance without sacrificing ride comfort. This method is more robust than a conventional linear sliding surface because it has no reaching mode. Moreover, since the ellipsoidal sliding surface is a smooth curve without a cusp, chattering is reduced compared with a conventional linear sliding surface. The energy consumption is also reduced. Since the ellipsoid is a closed loop curve, the state is perturbed by chattering that occurs near the equilibrium point and does not remain at the equilibrium point. To ensure that the state remains at the equilibrium point, we define an auxiliary linear sliding surface in the ellipsoidal area that is near the equilibrium point. In our previous paper [7], a linear time varying sliding surface that changes its gradient according to the degree of error was adopted. This method ensures that the state converges to zero near the equilibrium point. However, a cusp point is present at the changing point of the sliding surface. Thus, at the moment of a change from the ellipsoidal sliding surface to the auxiliary linear sliding surface a large jerk occurs. Unfortunately, this worsens the smoothness and the energy consumption performance. In this paper, this problem is improved as follows.

At the moment of changing sliding surface, in order to
achieve a smooth change, the tangent of the ellipsoidal sliding surface passing through the changing point is adopted for the auxiliary linear sliding surface. It enables smooth state convergence to the equilibrium point to parallel movement of the sliding surface according to the degree of the error.

Generally, applying a constant and thick boundary layer to prevent chattering makes it difficult to maintain a high error tracking response. On the other hand, applying a constant and thin boundary layer to maintain high error tracking response makes it difficult to prevent chattering. Therefore, we proposed a boundary layer adaptation law [8] for both chattering prevention and high error tracking response maintenance. To verify the advantages of the proposed method, we compare the proposed ellipsoidal sliding surface with the conventional linear sliding surface through simulation for an ACC system.

This paper is organized as follows: Section 2 introduces the vehicle model for the simulation of ACC systems. Section 3 outlines the adaptive cruise control system architecture. Section 4 describes the sliding mode controller design with an ellipsoidal sliding surface, an auxiliary time-varying linear sliding surface, and the boundary layer adaptation law to prevent chattering. Section 5 examines and compares the effectiveness of the proposed method with the conventional linear sliding surface through simulation for an ACC system. Section 6, some remaining issues of the proposed method are discussed. Finally, conclusions are provided in Section 7.

2. Vehicle Model

This section presents the subjective vehicle model. The design parameters are listed in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>1800 kg</td>
</tr>
<tr>
<td>$dm$</td>
<td>200 kg</td>
</tr>
<tr>
<td>$g$</td>
<td>9.8 m/s²</td>
</tr>
<tr>
<td>$\rho$</td>
<td>1.226 kg/m³</td>
</tr>
<tr>
<td>$C_d$</td>
<td>0.28</td>
</tr>
<tr>
<td>$A$</td>
<td>2 m²</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.01</td>
</tr>
<tr>
<td>$N$</td>
<td>Gear ratio</td>
</tr>
<tr>
<td>$T_m$</td>
<td>0.01 s</td>
</tr>
<tr>
<td>$R$</td>
<td>0.3 m</td>
</tr>
<tr>
<td>$\tau(t)$</td>
<td>Motor torque</td>
</tr>
<tr>
<td>$\tau_{max}$</td>
<td>300 Nm</td>
</tr>
<tr>
<td>$F(t)$</td>
<td>Longitudinal force</td>
</tr>
<tr>
<td>$x_s(t)$</td>
<td>Position of the subject vehicle</td>
</tr>
<tr>
<td>$v_s(t)$</td>
<td>Velocity of the subject vehicle</td>
</tr>
<tr>
<td>$x_p(t)$</td>
<td>Position of the preceding vehicle</td>
</tr>
<tr>
<td>$v_p(t)$</td>
<td>Velocity of the preceding vehicle</td>
</tr>
<tr>
<td>$x_r(t)$</td>
<td>Reference distance</td>
</tr>
<tr>
<td>$v_r(t)$</td>
<td>Reference relative velocity</td>
</tr>
<tr>
<td>$r$</td>
<td>Reference distance</td>
</tr>
<tr>
<td>$\epsilon_r(t)$</td>
<td>Distance error</td>
</tr>
<tr>
<td>$\epsilon_v(t)$</td>
<td>Relative velocity error</td>
</tr>
</tbody>
</table>

The longitudinal dynamics of the subjective vehicle can be represented by (1).

$\begin{align*}
    v_s(t) &= \dot{x}_s(t) \\
    \dot{v}_s(t) &= \frac{1}{m+dm} \left( [F(t) - f(v_s)] - \frac{1}{\tau} (\tau(t) - F(t)) \right) \\
    F(t) &= \frac{1}{m} (N + T_m) \\
\end{align*}

$ where, the third equation of (1) represents an actuator model that is approximated by the first-order transfer function.

The running resistance of the vehicle is represented as follows:

$$ f(v_s) = \frac{1}{2} \rho C_d A v_s^2 + \mu \left( v_s + dm \right) g $$

(2)

where, the first term of (2) denotes air resistance and the second term represents the rolling resistance of the tires.

3. Architecture of the ACC System

The ACC is a longitudinal vehicle control system for the reduction of driver task load. In this system, the control objective is to maintain a desired inter-vehicle distance between two vehicles.

A two degrees of freedom controller and the disturbance observer were applied for the inter-vehicle distance control system in [9]. In this study, the reference model in [9] is adopted and the error tracking controller is changed to the proposed sliding mode controller. The block diagram of the inter-vehicle distance control system is depicted in Fig. 1.

![Block Diagram](image)

Fig. 1 The block diagram of the inter-vehicle distance control.

It has been reported in [9] that error tracking characteristics of the vehicle distance control for which passengers do not feel strange require an overshoot. This result was derived by functional evaluation experiments. Hence, the canonical second-order transfer function is adopted as a vehicle distance reference model as follows:

$$ G_R(s) = \frac{\omega^2}{s^2 + 2\xi\omega s + \omega^2}. $$

(3)

In this study, we adopt the following parameters of the reference model: damping ratio $\xi = 0.7$; natural frequency $\omega = 0.6981$.

When a vehicle cuts in while the main vehicle follows a preceding vehicle as shown in Fig. 2, the errors of the integrators in the reference model cause a rapid change of the control input. In order to prevent this, the initial values of the integrators are reset to the relative distance and the relative velocity measured by the radar.

![Simulation Condition](image)

Fig. 2 Simulation condition.

4. Design of the Sliding Mode Controller

In this section, we introduce the sliding mode control law with an ellipsoidal sliding surface for a second-order uncertain dynamic system and a boundary layer width adaptation law to prevent chattering.
4.1 Sliding Mode Controller with an Ellipsoidal Sliding Surface

A model following sliding mode control is applied to a second-order system. First, we consider a single-input-single-output (SISO) second-order system:

$$\dot{x}_s(t) = Ax_s(t) + Bu(t) + \xi$$  \hspace{1cm} (4)

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1/(m + dm) \end{bmatrix}^T,$$

$$x_s(t) = \begin{bmatrix} x_r(t) & v_s(t) \end{bmatrix}^T, \quad u(t) = \frac{N}{R} \tau(t), \quad \xi = \begin{bmatrix} 0 & -f(v_s)/(m + dm) \end{bmatrix}^T.$$  \hspace{1cm} (5)

Here, the mechanical time constant of the motor $T_m$ is assumed small enough to be neglected. The reference model is given as follows:

$$\dot{x}_s(t) = A_s x_s(t) + B_s r(t)$$  \hspace{1cm} (6)

$$A_s = \begin{bmatrix} -\beta_r & -\sigma_r \\ 0 & 0 \end{bmatrix}, \quad B_s = \begin{bmatrix} 0 & 1/m \end{bmatrix}^T,$$

$$x_s(t) = \begin{bmatrix} x_r(t) & v_s(t) \end{bmatrix}^T.$$  \hspace{1cm} (7)

where, $\sigma_r = 2\omega \beta_r \sigma_r = a^2$, and $r(t)$ denotes the reference input. The tracking error is defined as follows:

$$e(t) := x_r(t) - (x_r(t) - x_s(t)).$$  \hspace{1cm} (8)

Meanwhile, the ellipsoidal sliding surface is defined as follows:

$$\sigma(t) = \frac{(e(t) - a)^2}{a^2} + \frac{\dot{e}(t)^2}{b^2} - 1.$$  \hspace{1cm} (9)

Differentiating (7) gives following:

$$\dot{\sigma}(t) = 2e(t) \left( \frac{1}{a^2} e(t) + \frac{1}{b^2} \dot{e}(t) - \frac{1}{a} \right).$$  \hspace{1cm} (10)

To derive the sliding mode control law, a Lyapunov function candidate is defined as follows:

$$V = \frac{1}{2} \sigma(t)^2.$$  \hspace{1cm} (11)

Differentiating (9) gives the following:

$$\dot{V} = \sigma(t) \dot{\sigma}(t) < 0.$$  \hspace{1cm} (12)

Here, if (10) is satisfied, it is guaranteed that the state is constrained to the sliding surface. Substituting (4)–(8) into (10) gives the following:

$$\left\{ \frac{(e(t) - a)^2}{a^2} + \frac{\dot{e}(t)^2}{b^2} - 1 \right\} \left\{ \left( e(t) \left( \frac{1}{a^2} e(t) + \frac{1}{b^2} \dot{e}(t) - \frac{1}{a} \right) \right) \right\} < 0.$$  \hspace{1cm} (13)

The terms in the left curly brackets on the left side of (11) represent the ellipsoid equation. Therefore, if the state exists on the inside of the ellipsoid, these terms take a positive value. In contrast, if the state exists on the outside of the ellipsoid, these terms take a negative value. To satisfy the condition in (11), each term in the left and right curly brackets on the left side of (11) must have different signs. Thus, the control law that constrains the state to the ellipsoidal sliding surface and leads the state to the equilibrium point is derived as follows:

$$u(t) = \left( \left[ \frac{e(t)}{a} - \beta_r \right] e(t) - \alpha_r \dot{e}(t) - \frac{a^2}{b^2} + r(t) \right) - K \text{sgn}\left( \dot{e}(t) \left( \frac{\alpha_r \sigma_r}{a^2} + \frac{\beta_r}{b^2} - 1 \right) \right)$$  \hspace{1cm} (14)

where, $K$ denotes the switching gain of the sliding mode controller, $\text{sgn}(\cdot)$ denotes the sign function. For more details about an ellipsoidal sliding surface, refer to [7].

4.2 Auxiliary Time-Varying Linear Sliding Surface

Since the ellipsoid is a closed loop curve, the state does not remain at the equilibrium point $O$. To make the state converge and remain at the equilibrium point, we define an auxiliary linear sliding surface in the ellipsoidal area $E_1$ that is near the equilibrium point $O$ as shown in Fig. 3. At the moment of changing the sliding surface, in order to achieve a smooth change, the tangent of the ellipsoidal sliding surface passing through the changing point is adopted for the auxiliary linear sliding surface. It allows a smooth state convergence into the equilibrium point to maintain parallel movement of the sliding surface according to the degree of the error. In Fig. 3, $E_1$ represents the ellipsoidal sliding surface. The ellipsoidal area $E_2$ is given as follows:

$$\frac{e(t)^2}{a^2} + \frac{\dot{e}(t)^2}{b^2} = q^2.$$  \hspace{1cm} (15)

In this paper, parameters ‘$a$’ and ‘$b$’ are pre-determined to achieve the desired smooth error response and the converge time based on that of the conventional method with a time invariant surface. Therefore, we can make it shape the error tracking trajectory smoother than that designed by the conventional method. The parameters of the ellipsoidal area $E_2$ in which auxiliary linear sliding surface is applied are ‘$a$’, ‘$b$’, and ‘$q$’. Here, the parameters ‘$a$’ and ‘$b$’ are the same as those of the ellipsoidal sliding surface. The scaling parameter ‘$q$’ is determined by the desired error tracking accuracy.

Moreover, to reduce the jerk near the equilibrium point, we adjust the linear sliding surface from $L_0$ to $L_1$ in the area which is near the equilibrium point. At first, the auxiliary linear sliding surface $L_0$ is given as the tangent of the ellipsoidal surface $E_1$ passing through the intersections of $E_1$ and $E_2$.

Here, $t_p$ is the passage time at the point $P$. The tangent point $P$ is represented as follows:

$$P = \left( \frac{aq^2}{2}, -bq \sqrt{1 - \frac{q^2}{2}} \right).$$  \hspace{1cm} (16)

The intersection at the vertical axis $\kappa(t)$ is adjusted by
\[
\kappa(t) = \frac{\sqrt{e(t)^2 + \dot{e}(t)^2}}{|OP|} \kappa(t_p). \tag{16}
\]

The time-varying sliding surface \([10]\) that moves in parallel is given in \((17)\) as follows:

\[
\sigma(t) = Se(t) + \kappa(t).
\]  

\(\sigma(t)\) is the sliding mode and \(S = [S_P - 1]\) are defined and then differentiated as follows:

\[
\dot{e}(t) = A_v e(t) + (A_v - A)x(t) + B_x r(t) - Bu(t) + d(t). \tag{18}
\]

Here, \(S_P\) represents the gradient of the sliding surface and \(\kappa(t)\) represents an intersection of the vertical axis of the sliding surface. Here, \(d(t)\) denotes the external disturbances. By assuming that \((A - A_v), B_x,\) and \(d(t)\) satisfy the matching condition, we can neglect these terms. Here, the matching condition means these uncertainties belong to the range space of \(B\). Differentiating \((17)\) and replacing \(B\) with \(B\), we obtain the following:

\[
\dot{\sigma}(t) = S[A_v e(t) + B_x r(t) + B_x u(t)] + \dot{\kappa}(t). \tag{19}
\]

To prove the existence of the sliding mode, we utilize a Lyapunov function. A Lyapunov function candidate is defined as follows:

\[
V(t) = \frac{1}{2} \sigma(t)^2 > 0. \tag{20}
\]

Differentiating \((20)\) with respect to time, the stability condition is given by:

\[
\frac{dV(t)}{dt} = \sigma(t) \dot{\sigma}(t) < 0. \tag{21}
\]

Thus, the control input is defined as \(u(t) = u_{eq}(t) + u_{sw}(t)\).

\[
u_{eq}(t) = (SB_v)^{-1} [S_A e(t) + SB_x r(t) + \dot{\kappa}(t)],
\]

\[
u_{sw}(t) = -(SB_v)^{-1} K \text{sgn} (\sigma(t)). \tag{22, 23}
\]

Here, \(u_{eq}(t)\) represents the equivalent control input, \(u_{sw}(t)\) denotes the switching control input, \(K\) indicates the switching gain, and \(\text{sgn}(\sigma(t))\) is the sign function. If the following condition is guaranteed, then \(\sigma(t)\) converges to zero, and the sliding mode is maintained as follows:

\[
\frac{dV(t)}{dt} = -K\sigma(t) \text{sgn}(\sigma(t)) < 0. \tag{24}
\]

### 4.3 Boundary Layer Adaptation Law

In this section, we introduce a boundary layer adaptation law proposed by authors \([8]\). In an actual application of SMC, the smoothing function or the saturation function is usually used to suppress chattering by approximating the sign function. In this paper, the smoothing function given by \((25)\) is adopted to suppress chattering.

\[
\text{sgn}(\sigma(t)) = \frac{\sigma(t)}{|\sigma(t)|} \approx \frac{\sigma(t)}{|\sigma(t)| + \delta}. \tag{25}
\]

The approximation of the sign function is shown in Fig. 4. In this figure, the dotted line represents the sign function and the solid line represents the approximated sign function.

Generally, applying a constant and thick boundary layer to prevent chattering makes it difficult to maintain a high error tracking response. On the other hand, applying a constant and a thin boundary layer to maintain a high error tracking response makes it difficult to prevent chattering. The boundary layer adaptation law has the capability to maintain both chattering prevention and a high error tracking response and to adjust the boundary layer automatically for the variation in uncertainty. The design parameter is only the upper and lower limits of the boundary layer.

Here, we consider adjusting the width of the boundary layer \(\delta(t) > 0\). Let \(\gamma(t) = 1/\delta(t)\) and the Lyapunov function candidate as follows:

\[
V(t) = \frac{1}{2} (|\sigma(t)| - \gamma(t))^2. \tag{26}
\]

Here, the time derivative of \((26)\) may satisfy \((27)\).

\[
\dot{V}(t) = (|\sigma(t)| - \gamma(t)) \left( \frac{\sigma(t)}{|\sigma(t)|} r(t) - \dot{\gamma}(t) \right) < -\eta(t) |\sigma(t)| - \gamma(t) \left( \frac{\sigma(t)}{|\sigma(t)|} (\sigma(t) - \dot{\gamma}(t)) \right) < -\eta(t). \tag{27}
\]

Here, \(\eta(t)\) is a positive variable that determines the convergence time of the Lyapunov function. Equation \((27)\) contains two sign functions. Hence, we can derive the boundary layer adaptation law as follows:

- If \(|\sigma(t)| < \gamma(t)\)
  - (a) If \(\sigma(t) > 0\) then \(\dot{\gamma}(t) > -\eta(t) + \dot{\sigma}(t)\)
  - (b) If \(\sigma(t) < 0\) then \(\dot{\gamma}(t) < -\eta(t) - \dot{\sigma}(t)\)
- If \(|\sigma(t)| > \gamma(t)\)
  - (a) If \(\sigma(t) > 0\) then \(\dot{\gamma}(t) < \eta(t) + \dot{\sigma}(t)\)
  - (b) If \(\sigma(t) < 0\) then \(\dot{\gamma}(t) > \eta(t) - \dot{\sigma}(t)\)

By applying the adaptation law, \(\delta(t)\) is adjusted faster than \(\sigma(t)\) by an amount of the contribution of \(\eta(t)\). Moreover, this adaptation law makes to be \(|\sigma(t)| \delta(t) = 1\) and the boundary layer \(\delta(t)\) is adjusted to be inversely proportional to the distance from the switching surface \(\sigma(t)\). Here, we need to define the upper and lower limits of the boundary layer. The maximum value in the range that satisfies the allowable value of the tracking error is chosen as the value for the upper limit. For the minimum uncertainty, the minimum value in a range that ensures that chattering does not occur is chosen as the value for the lower limit. The minimum value should be determined from the functional evaluation experiments of the ride comfort. In
order to ensure that the Lyapunov function converges to zero in accordance with \( \dot{\sigma}(t) \), (27) is applied.

\[
\eta(t) = \frac{|\sigma(t)|}{|\dot{\sigma}(t)| + \epsilon}
\]  

(28)

Here, \( \epsilon \) denotes a small positive constant to prevent division by zero. By applying (28), as the state moves away from the switching surface or moves slowly, the rate at which convergence occurs increases. Moreover, as the state approaches the switching surface or moves too fast, convergence becomes more gradual. Since the adjustment parameters are only the upper and lower limits of the boundary layer width, the tuning method is relatively easy.

5. Simulation

This section presents the simulation condition and results that demonstrate the advantages of the proposed method. Here, the proposed method is SMC with an ellipsoidal sliding surface with the application of a boundary layer adaptation law. In contrast, the conventional method is SMC with a time invariant linear sliding surface and a constant boundary layer. In this paper, the simulation is realized in the Matlab/Simulink environment.

5.1 Simulation Conditions

In this study, the advantages of the proposed method are verified using a scene where interruption occurs during vehicle distance control illustrated in Fig. 2. The details of the simulation condition are shown in the following. At \( t = 0 \), a vehicle A follows a preceding vehicle B traveling 30 m ahead at 70 km/h. At \( t = 2 \), a vehicle C that travels at 65 km/h in the adjacent lane 20 m ahead of vehicle A moves in between vehicles A and B. Here, the target inter-vehicle distance between the two vehicles is 30 m.

The design parameters for the proposed method and the conventional method are given in Table 2.

5.2 Simulation Results

Figure 5 shows the simulation results of the ACC system that is applied with the proposed method and the conventional method. In this figure, the dotted line represents the reference response, the solid line represents the response of the proposed ellipsoidal method, and the dashed line represents the response of the conventional method. The first figure represents the inter-vehicle distance, the second figure represents the relative velocity, and the third figure represents the velocity of the main vehicle. It is shown that the proposed method has good followability to change the target value and the smooth error tracking is achieved. In contrast, the conventional method has a slight overshoot in relative velocity.

Figure 6 shows the evolution of errors, \( e \) and \( \dot{e} \), in the time domain. Since the motor torque is insufficient to maintain the ellipsoidal sliding mode, reaching mode occurs. The conventional linear sliding surface method has a reaching phase (the dashed line \( 2 \leq t \leq 3.47 \)), whereas the proposed method has reaching phase (the solid red line \( 2 \leq t \leq 3.16 \)).

Figure 7 shows the phase portrait. In this figure, the dotted line is the designed sliding surface, and the dash-dot line is the region under the application of the auxiliary linear sliding surface in the vicinity of the equilibrium point in the proposed method. It is shown that the trajectory of the proposed method is along the designed ellipsoidal sliding surface after the short reaching mode. Although the motor torque is insufficient to maintain the ellipsoidal sliding mode, stability is maintained during the reaching mode.

Figure 8 shows the parameter \( \delta \) that determine the boundary
Fig. 7 Phase portraits.

Fig. 8 Boundary layer width.

Fig. 9 Acceleration, jerk, and control input.

layer width. In this figure, the solid line represents $\delta$ of the proposed ellipsoidal method with a boundary layer adaptation law, and the dashed line represents the constant $\delta$ of the conventional method.

Figure 9 shows the acceleration, the jerk, and the control input of the subject car. The proposed method can achieve a control objective while chattering less than the conventional method.

Figure 10 shows the energy consumption and the integral of absolute jerk. Here, the energy consumption $E_n(t)$ is calculated by

$$E_n(t) = \int_0^{T_{sim}} \omega_m \tau dt$$

where, $T_{sim}$ represents the total time of simulation; $\tau$ denotes the motor torque; and $\omega_m$ indicates the angular velocity of the motor. The energy consumption of the proposed method is lower than the conventional method. The jerk of the proposed method along with an ellipsoidal sliding surface is suppressed compared to the conventional method. As shown in Figs. 5–10, it is verified that the proposed method has advantages in terms of smoothness and low energy consumption than the conventional method. Moreover, the proposed method has the equivalent performance to the conventional method in terms of the error convergence.

6. Discussion

In this section, some remaining issues of the proposed method are discussed. Theoretically, the proposed method can eliminate the reaching mode completely. However, the reaching mode could not be eliminated completely on account of the limitation of the maximum actuator torque. As a result, the reaching mode of the proposed method is reduced to two thirds of the conventional method. In addition, the parameters of the ellipsoidal sliding surface are pre-determined based on the convergence time of the conventional method that has a time invariant surface in order to compare the control performance. The relative acceleration can be estimated from measured distance and relative velocity by using the Kalman filter. Thus, we could assign a suitable ellipsoidal sliding surface according to the states.

7. Conclusions

In this paper, a sliding mode control with an ellipsoidal sliding surface was applied for vehicle distance control. By applying the boundary layer adaptation law, chattering was prevented and a high error tracking response was maintained without loss
To verify the advantages of the proposed method, we compared the proposed ellipsoidal sliding surface with the conventional linear sliding surfaces through simulation of an Adaptive Cruise Control (ACC) system. It was shown through simulation of the ACC system that the sliding mode control with an ellipsoidal sliding surface proposed by the authors has advantages over the conventional sliding mode control with a linear sliding surface. The performance of the proposed method is smoother and has a lower energy consumption compared with the conventional method.

Future work includes an investigation on how to assign suitable ellipsoidal sliding surfaces according to the states.

References


---

Taichi Mizoshiri (Student Member)

He received his B.S. degree from the Tokyo Metropolitan Institute of Technology in 1998. In 2000, he received his M.S. degree from the Japan Institute of Science and Technology. Currently, he is a Ph.D. student at the Tokyo Metropolitan University and an employee of EV System Laboratory, Nissan Research Division, Nissan Motor Co., Ltd. His research interests include control system design and theory.

Yasuchika Mori (Member, Fellow)

He received his B.S. and Ph.D. degrees from Waseda University, Japan, in 1976 and 1981 respectively. He was a Professor at the Department of Mechanical System Engineering, National Defense Academy (1999–2002), Department of Electronic System Engineering, Tokyo Metropolitan Institute of Technology (2003–2004), Department of System Design, Tokyo Metropolitan University (2005–2014). Since 2015, he has been a Professor at the Department of Intelligent Mechanical Systems, Tokyo Metropolitan University. His research interests include control system design and theory. He is a member of ISCIE, and IEEE.