Derivation of Robust Stability Ranges for Disconnected Region with Multiple Parameters

Tadasuke MATSUDA *

Abstract: The aim of this paper is to give an extension of the paper [T. Matsuda et al. Proc. 33rd IASTED Modelling, Identification and Control, 809-004, 2014], which gives a robust stability condition for a system with disconnected stability regions. The considered system depends on only one uncertain parameter. In this paper, an explicit algorithm to derive the stability ranges for disconnected stability regions is given. We also extend the result to the case that the system depends on multiple uncertain parameters. A numerical example shows that the proposed method can be applied to robust stability analysis of the lateral dynamics of an aircraft even if all the coefficients of the characteristic polynomial vary. The numerical example also shows that the stability ranges derived by the proposed method are larger than those by a former method.

Key Words: robust control, stability analysis, parameter space, aircraft, stability feeler.

1. Introduction

Robust stability is an important characteristic for control systems with uncertainties because of the difference between the models and actual systems. Robust stability analysis methods have been developed especially based on a parametric approach [1]–[3]. Kharitonov’s theorem [4] gives a condition for robust Hurwitz stability of interval polynomials. Edge theorem shown in [5] implies that a polytope of polynomials is stable if and only if all the segment polynomials corresponding to the exposed edges are stable. A method for Hurwitz stability analysis of a segment polynomial is given in [6]. Schur stability of a segment polynomial is discussed in [7]. A method for stability analysis of delta-operator segment polynomial is presented in [8].

On the other hand, it is also required to obtain stability ranges of uncertain parameters in the case of design of a robust system, where “stability ranges” mean a subset of the ranges of uncertain parameters stabilizing the systems. There are some methods and tools to meet such a demand. Methods in [9]–[14] are based on Lyapunov stability theory, which can derive the stability ranges. A method in [15] is based on the result given in [6]. Methods based on the guardian maps are given in [16]–[18]. A method using LMI formulation is shown in [19]. The stability radius [20], [21] and the directional stability radius [22] are tools to derive the stability ranges of uncertain systems.

The stability feeler [23] also meets such a demand, which enables one to derive the stability ranges of a segment polynomial. The stability feeler can be applied to many types of stability analysis if the stability region in the complex plane is connected. Recently the presented conference paper [24] gives a robust stability condition based on the stability feeler for the case of disconnected stability regions in the complex plane. Stability analysis for disconnected stability regions is required as shown in [19], [25]. The advantage of using the stability feeler is that we can derive the exact stability ranges if the system depends on one uncertain parameter. If the system depends on multiple parameters, we can derive almost exact stability ranges by using the stability feeler, although they are not the exact stability ranges. “Almost exact stability ranges” are ranges which are not exact but can be considered to be exact practically1. Here, by “stability” we mean having all zeros of characteristic polynomials in prescribed regions in the complex plane. The phrase “stability region” means the above prescribed region in the complex plane, which may be disconnected. Therefore, “stability” in this paper includes performance of the system. This is a reason why stability analysis for the case of disconnected stability regions is required. An application to the lateral dynamics of an aircraft is also shown in the conference paper. However, an explicit algorithm to derive the stability ranges is not given in [24]. Moreover, it is assumed that uncertainty exists only in one term of the characteristic polynomial of the lateral dynamics of an aircraft.

The aim of this paper is to give extensions of [24]. An explicit algorithm to derive the stability ranges for disconnected stability regions is given. The case that characteristic polynomials depend on multiple uncertain parameters is also considered. The method presented in [23] enables one to derive the stability ranges. However, the method in [23] cannot be applied if the stability region in the complex plane is disconnected. In [24], systems with only one parameter are considered. In [26], a stability condition of systems for disconnected stability regions in the complex plane is given. However, methods to derive the stability range are not given in [26]. Contrary to them, the paper gives an algorithm to derive the stability ranges of systems with multiple parameters when the stability regions in the complex plane are disconnected. Table 1 shows what this paper deals with. A numerical example shows that the proposed method

1 Details are shown in Remark 2 of this paper.
can be applied to stability analysis of the lateral dynamics of an aircraft even if all the coefficients of the characteristic polynomial vary. This paper also compares the result by the proposed method and that by the method in [19]. Stability ranges derived by the proposed method turn out to be larger than those given in [19].

The notations used in the paper are as follows.

\( \mathbb{R} \): the set of real numbers.
\( \mathbb{R}^p \): the set of \( p \)-dimensional real column vectors.
\( \mathbb{R}^{p \times q} \): the set of \( p \)-by-\( q \) real matrices.
\( j \): the imaginary unit, i.e., \( j = \sqrt{-1} \).
\( A^T \): transposition of matrix \( A \).

2. Robust Stability Condition

In this section, a necessary and sufficient condition for robust stability given in [24] is shown. We consider a real characteristic polynomial family given by

\[
(f(s) + ag(s) \mid a \in \mathbb{R}),
\]

where \( f(s) = \sum_{i=0}^{n} f_i s^i \), \( f_0 \neq 0 \) and \( g(s) = \sum_{i=0}^{m} g_i s^i \) are fixed real polynomials. It is assumed that both of the leading coefficients of \( f(s) \) and \( g(s) \) are greater than zero without loss of generality. It is also assumed that the degree of \( f(s) \) does not drop. A characteristic polynomial family is said to be robustly stable if all the roots of all the polynomials in the polynomial family are in the prescribed open stability region in the complex plane. It is considered that the stability region in the complex plane can be disconnected. It is assumed that the number of crossing points greater than \( 0 \), which implies that there exists a root of polynomial \( f(s) + ag(s) \) on the stability boundary in the complex plane for all \( i = 1, 2, \ldots, m \). Define \( a_{m+1} := +\infty \).

Then the following robust stability condition is given:

Lemma 1. [24] All the polynomials in set \( \{ f(s) + ag(s) \mid a \in (a_i, a_{i+1}) \} \) for a given integer \( i \in \{0, 1, 2, \ldots, m\} \) are stable if and only if there exists a stable polynomial in \( \{ f(s) + ag(s) \mid a \in (a_i, a_{i+1}) \} \) for the same \( i \).

The following lemma gives a method to derive the above stability-boundary-crossing points \( a_1, \ldots, a_m \).

Lemma 2. [24] The set of stability-boundary-crossing points \( \{a_1, \ldots, a_m\} \) equals the union of sets

\[
\{a \mid a > a_0, \text{ } E_{c_{ij}}(f + ag) = 0, \text{ } x \in \partial S'_{s_i}, i = 1, 2, \ldots, \beta\}
\]

and

\[
\{a \mid a > a_0, \text{ } E_{c_{ij}}(f + ag) = 0, \text{ } x + jy \in \partial S''_{s_i}, x, y \in \mathbb{R}, y > 0, \text{ } i = 1, 2, \ldots, \beta\}
\]

The notations used in Lemma 2 are as follows: Coefficient vectors \( f \in \mathbb{R}^{n+1} \) and \( g \in \mathbb{R}^{n+1} \) are defined as

\[
f := [f_0 f_1 \cdots f_n]^T, \quad (4)
\]

\[
g := [g_0 g_1 \cdots g_n]^T, \quad (5)
\]

respectively. A vector \( e_x \in \mathbb{R}^{n+1} \) and a 2-by-(\( n + 1 \)) real matrix \( E_{c_{ij}} \in \mathbb{R}^{2(n+1)} \) are defined as

\[
e_x := [1 \ x^2 \ \cdots \ x^n]^T, \quad (6)
\]

\[
E_{c_{ij}} := \begin{bmatrix} h_{1_{c_{ij}}} & \cdots & h_{n_{c_{ij}}} \\ h_{n_{c_{ij}}} & \cdots & h_{1_{c_{ij}}}
\end{bmatrix}, \quad (7)
\]

respectively, where

\[
h_{1_{c_{ij}}} := [h_0 h_1 \cdots h_n], \quad (8)
\]

\[
h_{2_{c_{ij}}} := [0 h_0 h_1 \cdots h_{n-1}], \quad (9)
\]

\[
h_i := 2xh_{i-1} - (x^2 + y^2)h_{i-2}, \quad i = 2, \ldots, n, \quad (10)
\]

\[h_0 := 1, \quad h_1 := 2x. \quad (11)
\]

\( \partial S'_{s_i} \) and \( \partial S''_{s_i} \), \( i = 1, 2, \ldots, \beta \), are sets of real numbers and complex numbers on the boundary of connected region \( S_i \), \( i = 1, 2, \ldots, \beta \), respectively, where \( S_i \) are connected regions of which the given disconnected stability regions in the complex plane consist. Figure 2 shows an example of \( \partial S'_{s_i} \) and \( \partial S''_{s_i} \) in the complex plane.

3. Results

This section gives an algorithm to derive stability ranges based on Lemmas 1 and 2. We also extend the result to the multiple-parameter case. Finally, a numerical example is shown to compare the result by the proposed method and that by the method in [19].

3.1 Algorithm to Derive Stability Ranges

We give an algorithm to obtain the ranges of \( a \) such that the polynomial family (1) is robustly stable based on [24]. The stability regions in the complex plane can be disconnected. The
algorithm is given as Algorithm 1.

3.2 Multiple-Parameter Case

In this subsection we extend the above result to the multiple-parameter case. We consider the problem of determining ranges of a real uncertain parameter $K$ robustly stabilizing the following characteristic polynomial family with multiple uncertain parameters:

$$\mathcal{P} = \left\{ \hat{p}(s) + \sum_{i=0}^{l} a_i p_i(s) + Kp_K(s) \right\}$$

$$a_i \in [-r_i, r_i] \subset \mathbb{R}, \ i = 0, \ldots, l, \ K \in \mathbb{R},$$

where $\hat{p}(s)$, $p_i(s)$ and $p_K(s)$ are fixed real polynomials. We assume that the degree of $\mathcal{P}$ does not drop and the highest coefficient of $\mathcal{P}$ is positive for all $a_i \in [-r_i, r_i]$, $i = 0, \ldots, l$ when $K = 0$.

In the case of connected stability region in the complex plane, a method to solve the above problem is given in [23] as shown in Table 1. If Edge theorem [5] is satisfied for the case of disconnected stability regions, the method in [23] can also be applied to such a case. Fortunately, Edge theorem is satisfied for a class of disconnected stability regions as follows:

**Lemma 3.** [26] Assume that the complement of stability regions in the complex plane $\bigcup_{\beta} S_{\beta}$ is pathwise connected on the Riemann sphere. Then, polynomial family $\mathcal{P}$ is robustly stable if and only if its exposed edges are stable.

From the above lemma, one can derive stability ranges of $K$ by the same method in [23] if the complement of stability region in the complex plane is pathwise connected. The following theorem is therefore satisfied:

**Theorem 1.** Assume that the complement of stability regions in the complex plane $\bigcup_{\beta} S_{\beta}$ is pathwise connected on the Riemann sphere. Then, polynomial family $\mathcal{P}$ is robustly stable for all $K$ satisfying

$$K \in ([K_{i_{-1}}^{\alpha} + \Delta, K_{i_{-1}}^{\alpha} - \Delta] \cup [K_{i_{-1}}^{\alpha} + \Delta, K_{i_{-1}}^{\alpha} - \Delta] \cup \cdots \cup [K_{i_{-1}}^{\alpha} + \Delta, K_{i_{-1}}^{\alpha} - \Delta]), \ i_1, i_2, \ldots, i_{\beta} \in I,$$

and there exists an unstable polynomial in $\mathcal{P}$ for all $K$ satisfying

$$K \in([-M + \Delta, K_{i_{-1}}^{\alpha} + \Delta] \cup [K_{i_{-1}}^{\alpha} + \Delta, K_{i_{-1}}^{\alpha} - \Delta] \cup \cdots \cup [K_{i_{-1}}^{\alpha} + \Delta, K_{i_{-1}}^{\alpha} - \Delta], \ i_1, i_2, \ldots, i_{\beta} \in I,$$

where $K_{i_{-1}}^{\alpha} < K_{i_{-1}}^{\alpha} < K_{i_{-1}}^{\alpha} < \cdots < K_{i_{-1}}^{\alpha}$ and $I$ are derived by Algorithm 2.

**Remark 1.** The stability ranges of polynomials given in the lines 1, 4 and 5 of Algorithm 2 are determined by Algorithm 1.

**Remark 2.** The stability ranges (13) are almost exact. The word “almost” is used above because we cannot know whether $\mathcal{P}$ is robustly stable for $K \in (-\infty, -M + \Delta)$, $(K_{i_{-1}}^{\alpha} - \Delta, K_{i_{-1}}^{\alpha} + \Delta)$, $(M - \Delta, \infty)$, or not by the algorithm, where $d \in I$. Figure 3 shows an example of a derivable stability range by the proposed method when $I = \{d\}$. However, the obtained stability range can be considered to be exact practically because $M$ and $\Delta > 0$ can be set to be sufficiently large and small, respectively. Since $\Delta$ can be set to be sufficiently small, the ranges $(K_{i_{-1}}^{\alpha} - \Delta, K_{i_{-1}}^{\alpha} + \Delta)$, $(K_{i_{-1}}^{\alpha} - \Delta, K_{i_{-1}}^{\alpha} + \Delta)$, $d \in I$, which cannot be known whether stability ranges or not, can be sufficiently small as shown in Fig. 3. By the proposed method we can know the stability ranges in $[-M + \Delta, M - \Delta]$ instead of $(-\infty, \infty)$, except for the small ranges $(K_{i_{-1}}^{\alpha} - \Delta, K_{i_{-1}}^{\alpha} + \Delta)$, $(K_{i_{-1}}^{\alpha} - \Delta, K_{i_{-1}}^{\alpha} + \Delta)$, $d \in I$. The range $[-M + \Delta, M - \Delta]$ can be enlarged as shown in Fig. 3, because we can set $M$ and $\Delta$ to be sufficiently large and small, respectively.
Since this method is for robust stability test of polynomials with scalar uncertainty, a numerical check by gridding the parameter space can be an alternative way. However, the method using gridding has a risk to miss unstable polynomials. On the other hand, the proposed method does not have such a risk because the stability condition is sufficient (necessary and sufficient when $M$ and $\Delta$ are sufficiently large and small, respectively).

Note that our method cannot be applied if the complement of the stability region in the complex plane is not pathwise connected. The example in the next subsection satisfies the condition that the complement is pathwise connected.

### 3.3 Robust Stability Analysis of an Aircraft

In this subsection, robust stability analysis for the lateral dynamics of an aircraft taken from [19],[25] is given. This example shows that one can derive the stability range of an uncertain parameter by using Algorithm 2. The example also shows that the stability range derived by the proposed method are larger than those given in [19].

The nominal characteristic polynomial of the lateral dynamics of an aircraft is given as

$$\hat{p}(s) := 52 + 154s + 122.5s^2 + 50s^3 + 10.5s^4 + s^5. \quad (15)$$

The zeros of $\hat{p}(s)$ are

$$-0.5, \quad -2 \pm j2, \quad -3 \pm j2. \quad (16)$$

Unlike in [24], we consider the case that all the coefficient of the characteristic polynomial varies. The characteristic polynomial family is given by

$$\mathcal{P} = \left\{ \hat{p}(s) + \sum_{i=0}^{4} \alpha_i p_i(s) + Kp_K(s) \right\}$$

$$\alpha_i \in [-r_i, r_i] \subset \mathbb{R} \right\}, \quad (17)$$

where

$$p_i(s) := s^i, \quad i = 0, 1, 2, 3, 4,$$

$$p_K(s) := s^5,$$

$$r_0 := 52 \times 0.01k,$$

$$r_1 := 154 \times 0.001k,$$

$$r_2 := 122.5 \times 0.002k,$$

$$r_3 := 50 \times 0.007k,$$

$$r_4 := 10.5 \times 0.02k. \quad (18)$$

This characteristic polynomial family has six uncertain parameters: $\alpha_0$, $\alpha_1$, $\alpha_2$, $\alpha_3$, $\alpha_4$ and $K$. The stability region in the complex plane is inside the disks of radius 0.4 around the zeros of $\hat{p}(s)$ as shown in Fig. 4. The stability region satisfies the condition that its complement is pathwise connected on the Riemann sphere.

If we assume that $K \in [-1 \times 0.05k, 1 \times 0.05k]$, the polynomial family $\mathcal{P}$ given in (17) coincides with the polynomial family considered in Example 1 of [19]. In [19], it is shown that the system is robustly stable if $k \leq 0.241$. In this paper, we show that the proposed method enables one to derive a larger stability range.

Now we define that $k := 0.25$ and derive ranges of $K$ stabilizing $\mathcal{P}$. First, we execute the line 1 of Algorithm 2. In order to derive $K$ stabilizing

$$\left\{ \hat{p}(s) + \sum_{i=0}^{4} r_i p_i(s) + Kp_K(s) \right\}$$

we derive $K$ stabilizing the following $2^5$ polynomials:

$$\hat{p}(s) - r_0 p_0(s) - r_1 p_1(s) - r_2 p_2(s) - r_3 p_3(s) - r_4 p_4(s) + Kp_K(s)$$

by Algorithm 1 and obtain the intersection of each stability range. Stability-boundary-crossing points of the first polynomial of (20) are derived as follows. Define

$$f(s) := \hat{p}(s) - r_0 p_0(s) - r_1 p_1(s) - r_2 p_2(s) - r_3 p_3(s) - r_4 p_4(s) + Kp_K(s),$$

$$g(s) := p_K(s). \quad (21)$$

The coefficient vectors are then given by

$$f = [51.87 \quad 153.9615 \quad 122.43875 \quad 49.9125 \quad 10.4475 \quad 1]^T,$$

$$g = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1]^T. \quad (22)$$
Therefore \( a_0 \) is defined as \( a_0 = -f_1/g_5 = -1/1 = -1 \) in the line 2 of Algorithm 1. The connected regions of which stability region in the complex plane consist are given as

\[
S_1 := \{ x + jy \mid (x + 0.5)^2 + y^2 < 0.4^2 \}, \\
S_2 := \{ x + jy \mid (x + 2)^2 + (y - 2)^2 < 0.4^2 \}, \\
S_3 := \{ x + jy \mid (x + 2)^2 + (y + 2)^2 < 0.4^2 \}, \\
S_4 := \{ x + jy \mid (x + 3)^2 + (y - 2)^2 < 0.4^2 \}, \\
S_5 := \{ x + jy \mid (x + 3)^2 + (y + 2)^2 < 0.4^2 \},
\]

as shown in Fig. 4, where \( x \) and \( y \) are real numbers. Then, the set of points on the boundaries are given as

\[
\partial S_i = \{ -0.1, -0.9 \}, \quad i = 2, 3, 4, 5, \\
\partial S_i = \{ x + jy \mid (x + 0.5)^2 + y^2 = 0.4^2, \ y \neq 0 \}, \quad i = 2, 3, 4, 5, \\
\partial S_i = \{ x + jy \mid (x + 2)^2 + (y - 2)^2 = 0.4^2 \}, \quad i = 2, 3, 4, 5, \\
\partial S_i = \{ x + jy \mid (x + 2)^2 + (y + 2)^2 = 0.4^2 \}, \quad i = 2, 3, 4, 5, \\
\partial S_i = \{ x + jy \mid (x + 3)^2 + (y - 2)^2 = 0.4^2 \}, \quad i = 2, 3, 4, 5, \\
\partial S_i = \{ x + jy \mid (x + 3)^2 + (y + 2)^2 = 0.4^2 \}, \quad i = 2, 3, 4, 5,
\]

where \( \phi \) is the empty set. Since \( \partial S_1 = \{ -0.1, -0.9 \} \), we derive the values of \( a \) which satisfies \( e^i_s(f + ag) = 0 \) for the case that \( x = -0.1 \) and \( y = -0.9 \) in order to derive \( A_1' \) in the line 3 in Algorithm 1. In the case that \( x = -0.1 \), the equation \( e^i_s(f + ag) = 0 \) has the solution

\[
a = 3.765 \times 10^6.
\]

In the case that \( x = -0.9 \), the equation \( e^i_s(f + ag) = 0 \) has the solution

\[
a = -29.88.
\]

However, \( -29.88 \notin A_1' \) because \(-29.88 < a_0 = -1 \). \( A_1' \) is therefore determined as

\[
A_1' = (3.765 \times 10^6).
\]

On the other hand,

\[
A_1' = \phi, \quad i = 2, 3, 4, 5
\]

because \( \partial S_i = \phi, \quad i = 2, 3, 4, 5 \). The sets \( A_i' \) given in the line 4 in Algorithm 1 is derived as follows. To derive \( A_1' \), we solve the following system of equations

\[
\begin{cases}
E_{c+2}(f + ag) = 0 \\
(x + 0.5)^2 + y^2 = 0.4^2.
\end{cases}
\]

The solution of the above is obtained as

\[
a = -1.142 \times 10^6.
\]

However, \(-1.142 \times 10^6 \notin A_1' \) because \(-1.142 \times 10^6 < a_0 \). We therefore obtain

\[
A_1' = \phi.
\]

\( A_2' \) can be derived by solving

\[
\begin{cases}
E_{c+2}(f + ag) = 0 \\
(x + 2)^2 + (y - 2)^2 = 0.4^2.
\end{cases}
\]

The solutions of the above are obtained as

\[
a = -0.2056, 0.0802.
\]

Both of the solutions of \( a \) is greater than \( a_0 = -1 \). Therefore \( A_2' \) is given as

\[
A_2' = \{ -0.2056, 0.0802 \}.
\]

Similarly, \( A_4' \) is derived as

\[
A_4' = \{ -0.0445, 0.0232 \}.
\]

It is obvious that

\[
A_2' = A_2' = \空
\]

because there are no \( y > 0 \) satisfying \( x + jy \in \partial S_i \) for \( i = 3 \) or \( 5 \). Form the above, set \( \bigcup_{i=1}^5 (A_i' \cup A_i') \) is equal to \(-0.2056, -0.0445, 0.0232, 0.0802, 3.765 \times 10^6 \). Therefore the stability-boundary-crossing points are given as

\[
a_1 = -0.2056, \\
a_2 = -0.0445, \\
a_3 = 0.0232, \\
a_4 = 0.0802, \\
a_5 = 3.765 \times 10^6.
\]

Now we derive the stability ranges of the first polynomial of (20) by using the above stability-boundary-crossing points. One can see that \( f(s) + 4s^4g(s), f(s) + 4s^4g(s), f(s) + 4s^4g(s), f(s) + 4s^4g(s), f(s) + 4s^4g(s) \) and \( f(s) + 4s^4g(s) \) are not stable and that \( f(s) + 4s^4g(s) \) is stable. It is concluded that the first polynomial of (20) is stable for all

\[
K \in (a_2, a_5) = (-0.0445, 0.0232).
\]

The stability ranges of all the polynomials of (20) can be derived by the same way. The intersection of these stability ranges is given by

\[
(K_{11}^c, K_{11}^c) = (-0.0170, 0.0181),
\]

which is equal to the range of \( K \) stabilizing (19).

Now we execute the lines 2–12 of Algorithm 2. Stability of

\[
\dot{p}(s) + \sum_{i=1}^{c-1} r_{ia} p_i(s) + \sum_{i=1}^{l} r_{ia} p_i(s) + (K_e^d - \Delta c)p_k(s) + \alpha_e p_k(s)
\]

needs to be examined. It is possible by using Algorithm 1. As a result, the polynomial is stable for all \( \alpha_e \in [-r_e, r_e], r_e \in [-r_e, r_e], c \in [-1, +1], d \in I, e \in [1, \ldots, l], \) where \( I = \{ 1 \} \) and \( \Delta = 1 \times 10^{-4} \). Therefore the range of \( K \) stabilizing the system is given by

\[
[-0.0170 + \Delta, 0.0181 - \Delta] = [-0.0169, 0.0180].
\]

Now let us compare the result in [19] and that by the proposed method. The result in [19] is as follows. The system is stable if \( k \leq 0.241 \), i.e.,

\[
\dot{p}(s) + \sum_{i=0}^{4} \alpha_i p_i(s) + K p_k(s)
\]
is stable for all
\[ \begin{align*}
\alpha_0 & \in [-0.12532, 0.12532], \\
\alpha_1 & \in [-0.037114, 0.037114], \\
\alpha_2 & \in [-0.059045, 0.059045], \\
\alpha_3 & \in [-0.08435, 0.08435], \\
\alpha_4 & \in [-0.05061, 0.05061], \\
K & \in [-0.01205, 0.01205].
\end{align*} \]

(58)

On the other hand, the result by the proposed method shows that (57) is stable for all
\[ \begin{align*}
\alpha_0 & \in [-0.13, 0.13], \\
\alpha_1 & \in [-0.0385, 0.0385], \\
\alpha_2 & \in [-0.06125, 0.06125], \\
\alpha_3 & \in [-0.0875, 0.0875], \\
\alpha_4 & \in [-0.0525, 0.0525], \\
K & \in [-0.0169, 0.0180].
\end{align*} \]

(59)

The obtained stability ranges are larger than those shown in [19].

4. Conclusion

This paper has given extensions of [24]. The algorithm shown in Subsection 3.1 enables one to derive stability ranges for disconnected stability region. We also have shown another algorithm to derive stability ranges in the case that characteristic polynomials depend on multiple uncertain parameters in Subsection 3.2. The numerical example shows that the proposed method can be applied to stability analysis of the lateral dynamics of an aircraft even if all the coefficients of the characteristic polynomial vary. It is shown that stability ranges derived by the proposed method are larger than those given in [19].

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References

Tadasuke Matsuda (Member)

He received his B.S., M.S., and Ph.D. degrees from Kyoto Institute of Technology, Japan, in 2003, 2005, and 2008, respectively. In 2016, he joined Chiba Institute of Technology, where he is currently an Associate Professor of the Department of Electrical and Electronic Engineering, Faculty of Engineering. His research interests include robust stability theory and power systems. He is a member of ISCIE and IEEJ.