Constrained Optimal Flyby Guidance Algorithm by Iterative Two-Stage Stochastic Programming

Naoya OZAKI *, Yosuke KAWABATA **, Hiroshi TAKEUCHI ***, Tsutomu Ichikawa ***, Ryu Funase *, and Yasuhiro KAWAKATSU ***

Abstract: Interplanetary micro-spacecraft have been recently demonstrated for various missions. Orbital control of micro-spacecraft is challenging because the spacecraft systems have severe constraints. This paper presents a constrained optimal flyby guidance algorithm. We iteratively introduce two-stage stochastic programming and achieve rapid and global optimization by using delta-v mappings and cross-correlation technique, which is well-known in the field of image and signal processing. Finally, numerical examples and Monte-Carlo simulation show that the proposed algorithm is efficient in terms of the expected total delta-v.

Key Words: spacecraft trajectory optimization, flyby guidance, stochastic control, stochastic programming.

1. Introduction

In the background of the recent rapid growth of small spacecraft, PROCYON was launched on 3rd December 2014. PROCYON is an interplanetary micro-spacecraft developed by the University of Tokyo and ISAS/JAXA, and it is the world’s first deep space micro-spacecraft [1]–[3]. The primary mission objectives are the technology demonstrations of a micro-spacecraft bus system for deep-space exploration, and the secondary objectives include a number of engineering and science experiments. As one of the engineering experiments, we have conducted the flyby guidance experiment toward the Earth. The previous paper is focused on the flight experiments and their results [4]. The associated paper mainly describes the theoretical part of the constrained optimal flyby guidance algorithm.

Although orbital mechanics of micro-spacecraft is the same as larger spacecraft, there are more severe constraints in the spacecraft systems. The flyby guidance problems must deal with the constrained problems. For the constrained flyby guidance problem, the previous literature [5],[6] shows that the offset on the target plane improves the performance and robustness. Most of these navigation analyses are based on heuristic methods and therefore it is time-consuming work.

This paper finds the optimal offset sequence in constrained optimal flyby guidance problems by two-stage stochastic programming [7]. The algorithm introduces the pre-computation of delta-v mappings and matrix cross-correlations to reduce the computational time. The delta-v mappings also visually support the intention of the computational process. Although this paper deals with the simple constraints on the solar angle, the proposed algorithm is also applicable for complicated flyby problems such as a risk-sensitive flyby guidance problem [8].

2. PROCYON Guidance, Navigation, and Control

2.1 Overview

To guide spacecraft to a target point precisely, we alternately execute Orbit Determination (OD) and Trajectory Correction Maneuver (TCM). In the PROCYON spacecraft, OD is based on radiometric navigation (Range and Range-Rate: RARR and Delta Differential One-way Ranging: DDOR) [9],[10], and TCM is designed using the previous OD and controlled by the cold gas jet thrusters [11].

2.2 Navigation: Orbit Determination

OD of PROCYON has some difficulties; 1) fewer operations make less number of RARR observations than usual interplanetary spacecraft, and 2) flexible attitudes for other experiments make not only the dynamical system but also Doppler observations noisy. The details are discussed in the previous paper [4].

2.3 Control

PROCYON is equipped with the combined propulsion system of the ion thruster and the cold gas jet thrusters, which share Xenon propellant in the same tank [11]. The ion thruster achieves the large delta-v but low-thrust, and it takes long time to accelerate enough. The cold gas jet thrusters are complementarily used to achieve higher thrust. TCM uses the cold gas jet thrusters, whose arrangements are illustrated in Fig. 1. 8 thrusters can generate 3-DOF torques for the attitude control and 2-DOF translational acceleration for the orbital control. However, the flight operations let us notice that $+X$-translational acceleration (by CT-5,6/CT-7,8 in Fig. 1) caused larger angular momentum than expected. We suspect that the large disturbances are caused by the reflection of the exhaust gas on the backside of the solar array panels. Hence, the flight operations use only $+Z$/Z translational acceleration (by CT-1,2/CT-3,4 in Fig. 1), and it results in the severe constraints on the thrust direction because the solar arrays must direct to the Sun.
3. Constrained Optimal Flyby Guidance Algorithm

3.1 Variable Time of Arrival Guidance Law

PROCYON is guided to a virtual target point on the target plane by the Variable Time-of-Arrival (VTA) guidance law [12]. The target plane is defined as a plane normal to the relative velocity as shown in Fig. 2. The VTA guidance law has 1-DOF to determine the maneuver (control input), and therefore the maneuver is designed to minimize the delta-v magnitude subject to the thrust direction constraints which comes from the solar angle limitation.

Let us define a state vector $X(t) := [r(t), v(t)]^T$, where $r(t) \in \mathbb{R}^3$ and $v(t) \in \mathbb{R}^3$ are the position and velocity of the spacecraft. The State Transition Matrix (STM) $\Phi(t, t_0) \in \mathbb{R}^{6 \times 6}$ is defined as the partial derivative of $X(t)$ with respect to $X(t_0)$. Hence,

$$\Phi(t, t_0) := \frac{\partial X(t)}{\partial X(t_0)} = \begin{bmatrix} \Phi_{rr}(t, t_0) & \Phi_{rv}(t, t_0) \\ \Phi_{vr}(t, t_0) & \Phi_{vv}(t, t_0) \end{bmatrix}. \quad (1)$$

In other words, STM is the matrix to describe the linear sensitivity of the variation of $\delta X(t)$ by the variation $\delta X(t_0)$:

$$\begin{bmatrix} \delta r(t) \\ \delta v(t) \end{bmatrix} \approx \begin{bmatrix} \Phi_{rr}(t, t_0) & \Phi_{rv}(t, t_0) \\ \Phi_{vr}(t, t_0) & \Phi_{vv}(t, t_0) \end{bmatrix} \begin{bmatrix} \delta r(t_0) \\ \delta v(t_0) \end{bmatrix}. \quad (2)$$

STM is numerically computed by integrating the dynamical system [13].

Let us design the $k$-th TCM $\Delta v_k$ at an epoch $t_k$ to correct the position on the target plane. The $k$-th OD estimates the state vector as $X_{k}^{est}$, and the final state vector $X(t_f)$ and the STM $\Phi(t_f, t_k)$ can be propagated using $X_{k}^{est}$.

For the target body, whose position and velocity vectors are $r^{tar}(t_f)$ and $v^{tar}(t_f)$ at $t_f$, the relative position of the target body from the spacecraft at time $t_f + \delta t$ is

$$r_{rel}(t_f + \delta t) = r^{tar}(t_f + \delta t) - r^{tar}(t_f)$$

Therefore, the VTA guidance designs the TCM $\Delta v_k$ as

$$\Delta v_k = \Phi_{rr}(t_f, t_k)^{-1}\left[ r_{rel}(t_f + \delta t) - r_{rel}(t_f) \right], \quad (5)$$

where $\delta t$ is an arbitrary parameter representing the time difference. As shown in Fig. 2, $\Delta v_k$ in Eq.(5) causes the arbitriness on the terminal relative position along the relative velocity.

Let us determine the parameter $\delta t$ to minimize $\|\Delta v_k\|$ subject to the solar angle constraint:

$$\|\Delta v_k \cdot s\| \geq \|\Delta v_k\| \cos \alpha, \quad (6)$$

where $s$ is the solar direction with respect to the spacecraft at $t_k$, and $\alpha$ is the maximum solar angle (nominally 45° to 55° for PROCYON). The left hand side of the equation takes the absolute value because the spacecraft can accelerate either plus or minus Z direction. Introducing $a \in \mathbb{R}^3$ and $b \in \mathbb{R}^3$ such that $\Delta v_k = a + b\delta t$ and applying orthogonalization yields the optimal parameter $\delta t^*$ as $-(a \cdot b)/(\|b\|^2)$ if $\delta t^*$ satisfies the solar angle constraint (6). If $\delta t^*$ does not satisfy the constraint, we find the optimal parameter with the active constraint $\|\Delta v_k \cdot s\| = \|\Delta v_k\| \cos \alpha$. The parameter is obtained by solving the quadratic equation for $\delta t$

$$\left\{ \begin{array}{l} (a \cdot s)^2 - \|a\|^2 \cos^2 \alpha + 2(a \cdot b)(b \cdot s) - (a \cdot b) \cos^2 \alpha \|b\|^2 \cos^2 \alpha \delta t^* = 0 \\
\end{array} \right.$$
For a decision variable $y \in \mathbb{R}^n$ and a random variable $\xi$, a two-stage-stochastic programming problem is generally defined by
\[
\min_y \left\{ f(y) + E_\xi [Q(y, \xi)] \right\},
\]
where $E_\xi \{ \}$ denotes the expectation with respect to $\xi$, and $Q(y, \xi)$ is the optimal value of the second-stage problem
\[
Q(y, \xi) = \min_y \{ q(y, \xi) \}. \tag{8}
\]

The optimal offset problem introduces the offset vectors $y_k = [y_{1k}, y_{2k}, ...] \in \mathbb{R}^2, k = 1, ..., N$. Assuming $y_{k-1}$ and $y_{k+1}$ are given, let us solve a two-stage stochastic programming problem
\[
\min_{y_k} \left\{ \Delta V_k(y_k) + E_\xi [\Delta V_{k+1}(y_k, \xi)] \right\}, \tag{9}
\]
where $\Delta V_k(\cdot)$ is the deterministic delta-v from $y_{k-1}$ to $y_k$ and $\Delta V_{k+1}(\cdot)$ is the stochastic delta-v from $y_k$ to $y_{k+1}$. Figure 5 illustrates the schematic of the two-stage stochastic programming problem. The proposed algorithm iteratively solves the two-stage stochastic programming problems by computing the deterministic and stochastic delta-v using the delta-v mappings.

The optimal offset problem introduces the offset vectors $y_k = [y_{1k}, y_{2k}, ...] \in \mathbb{R}^2, k = 1, ..., N$. Assuming $y_{k-1}$ and $y_{k+1}$ are given, let us solve a two-stage stochastic programming problem
\[
\min_{y_k} \left\{ \Delta V_k(y_k) + E_\xi [\Delta V_{k+1}(y_k, \xi)] \right\}, \tag{9}
\]
where $\Delta V_k(\cdot)$ is the deterministic delta-v from $y_{k-1}$ to $y_k$ and $\Delta V_{k+1}(\cdot)$ is the stochastic delta-v from $y_k$ to $y_{k+1}$. Figure 5 illustrates the schematic of the two-stage stochastic programming problem. The proposed algorithm iteratively solves the two-stage stochastic programming problems by computing the deterministic and stochastic delta-v using the delta-v mappings.

Assumption 1. The difference between the nominal state vector $X_{k}^{\text{nom}}$ and the estimated state vector $X_{k}^{\text{est}}$ by k-th OD does not make large differences on $\Phi_{\text{ref}}(t, t_k)$ and $v_{\text{ref}}$ in Eq.(5).

Therefore the delta-v mapping $\Delta V_k(\cdot)$ based on $X_{k}^{\text{nom}}$ can approximate the upcoming delta-v mapping based on $X_{k}^{\text{est}}$. This assumption is reasonable for high relative velocity flyby problems.

For the offset vectors $y_{k-1}$ and $y_k$, the deterministic delta-v by k-th TCM is obtained by the delta-v mapping as
\[
\Delta v_k = \Delta V_k(y_{k-1} - y_k). \tag{10}
\]

The position on the target plane will be perturbed by the navigation and execution errors of the k-th TCM. Let us define the perturbed offset vector as $Y_k = \mathcal{N}(Y_k, \Sigma_k)$, where $\Sigma_k$ is the covariance matrix of the guidance error by k-th TCM. Hence, we can derive the stochastic delta-v by $(k+1)$-st TCM to target to $y_{k+1}$
\[
\Delta v_{k+1} = \Delta V_{k+1}(Y_k - y_{k+1}). \tag{11}
\]

Substituting Eq.(10) and Eq.(11) to Eq.(9) yields the value function for the optimal offset guidance problem
\[
\Delta V_k (y_{k-1} - y_k) + E [\Delta V_{k+1}(Y_k - y_{k+1})]. \tag{12}
\]

This value function depends on $y_{k-1}$, $y_k$, and $y_{k+1}$. Let us introduce the iterative computation to find the optimal offset sequence $\{y_1, y_2, ..., y_N\}$. For each iteration step, we assume that $y_{k-1}$ and $y_{k+1}$ are given, and find the optimal offset $y_k^*$ to minimize Eq.(12). Note that the offset sequence is assumed as deterministic values.

Algorithm 1 (Iterative Two-Stage Stochastic Programming).
Find the offset sequence $\{y_1, y_2, ..., y_N\}$ by solving the following problems iteratively until convergence. For each $k = 1, ..., N-1$, given $y_{k-1}$ and $y_{k+1}$, find the optimal offset $y_k^*$ by solving
\[
\min_{y_k} \{ \Delta V_k (y_{k-1} - y_k) + E [\Delta V_{k+1}(Y_k - y_{k+1})]\}, \tag{13}
\]
where $y_1, ..., y_{N-1}$ should be iteratively updated as the optimal offset obtained in Eq.(13), $y_0$ is a given initial condition, $y_N = 0$ because there is no further TCM, and $N$ (the number of TCM) is fixed.

Note that this algorithm has good convergence for the typical problems as shown in the next section, but the convergence for general problems is not proven.

The expected value in Eq.(13) is rapidly and globally computed by using the delta-v mapping $\Delta V_k(\cdot)$ and the cross-correlation technique, which is well-known in the field of the signal and image processing. Given the probability distribution of $Y_k$ and the delta-v mapping $\Delta V_{k+1}(\cdot)$, the expected value $E[\Delta V_{k+1}(Y_k - y_{k+1})]$ can be computed by the cross-correlation between the two matrices, such as xcorr2( ) in Matlab. Figure 6 illustrates the contour plots of two matrices $M_1$ and $M_2$ corresponding to the probability distribution of $(Y_k - y_k)$ and the delta-v mapping $\Delta V_{k+1}(\cdot)$, respectively. We can compute the expected value $E[\Delta V_{k+1}(\cdot)]$ for each $y_k$ by taking the cross-correlation of $M_1$ and $M_2$
\[
C = \text{xcorr2}(M_1, M_2), \tag{14}
\]

Fig. 4 Guidance errors on the delta-v mapping.

Fig. 5 Two-stage stochastic programming for the guidance problem.

Fig. 6 Cross-correlation to compute $E[\Delta V_{k+1}(\cdot)]$. 
where \( C \) is the matrix corresponding to the expected value \( E[\Delta V_{k+1}(Y_k - y_{k+1})] \) for each \( y_k \) and the contour of the matrix \( C \) is shown in the right part of Fig. 7.

In Eq.(13), the first term \( \Delta V_k(y_k - y_{k-1}) \) is computed by the translation and the horizontal and vertical flip of the delta-v mapping \( \Delta V_k(\cdot) \), and the second term \( E[\Delta V_{k+1}(\cdot)] \) is computed as Eq.(14). The left and right parts of Fig. 7 show the first and second terms of Eq.(13), respectively. Finding the minimum value of the sum of the both figures yields the optimal offset vector \( y_k^* \).

![Fig. 7 Computation of Eq.(13) based on the delta-v mapping.](image)

### 4. Numerical Example

#### 4.1 Numerical Results

This section shows the numerical examples of the proposed guidance algorithm. The simulation conditions are shown in Table 1, and the optimal offset sequences are shown in Fig. 8. Starting from the initial guess, where all offset vectors are set as zeros, the optimization problems usually converge within about 3 iterations. As a practical issue, the delta-v mapping has singular points, and therefore, imposing the maximum penalty avoids the singularity such that \( \Delta V_k(\cdot) = 10\text{m/s} \) if \( \Delta V_k(\cdot) > 10\text{m/s} \). As shown in Fig. 8, we realize that the offset guidance laws have significant differences when the solar angle constraint is 45° or 50°.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Position</td>
<td>(-800.0, 0.0)</td>
<td>km</td>
</tr>
<tr>
<td>Number of TCMs ( N )</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Angle Constraint</td>
<td>45.0, 50.0, 55.0, N/A</td>
<td>deg</td>
</tr>
<tr>
<td>Max. Penalty</td>
<td>10.0</td>
<td>m/s</td>
</tr>
<tr>
<td>Epoch</td>
<td>2015/11/07 03:00:00 UTC @TCM1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2015/11/15 03:00:00 UTC @TCM2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2015/11/22 08:00:00 UTC @TCM3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2015/11/28 08:00:00 UTC @TCM4</td>
<td></td>
</tr>
<tr>
<td>Guidance Error ((\sigma_T, \sigma_R))</td>
<td>(35.0, 100.0) @TCM1</td>
<td>km</td>
</tr>
<tr>
<td></td>
<td>(15.0, 50.0) @TCM2</td>
<td>km</td>
</tr>
<tr>
<td></td>
<td>(5.0, 30.0) @TCM3</td>
<td>km</td>
</tr>
<tr>
<td></td>
<td>(2.0, 8.0) @TCM4</td>
<td>km</td>
</tr>
</tbody>
</table>

#### 4.2 Monte-Carlo Simulations

This section shows two scenarios of the Monte-Carlo simulations to demonstrate the efficiency of the proposed algorithm.

In the first scenario, the solar angle is constrained within 45°. Figures 9 and 10 show the flight paths of the optimal offset guidance and the no-offset guidance, respectively. In the Monte-Carlo simulation, every sample experiences the uncertainties coming from the guidance errors defined in Table 1. We realize that the proposed optimal offset guidance avoids the unfavorable direction. Figure 11 shows the histograms of the total delta-v for both cases. Although the no-offset guidance saves the nominal delta-v, the optimal offset guidance is more efficient in terms of the expected delta-v and 3-\( \sigma \) delta-v. These results show that the effectiveness of the proposed algorithm.

The second scenario considers the case of the 55° solar angle constraint. As shown in Fig. 8, the nominal flight path of the optimal offset guidance is almost the same as that of the no-offset guidance. Figure 12 illustrates the histogram of the total delta-v. We notice that the optimal offset guidance is slightly efficient in terms of the expected delta-v, but the difference is almost negligible for practical purposes.

### 5. Flight Result

We have conducted the flight experiment of flyby guidance by PROCYON. The guidance accuracy has aimed less than
100 km at about 3,000,000 km distance from the Earth, which is more strict than the required guidance accuracy at the Earth flyby for the nominal missions to the asteroid 2000DP107. The flight experiment has adopted the no-offset guidance law since we have allowed 55° solar angle and the difference between the no-offset and optimal offset guidance is trivial. Three TCMs have been executed in total, and PROCYON has finally achieved 100 km guidance accuracy from the target point by the final TCM. The details of the flight experiments are described in the previous paper [4].

Figure 13 shows the actual flight path and the guidance errors on the target plane. The flight path is estimated by the post analyses using the whole tracking data. The guidance errors include the execution errors and the orbit determination errors with consider parameters in the solar radiation pressure. Table 2 shows the overall planning of TCMs. TCMs are designed based on the latest OD results. The guided points at OD3 and OD4 are slightly out of the 3-σ guidance errors, i.e., the guidance errors estimated through the orbit determination are rather optimistic.

We simulate the flight result by the Monte-Carlo method with flight covariance matrices. The flight covariance matrices are obtained by propagating the navigation errors with the TCM execution errors as illustrated in Fig. 13 and Table 3. Figure 14 illustrates the histogram of the total delta-v, and we realize that the total delta-v of flight result (1.023 m/s) is around the 3-σ value of the histogram. The flight path in Fig. 13 also shows that the spacecraft has flown around the 3-σ guidance errors.

Table 2 Overall planning of TCM1 to TCM3.

<table>
<thead>
<tr>
<th>TCM</th>
<th>ΔV (m/s)</th>
<th>Epoch UTC</th>
<th>+Z Sun (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TCM1</td>
<td>0.540</td>
<td>2015/11/07 03:00:00</td>
<td>41.2</td>
</tr>
<tr>
<td>TCM2</td>
<td>0.223</td>
<td>2015/11/15 05:30:00</td>
<td>52.9</td>
</tr>
<tr>
<td>TCM3</td>
<td>0.260</td>
<td>2015/11/22 08:00:00</td>
<td>52.5</td>
</tr>
</tbody>
</table>

Table 3 Flight covariance matrices on the target plane.

<table>
<thead>
<tr>
<th>TCM</th>
<th>T(km)</th>
<th>R(km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TCM1</td>
<td>(0.47162 \times 10^3)</td>
<td>(-0.03173 \times 10^4)</td>
</tr>
<tr>
<td>TCM2</td>
<td>(0.97862 \times 10^3)</td>
<td>(0.63401 \times 10^4)</td>
</tr>
<tr>
<td>TCM3</td>
<td>(0.26968 \times 10^3)</td>
<td>(0.15311 \times 10^3)</td>
</tr>
</tbody>
</table>

Fig. 14 Histogram of Monte-Carlo simulation based on flight result.

6. Conclusion

This paper presents the flyby guidance algorithm for the interplanetary micro-spacecraft PROCYON. Orbital controls of interplanetary micro-spacecraft are challenging because of severe restrictions on spacecraft systems. We introduce a constrained optimal flyby guidance strategy by iterative two-stage stochastic programming. The proposed algorithm achieves the rapid and global computation by the delta-v mappings and the cross-correlation technique. The numerical examples and Monte-Carlo simulation show the effectiveness of the proposed algorithm.

Acknowledgments

The flight experiments were supported by Fujitsu Limited and the authors thank Sho Taniguchi and Tomoko Yagami for support and collaborations. This work has been supported by JSPS Grant-in-Aid for JSPS Fellows Number 15J05999.
References


Naoya Ozaki (Student Member)
He received his B.S., M.S. degrees from The University of Tokyo, Japan. He is currently a Ph.D. student of the Department of Aeronautics and Astronautics in The University of Tokyo. His research interests include orbital mechanics, spacecraft trajectory designs, robust and stochastic optimal control.

Yosuke Kawabata
He is a Ph.D. student in Graduate School of Frontier Sciences, The University of Tokyo. He received his B.S. and M.S. degrees from Toyota Technological Institute and The University of Tokyo, Japan, in 2012 and 2014, respectively. His research interests include orbit determination, trajectory design, and autonomous navigation.

Hiroshi Takeuchi
He received his Ph.D. in Physics & Applied Physics from Waseda University, Tokyo, Japan in 2000. In 2006, he joined ISAS/JAXA, where he is currently an Associate Professor at ISAS. He is leading development of JAXA’s deep space orbit determination software, media calibration system, and Delta-DOR measurement system.

Tsutomu Ichikawa (Member)
He has worked at ISAS, JAXA since 1987. He has been working to develop the orbit determination system and analysis to JAXA’s several spacecrafts for the navigation. He is currently researcher of JAXA. His research includes modern control and estimation theory. He received Ph.D. degree in 2006. He is member of IEEE.

Ryu Funase
He is currently an Associate Professor of The University of Tokyo, Japan. He received the B.E., M.E., and Ph.D. degrees from The University of Tokyo. He initiated the project to develop ultra-small deep space probe PROCYON as the project manager. He has been engaged in the researches on nano or micro spacecraft systems, and guidance, navigation, and control of spacecraft.

Yasuhiro Kawakatsu
He received his B.S., M.S., and Ph.D. degrees from The University of Tokyo, Japan. In 1997 and 2003, he joined NASDA and ISAS, JAXA, respectively. He is currently an Associate Professor of ISAS, JAXA. His research interests include astrodynamics and spacecraft mission design. He is a member of JSASS.