Gain-Scheduled Control/Steering Design for a Spacecraft with Variable-Speed Control Moment Gyros

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Abstract: In this paper, we establish the dynamics of a spacecraft with Variable-Speed Control Moment Gyros (VSCMGs). Based on this dynamics, we develop an easy-to-use Linear Parameter-Varying (LPV) model, in which we deal with 3-axis attitude control of a spacecraft with pyramid-array VSCMGs. Although it is hard to consider overall stability and control performance of attitude control at the same time with conventional Lyapunov function-based controllers, this paper attains both of them via LPV control theory. And also we design two types of Gain-Scheduled (GS) steering laws for singularity avoidance and escape. Through numerical examples, the efficiency of the proposed GS controller and the GS steering laws is demonstrated.

Key Words: spacecraft, attitude control, Variable-Speed Control Moment Gyro, GS control, GS steering.

1. Introduction

Reaction Wheels (RWs) are often used for an attitude control of spacecraft as attitude actuators. A RW is constructed by a variable-speed flywheel. When the wheel speed is changed, a RW can generate a required torque for attitude control. Control Moment Gyros (CMGs) have been also used for an attitude control of spacecraft. A CMG has a constant-speed flywheel with a gimbal. When the gimbal of the CMG rotates, it can generate a gyroscopic torque which is larger than that of a RW. Variable-Speed CMGs (VSCMGs) can be regarded as a kind of hybrid system which consists of a RW and a CMG. The extra degree of freedom (DOF) of the wheel spin rate changes can be easily used to avoid singularities [1]–[4].

Satellite dynamics is described by a nonlinear differential equation. Most of recent studies about attitude control have been using non-linear controllers such as Lyapunov function-based controllers [1],[2]. In these studies, although overall stability of attitude control is always guaranteed, control performance is ignored in most cases. To overcome this problem, we applied Linear Parameter-Varying (LPV) control theory [5]–[7] to attitude control problem [8]–[10]. In LPV control theory, we modeled dynamics of spacecraft as an LPV system and applied a Gain-Scheduled (GS) controller to this model using Linear Matrix Inequalities (LMIs).

A CMG system has a singularity problem. In the case of a VSCMG system, it is easy to avoid this problem to switch between a RW mode and a CMG mode with a weighting matrix [1]–[3]. However, when both RW mode and CMG mode are close to singularity, this weighting matrix may not work well. Therefore, in this paper, singularity avoidance steering laws not only with conventional-like weighting matrix approach but also with singular value decomposition (SVD) one are proposed.

In this paper, first, we establish the dynamics of a spacecraft with VSCMGs and develop an easy-to-use LPV model, while redefining the control input to design a GS controller. Then, we design two types of GS steering laws for singularity avoidance and escape. Through numerical examples, we demonstrate the efficiency of the proposed controller and the steering laws.

2. Spacecraft Model

2.1 Dynamics

The total angular momentum $H$ of a spacecraft with multi-VSCMGs can be described as follows:

$$ H = J\omega + G_i l_i \delta + G_s I_{ws} \Omega, $$

(1)

where $J \in \mathbb{R}^{3 \times 3}$ is the inertia matrix of a spacecraft including VSCMGs, $\omega \in \mathbb{R}^3$ the angular velocity of the spacecraft, the matrices of the spin axes and the gimbal axes are denoted by $G_i = [s_1, \ldots, s_n] \in \mathbb{R}^{3 \times n}$ and $G_s = [g_1, \ldots, g_n] \in \mathbb{R}^{3 \times n}$, respectively. $l_{ws}$ or $l_i$ is the moment of inertia matrix of the VSCMGs about the wheel or gimbal axes, respectively, $\Omega = [\Omega_1, \ldots, \Omega_n]^T \in \mathbb{R}^n$ the wheel spin rate vector and $\delta = [\delta_1, \ldots, \delta_n]^T \in \mathbb{R}^n$ the gimbal angle vector. Assuming that no external torque is applied to the spacecraft body, the dynamics is given by

$$ H + \omega^T H = 0, $$

(2)

where the notation $\omega^T$ denotes the following skew-symmetric matrix:

$$ \omega^T := \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}, \quad \omega = [x_1, x_2, x_3]^T. $$

Substituting Eq. (1) into Eq. (2), we have

$$ J\dot{\omega} + \frac{d}{dt} G_i l_i \delta + \frac{d}{dt} G_s I_{ws} \Omega = \omega^T J\omega $$

$$ + \omega^T G_i l_i \delta + \omega^T G_s I_{ws} \Omega = 0. $$

(3)

where $F_b$ denotes a body-fixed frame. The second term of the LHS in Eq. (3) is related to the gimbals of the VSCMGs.
the gimbal axes are fixed to a spacecraft, we need not take into account the gimbal rotations in the differentiation of $\mathcal{F}_B$. This is shown as follows:

$$
\frac{d}{dt}|_{F_B} (G_i I_{\delta}) = \sum_{i=1}^{n} \frac{d}{dt}|_{F'_i} (\hat{g}_i I_{\delta_i}) = G_i I_{\delta_i} \dot{\delta}_i.
$$

(4)

The third term of the LHS in Eq. (3) is related to the wheel spin rates of the VSCMGs. Therefore, we must take into account the gimbal rotations. This is shown as follows:

$$
\frac{d}{dt}|_{F_B} (G_i l \Omega) = \sum_{i=1}^{n} \frac{d}{dt}|_{F'_i} (\hat{g}_i l i \Omega_i) + \sum_{i=1}^{n} (\hat{g}_i I_{\delta_i} l_i \Omega_i) = G_i l \Omega_i + G_i l_i \Omega_i \, \text{diag}[\Omega] \hat{\delta}_i.
$$

(5)

where $\mathcal{F}_B$ is a gimbal rotation frame and $G_i = \{t_1, \ldots, t_n\} \in \mathbb{R}^{3 \times n}$ is the matrix that consists of the torque direction vectors $\hat{t}_i = \hat{g}_i \hat{\delta}_i$ for each VSCMG. In summary, Eq. (3) can be rewritten as

$$
J \dot{\omega} + G_i l u \dot{\Omega} + G_i l_i \Omega_i \, \text{diag}[\Omega] \dot{\delta} + G_i l_i \hat{\delta} + \omega \times J \omega + \omega \times G_i l_i \dot{\delta} + \omega \times G_i l_i \dot{\Omega} = 0.
$$

(6)

In this paper, we deal with pyramid-array VSCMGs allocation depicted as in Fig. 1. Each direction matrix $G_s$, $G_d$ and $G_i$ is given by

$$
G_s = \begin{bmatrix}
0 & 0 & s\beta & 0 \\
0 & 0 & 0 & -s\beta \\
s\beta & 0 & 0 & 0 \\
0 & -s\beta & 0 & 0
\end{bmatrix},
$$

$$
G_d s = \begin{bmatrix}
-c\beta \sin \delta_1 & -c\beta \sin \delta_2 & c\beta \sin \delta_3 & c\beta \cos \delta_1 \\
c\beta \sin \delta_1 & -c\beta \sin \delta_2 & -c\beta \sin \delta_3 & c\beta \cos \delta_1 \\
 s\beta \sin \delta_1 & s\beta \sin \delta_2 & s\beta \sin \delta_3 & s\beta \cos \delta_1 \\
 s\beta \cos \delta_1 & s\beta \cos \delta_2 & s\beta \cos \delta_3 & -s\beta \sin \delta_1 \\
n\beta \cos \delta_1 & s\beta \cos \delta_2 & -s\beta \cos \delta_3 & c\beta \sin \delta_1 \\
 s\beta \cos \delta_1 & -s\beta \cos \delta_2 & s\beta \cos \delta_3 & c\beta \sin \delta_1 \\
 s\beta \cos \delta_1 & s\beta \cos \delta_2 & -s\beta \cos \delta_3 & c\beta \sin \delta_1 \\
 s\beta \cos \delta_1 & s\beta \cos \delta_2 & -s\beta \cos \delta_3 & -c\beta \sin \delta_1 \\
\end{bmatrix},
$$

$$
G_i = \begin{bmatrix}
-c\beta \cos \delta_1 & c\beta \cos \delta_2 & -c\beta \cos \delta_3 & c\beta \sin \delta_1 \\
-c\beta \cos \delta_1 & c\beta \cos \delta_2 & c\beta \cos \delta_3 & -c\beta \sin \delta_1 \\
 s\beta \cos \delta_1 & s\beta \cos \delta_2 & s\beta \cos \delta_3 & s\beta \cos \delta_1 \\
 s\beta \cos \delta_1 & s\beta \cos \delta_2 & -s\beta \cos \delta_3 & c\beta \sin \delta_1 \\
\end{bmatrix},
$$

where $\sin \beta$ and $\cos \beta$ are abbreviated to $s\beta$ and $c\beta$.

2.2 Kinematics

The quaternion consists of the vector part and the scalar one. Given the principal rotation axis $\hat{\alpha} = [\alpha_x, \alpha_y, \alpha_z]^T$ with $\hat{\alpha} \times \hat{\alpha} = 1$ and the rotation angle $\Theta$, the quaternion (Euler Parameters) is (are) defined by

$$
q = \begin{bmatrix}
\hat{\alpha} \\
\cos \frac{\Theta}{2}
\end{bmatrix} = \begin{bmatrix}
\hat{\alpha} \\
\cos \frac{\Theta}{2}
\end{bmatrix}.
$$

(7)

with the constraint:

$$
q^T q = \alpha^T \alpha \sin^2 \frac{\Theta}{2} + \cos^2 \frac{\Theta}{2} = 1.
$$

(8)

To formulate the attitude tracking problem of a spacecraft, we need the error quaternion $q_e = q_d - q$, where $q$ denotes the current quaternion and $q_d$ denotes the desired quaternion with $\dagger$ meaning the conjugate operation. The kinematics equation is given by

$$
\begin{bmatrix}
\dot{q}_e \\
\dot{q}_{we}
\end{bmatrix} = G(q_e) \omega, \quad G(q_e) := \begin{bmatrix}
\frac{\partial q}{\partial \omega} & 0 \\
0 & \frac{\partial q}{\partial q}
\end{bmatrix} = \begin{bmatrix}
q_{we} I_3 + q_e^c \\
0
\end{bmatrix}.
$$

(9)

3. GS Controller Design

The Jacobian linearization of Eqs. (6) and (9) around the equilibrium point $(\omega_{eq} = 0, \Omega_{eq} = 0, \delta_{eq} = 0, q_{weq} = 0, q_{seq} = 1)$ leads to the LPV plant for complete attitude control as follows:

$$
\begin{bmatrix}
\dot{\omega} \\
\dot{q}_e
\end{bmatrix} = A(\rho) \begin{bmatrix}
\omega \\
q_e
\end{bmatrix} + B(\rho) u,
$$

(10)

where $u = [\Omega^T \delta]^T$ and

$$
A(\rho) = J^{-1}[G_i l u \Omega]^T, \quad B(\rho) = J^{-1}[C D],
$$

together with

$$
C = G_s, \quad D = G_d \text{diag}[\Omega],
$$

(11)

and $\rho = [\Omega^T \delta]^T$ is the scheduling parameter vector. If $\rho$ is rewritten by $\rho = \rho_1 \sin \delta_1 \sin \delta_2 \sin \delta_3 \cos \delta_4 \cos \delta_7 \cos \delta_1 \cos \delta_3$, this system is covered with a convex hull which has 4096 ($= 2^{12}$) extreme points or vertices. The convex hull of this LPV model in Eq. (10) has too many vertices to perform the GS controller design. To overcome this problem, we shall propose a method to reduce the number of vertices. We embed the part which depends on the scheduling parameters of the coefficient matrix $B(\rho)$ into a new input $u'$ as follows:

$$
\begin{bmatrix}
B := -J^{-1} l_{we} \\
u' := Q(\rho) u
\end{bmatrix}
$$

(12)

together with

$$
Q(\rho) = [C D].
$$

(13)

After this operation, the state-space representation for complete attitude control is rewritten as follows:

$$
\begin{bmatrix}
\dot{\omega} \\
\dot{q}_e
\end{bmatrix} = A(\rho) \begin{bmatrix}
\omega \\
q_e
\end{bmatrix} + B \begin{bmatrix}
\omega \\
q_e
\end{bmatrix} u'
$$

(14)

$$
\Rightarrow \dot{x} = A(\rho)x + B \dot{x} u'.
$$

(15)

After these operations, the number of vertices has been reduced into 8 $(= 2^3)$. Setting $p = 8$ as the number of the vertices, the LPV system and the GS controller can be expressed by the following polytopic representation:
\[ A_e(p) = \sum_{i=1}^{p} \lambda_i(p) A_{ai}, \quad \text{(16)} \]
\[ K(p) = \sum_{i=1}^{p} \lambda_i(p) K_i, \quad u' = K(p)x, \quad \text{(17)} \]
\[ \lambda_i(p) \geq 0, \quad \sum_{i=1}^{p} \lambda_i(p) = 1. \quad \text{(18)} \]

Figure 2 shows the diagram of polytopic system in Eqs. (16) and (17) (in case of \( p = 8 \)).

Now, we consider a GS controller \( K(p) \) that guarantees overall stability and achieves \( H_2 \) performance for the LPV model as in Eq. (18). First, we introduce the generalized plant for problem [7]:

\[ \inf_{w, x, \rho} \left[ \text{Trace } (Z) \right] \quad \text{subject to} \]
\[ \left[ \begin{array}{c|c} (A_{ai} X - B_i W_i) + (\cdot)^T & \ast \\ \hline C X - D W_i & -I \end{array} \right] < 0, \quad \text{(20)} \]
\[ \left[ \begin{array}{cc} X & \ast \\ E^T & Z \end{array} \right] > 0 \quad \text{for all } 1 \leq i \leq p. \quad \text{(21)} \]

Using the optimal solution sets \( X, W_i \) to the problem in Eqs. (20) and (21), we have the extreme controllers at each vertex in Fig. 2:

\[ K_i = W_i X_i^{-1}, \quad 1 \leq i \leq p. \quad \text{(22)} \]

Then, the GS controller to the simple LPV model in Eq. (22) is constructed by substituting Eq. (22) into Eq. (17).

Note that the common Lyapunov solution \( X > 0 \) was used in the past GS controller design and resulted in conservatism. As an alternative, we use another method [7], in which the distinct Lyapunov solutions \( X_i > 0 \) are adopted. Then, we have

\[ \inf_{w, x, z} \left[ \text{Trace } (Z) \right] \quad \text{subject to} \]
\[ \left[ \begin{array}{c|c} (A_{ai} X - B_i W_i) + (\cdot)^T & \ast \\ \hline C X - D W_i & -I \end{array} \right] < 0, \quad \text{(23)} \]
\[ \left[ \begin{array}{cc} X_i & \ast \\ E_i^T & Z_i \end{array} \right] > 0 \quad \text{for each } 1 \leq i \leq p. \quad \text{(24)} \]

Using the optimal solution sets \( X_i, W_i \), less conservative extreme controllers can be obtained. Using these extreme controllers \( K_i = W_i X_i^{-1}, 1 \leq i \leq p \), a GS controller is again constructed. In order to guarantee overall stability and control performance in a whole operation range, we seek a common Lyapunov solution \( X_c > 0 \) that satisfies the following problem:

\[ \inf_{w, x, z} \left[ \text{Trace } (Z) \right] \quad \text{subject to} \]
\[ \left[ \begin{array}{c|c} (A_{ai} X_c - B_i K_i) X_c + (\cdot)^T & \ast \\ \hline (\hat{C} - D K_i) X_c & -I \end{array} \right] < 0, \quad \text{(25)} \]
\[ \left[ \begin{array}{cc} X_c & \ast \\ E^T & Z \end{array} \right] > 0 \quad \text{for all } 1 \leq i \leq p. \quad \text{(26)} \]

for a set of previously designed extreme controllers \( K_i, 1 \leq i \leq p \). Therefore, we can get the GS controller \( K(p) \) constructed by polytopic system in Eq. (26).

When the LMIs in Eqs. (25) and (26) are infeasible, we can check just the overall stability of the closed-loop system in a whole operation range, while replacing them by the following inequalities:

\[ \exists X_c > 0, \quad (A_{ai} - B_i K_i) X_c + X_c (\cdot)^T < 0 \quad \text{(27)} \]

for all \( 1 \leq i \leq p \).

If LMIs in Eqs. (25) and (26) are feasible, LMIs in Eq. (27) are always feasible. The \( H_2 \) cost is not guaranteed only when LMIs in Eq. (27) are feasible. In this case, to evaluate \( H_2 \) cost, we need to divide the convex hull as shown in Fig. 2 until feasibly solving LMIs in Eqs. (25) and (26).

4. Steering Law Design

The pyramid array VSCMG system has singularities. In this section, we define three types of singular measures and propose two types of GS steering laws for singularity avoidance.

4.1 Singularity of VSCMGs

The steering law is related to the equation to describe the relationship between the actuator input \( u \) and the control input \( u' \). It is defined as a map from \( u' \) to \( u \). From Eq. (12), the steering law of a spacecraft with the pyramid array VSCMG system is derived from

\[ u' = Qu, \quad \text{(28)} \]
\[ Q = [C \ D], \quad \text{(29)} \]

where \( u' \) is the control input to be determined by the GS controller in Eq. (17), and \( u \) is the actuator input given by \( u = [\Omega^T \ \delta^T]^T \) as in Eq. (10). The steering law in Eq. (28) has a singularity problem. If rank \( (Q) = 3 \), we can always solve Eq. (28) with respect to \( u \) for any control input \( u' \). In rank \( (Q) \neq 3 \), we cannot solve it. This is called singularity. The singular measures of the VSCMG, RW and VSCMG system are given by
respectively. When \( m \) is equal to 0, the system falls into the singularity. In Ref. [1], only \( m_{CMG} \) is considered for singularity avoidance. In this paper, we also consider \( m_{VSCMG} \) and \( m_{RW} \) in addition to \( m_{CMG} \). This is the key point of this paper.

4.2 GS Steering Law

In this paper, by switching the operating mode from a CMG mode to a VSCMG mode, singularity avoidance is attained. We introduce a weighting matrix [1] as follows:

\[
W = \begin{bmatrix} aI & 0 \\ 0 & I \end{bmatrix},
\]

where \( a \) is the positive scalar and the singular measure value is given by \( q_1 = m_{CMG} \). Then, the GS steering law [1] is given by

\[
u = WQ^T(QWQ^T)^{-1}u'.
\] (34)

In Ref. [1], the VSCMGs operate as CMGs to take full advantage of the torque amplification effect under normal condition (\( a \) is small) but as the singularity is approached, \( a \) becomes large and the VSCMGs smoothly switch to a RW mode [1]. However, a singularity condition of a RW mode is not considered. To overcome this problem, we introduced the singularity measure as follows:

\[
q_1 = \frac{m_{CMG}}{m_{RW}} = \frac{\sqrt{\det(DD^T)}}{\sqrt{\det(CC^T)}}.
\] (35)

This measure considers singularity conditions of both a CMG mode and a RW mode.

4.3 SVD-Based GS Steering Law

The singularity avoidance in Subsection 4.2. is considered a singularity condition of both a CMG mode and a RW mode. This method is not efficient when both \( m_{CMG} \) and \( m_{RW} \) are small. To avoid this situation, in this Subsection, we introduce a singular value decomposition (SVD) method which hires orthonormal matrices \( U \in \mathbb{R}^{3 \times 3} \) and \( V \in \mathbb{R}^{8 \times 8} \) to assist a steering law with a weighting matrix.

In Eq. (35), the matrix \( Q \) can be decomposed into

\[
Q = U\Sigma V^T,
\] (36)

where

\[
\Sigma = \begin{bmatrix} \sigma_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_3 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},
\]

\[
U = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix},
\]

\[
V = \begin{bmatrix} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 \end{bmatrix},
\]

which yields

\[
Q = \sum_{i=1}^{3} \sigma_i u_i v_i^T.
\] (37)

where \( \sigma_1 > \sigma_2 > \sigma_3 \geq 0 \), \( 1 \leq i \leq 3 \) are the singular values of \( Q \). If \( \det(QQ^T) \neq 0 \), then \( \sigma_i \geq 0 \), \( 1 \leq i \leq 3 \) and

\[
Q^{-1} = V\Sigma^T U^T, \quad \Sigma^T = \Sigma'(\Sigma\Sigma'^T)^{-1},
\] (38)

where

\[
Q^{-1} = \sum_{i=1}^{3} \left( \frac{1}{\sigma_i} \right) v_i u_i^T.
\] (39)

Therefore, an SVD-based GS steering law is given by

\[
u = WQ^T(QWQ^T)^{-1}u' + \beta v_1,
\] (40)

with

\[
\beta = k_1 \exp(-k_2 q_2),
\] (41)

where \( k_1 \) and \( k_2 \) are chosen as positive scalars and \( q_2 = m_{VSCMG} = \sqrt{\det(QQ^T)} \). The second term in the R.H.S. of Eq. (40) is relative to singularity escape. When \( q_2 \) is big enough, Eq. (40) approaches Eq. (34). The vector \( v_1 \) is employed to output the maximum torque in the direction orthogonal to the singularity surface and to escape rapidly from the singular point [11]. This method is efficient when both \( m_{CMG} \) and \( m_{RW} \) are small.

5. Numerical Simulation

This section presents some numerical simulations of the conventional steering law in Eq. (34) with \( q_1 = m_{CMG} \) and proposed ones in Eq. (34) with \( q_1 = m_{CMG}/m_{RW} \) and Eq. (40). The simulation parameters, the initial condition, and the desired laws. From this figure, each steering law can avoid their singularity measure in these three types of steering laws. From this figure, each steering law can avoid their singularity (these singularity measures can avoid 0 status successfully). Finally, Figs. 22 and 23 show total input to wheels and gimbals, respectively. From these figures, the efficiency of the proposed method is demonstrated, compared with the conventional method (total input of the proposed method is smaller than the conventional one).
Total wheel input $T_{RW}$ and total gimbal input $T_{CMG}$ are given by

$$T_{RW} = \int ||\vec{\Omega}|| \, dt, \quad T_{CMG} = \int ||\vec{\delta}|| \, dt,$$

(42)

respectively.

6. Conclusion

In this paper, a GS controller has been designed to attain overall stability and control performance of attitude control at the same time via LPV control theory. Then, we have proposed two types of the GS steering laws for singularity avoidance and escape. Through some numerical simulations, the performance of the proposed methods has been improved compared with the conventional method.

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