A Randomized Algorithm for Chance Constrained Optimal Power Flow with Renewables

Takayuki WADA *, Ryosuke MORITA **, Toru ASA I ****, Izumi MASUBUCHI ***, and Yasumasa FUJISAKI *

Abstract: A chance constrained AC optimal power flow is to find the optimal economic operation plan whose probability satisfying AC power flow equations and various inequality constraints on operating limits of the power system is greater than a specified probability level. Even if a constraint condition for each uncertain power supply value is convex with respect to decision variables, the chance constrained problem is not convex in general. Thus, it is difficult to solve the problem within reasonable computational time. Employment of randomization techniques for this issue is proposed in this paper. A main advantage of the framework leads to a solution with a theoretical guarantee. Its sample complexities are of polynomial order for parameters of a given accuracy. The efficiency of this algorithm is demonstrated by applying it to the Japanese power system models.

Key Words: Randomized algorithms, optimal power flow, chance constrained programming, renewable power sources.

1. Introduction

The alternate current optimal power flow (AC-OPF) [1],[2] is to find the optimal economic operation plan satisfying power flow equations and inequalities which describe power system limitations. For example, Independent System Operators (ISO) have to solve the problem every day, every hour, or every 5 minutes, for keeping reliability and economic efficiency of power systems [3]. Notice that the problem does not have convexity, although there are several algorithms for solving the original standard optimal power flow (OPF) problem [4].

On the other hand, high penetration of renewable production, such as wind and solar power, leads to uncertainty and indeterminacy in power systems. This is because, although amount of power generation of renewables heavily depends on weather, it is difficult to exactly predict nature. However, depletion of fossil fuel and degradation of global environment require us to embed renewable energy into our power systems. Thus, how to make risk-averse operation plans for uncertain power systems is a fundamental research topic for system operators and planners.

Employment of heuristic margin [5] is an easy way to realize reliable power systems. Since its margin could be conservative, operation plans of this approach could not be cost efficient either. To solve its conservativeness, robust optimization [6] and chance constrained optimization [7] are introduced for the optimal power flow with renewables.

The robust optimal power flow is to find the optimal economic operation plan subject to various constraints on the active and reactive power of each generator, the bus voltage, the capacity of the transmission lines, and other factors, for any amount of renewable power generation. In [8], an adjustable robust direct current optimal power flow (DC-OPF) whose operation plan is affinely adjustable with respect to renewable power generation, is formulated. A similar strategy is also employed for security-constrained OPF [9]. Multi-period version for the problem with storage and renewables is presented in [10]. However, a robust version of AC-OPF is not mentioned due to its heavily computational requirement.

The chance constrained optimal power flow is to find the optimal economic operation plan whose probability satisfying the parameter dependent constraints of a robust optimal power flow with respect to renewable generations is greater than a specified value. That is, we suppose that renewable power generations are random variables. This is a relaxation of the robust optimal power flow in a probabilistic sense. However, this relaxation does not work well from a viewpoint of computational complexity of the problem. This is because, in general, it is well known that a chance constrained version of a convex problem does not keep convexity [7]. To avoid this difficulty, several assumptions are needed. For example, in papers [11],[12], they suppose that parameters are according to normal distributions and they deal with a chance constrained DC-OPF. In fact, since a chance constrained DC-OPF with the Gaussian setting has convexity, we can efficiently solve this problem. On the other hand, a chance constrained AC-OPF is presented in [13]. In that paper, they also suppose that uncertain parameters are normal random variables. However, even if we introduce the above assumption, we have to solve nonconvex optimization for a chance constrained AC-OPF due to nonlinearity of the standard AC power flow equations. Then, they employ a backmapping approach and linear approximation of the constraints. Since its linearization is computed at the mean value of uncertain parameters, they implicitly suppose that uncertain parameters have small variance. Furthermore, they suppose that amount of renewable generation is according to a normal distribution.
However, notice that wind power generation is determined by a nonlinear function of wind speed [14]. This implies that, even if forecast error of wind speed is a normal random variable, prediction error of wind power generation is not Gaussian. Furthermore, forecast error of wind speed is not a normal random variable in practice [15]. That is, we need algorithms for solving a chance constrained AC-OPF under non-Gaussian settings.

In this paper, we apply randomization techniques [16] to a chance constrained AC-OPF. We would like to emphasize that a suboptimal solution which is an output of our algorithm satisfies chance constraints with prespecified probability. That is, our algorithm has theoretical guarantee. Notice that there is no theoretical guarantee on algorithms in [13]. The randomization technique is also applied for robust convex optimization [17],[18] and there is an application to a robust DC-OPF [19]. However, a constraint of AC-OPF for each renewable power generation is nonconvex. This is because the alternate current power flow is nonlinear. In fact, since uncertain AC power flow is a difficult but important problem, there are several researches [20],[21] for solving this problem. In this paper, we solve this issue by using a pure randomized algorithm, that is, our algorithm is based on random samples from both set of decision variables and uncertain parameters. In fact, introducing probability distributions for both sets, we firstly generate a random sample from the set of decision variables. Then, we generate a lot of random samples from the set of uncertain parameters, and we solve AC power flow equations for a random sampled decision variable and each randomly sampled uncertain parameter. If all of the constraints are satisfied for any random sampled parameters, we set the random sampled decision variable as a feasible solution of the problem. Otherwise, we iterate the same procedure, that is, we generate new random samples from the set of decision variable and we drop many random samples from the set of uncertain parameters. Then, we solve the AC power flow equations and we check the constraints. We show that it leads to a suboptimal solution satisfying the chance constraint with high probability. In fact, we clarify a necessary number of iterations for getting a feasible solution with specified probability. Furthermore, it is shown that the sample complexity of our algorithm is of polynomial order for parameters of specified accuracy. In addition to this, we guarantee the suboptimality of the obtained solution. Note that our algorithm allows us to select any probability distributions for both sets, we firstly generate a random samples from the set of uncertain parameters. Then, our algorithm is based on random samples from both set of parameters, we set the random sampled decision variable at each bus.

This paper is organized as follows. In Section 3, we formulate a chance constrained AC-OPF. In Section 4, we propose a randomized algorithm for this problem. Next, in Section 5, we apply our algorithms to the Japanese power system model. We present our concluding remarks in Section 6.

2. Symbols

\[ B = \{1, 2, \ldots, n\} \] The set of buses
\[ \mathcal{L} \subseteq B \times B \] The set of transmission lines
\[ \mathcal{G} \subseteq B \] The set of controllable generators
\[ n_{\mathcal{G}} \] The number of controllable generators
\[ D \subseteq B \] The set of demands
\[ R \subseteq B \] The set of renewables
\[ n_R \] The number of renewables
\[ P_{gi} \] Active power generation of generator \( i \in \mathcal{G} \)
\[ p_{\max}^{gi} \] Upper limit of active power generation of generator \( i \in \mathcal{G} \)
\[ p_{\min}^{gi} \] Lower limit of active power generation of generator \( i \in \mathcal{G} \)
\[ Q_{gi} \] Reactive power generation of generator \( i \in \mathcal{G} \)
\[ Q_{\max}^{gi} \] Upper limit of reactive power generation of generator \( i \in \mathcal{G} \)
\[ Q_{\min}^{gi} \] Lower limit of reactive power generation of generator \( i \in \mathcal{G} \)
\[ P_{ni} \] Active power generation of renewable plant \( i \in \mathcal{R} \)
\[ Q_{ni} \] Reactive power generation of renewable plant \( i \in \mathcal{R} \)
\[ V_i \in \mathbb{C} \] Complex voltage at bus \( i \in \mathcal{B} \)
\[ Y \in \mathbb{C}^{\mathcal{N} \times \mathcal{N}} \] The admittance matrix
\[ Y_{ij} \] The \((i, j)\) element of admittance matrix \( Y \)
\[ f_i(x) \] The cost function of generator \( i \in \mathcal{G} \)
\[ \mathcal{X} \] The set of decision variables
\[ \mathcal{Z} \] The set of uncertain parameters
\[ P_{\mathcal{X}} \] The uniform measure on \( \mathcal{X} \)
\[ P_{\mathcal{Z}} \] The probability measure on \( \mathcal{Z} \)

3. Chance Constrained AC Optimal Power Flow

Chance constrained AC optimal power flow is described by

\[
\min_{x, \Psi} \; f(x) \quad \text{subject to} \quad \mathcal{P}_{\mathcal{Z}}(\zeta \in \mathcal{Z} : \omega(x, y(\zeta), \zeta) = 0, g(x, y(\zeta), \zeta) \leq 0) \geq 1 - \alpha_c.
\]

The decision variables are \( x \in \mathbb{R}^{2n_y} \) and \( y(\zeta) \in \mathbb{R}^{2n_y} \). The uncertain vector \( \zeta \in \mathbb{R}^{2n_z} \). Note that \( y(\zeta) \in \mathbb{R}^{2n_y} \) depends on \( \zeta \). In fact, the variable \( x \) consists of active power generation \( P_{gi} \in \mathbb{R} \) of controllable generator at bus \( i \in \mathcal{G} \) and voltage magnitude \( |V_i| \) of the complex voltage \( V_i \in \mathbb{C} \) at bus \( i \in \mathcal{G} \). The parameter dependent decision variable \( y(\zeta) \) contains reactive power generation \( Q_{gi} \) at bus \( i \in \mathcal{G} \), \( |V_i| \) at bus \( i \in \mathcal{B} \setminus \mathcal{G} \), and argument \( \arg(\mathcal{V}_i) \) of \( V_i \) at each bus \( i \in \mathcal{B} \). The uncertain vector \( \zeta \) consists of uncertain active and reactive power generation \( P_{ni} \) and \( Q_{ni} \) at each bus \( i \in \mathcal{R} \). We suppose that \( \zeta \) belongs to a given set \( \mathcal{Z} \subseteq \mathbb{R}^{2n_z} \) in which \( \zeta \) is a random variable according to the probability measure \( P_{\mathcal{Z}} \). That is, we assume that \( P_{\mathcal{Z}} \) is known. For example, distribution of the forecast error for wind speed is investigated in [15].

This problem is to minimize the objective function \( f(x) \) subject to the chance constraint of the problem (1) for prespecified level \( \alpha_c \in (0, 1) \). The objective function is summation of cost
$f_i$ of the controllable generator $i \in \mathcal{G}$:
$$f(x) := \sum_{i \in \mathcal{G}} f_i(P_g).$$

We do not assume convexity of $f(x)$. In fact, cost function of controllable generators is nonconvex in general [2].

The equality constraint $\omega(x, y(\zeta), \zeta) = 0$ corresponds to power flow equations. Each equation is defined by
$$P_{gi} - P_{di} - P_{ui} - \text{Real}\left\{ V_i \left( \sum_{j=1}^{n} Y_{ij} V_j \right) \right\} = 0, \quad \forall i \in \mathcal{B},$$
$$Q_{gi} - Q_{di} - Q_{ui} - \text{Imag}\left\{ V_i \left( \sum_{j=1}^{n} Y_{ij} V_j \right) \right\} = 0, \quad \forall i \in \mathcal{B},$$

where $P_{gi}$ and $Q_{gi}$ are the active and reactive power generation of a controllable generator which is connected to bus $i \in \mathcal{B}$. If $i \not\in \mathcal{G}$, $P_{gi} = 0$ and $Q_{gi}$ are also fixed by 0. Then, $P_{di}$ and $Q_{di}$ are also active and reactive power demand at bus $i \in \mathcal{B}$. If $i \not\in \mathcal{D}$, we set $P_{ui} = 0$ and $Q_{ui} = 0$. The symbols $P_{gi}$ and $Q_{gi}$ also mean active and reactive power generation of renewable at bus $i \in \mathcal{B}$. If $i \not\in \mathcal{R}$, $P_{ii} = 0$ and $Q_{ii} = 0$.

On the other hand, $g(x, y(\zeta), \zeta) \leq 0$ consists of the limitation on active and reactive power at each controllable generator:
$$P_{\text{min}} \leq P_g \leq P_{\text{max}},$$
$$Q_{\text{min}} \leq Q_g \leq Q_{\text{max}},$$

the constraints on the voltage:
$$V_{\text{min}} \leq |V| \leq V_{\text{max}},$$

and the line MW flow limits on each transmission line:
$$P_{\text{ij}} \leq \text{Real}(V_i V_j^* Y_{ij} + V_j V_i^* Y_{ij} - V_j V_i Y_{ij}) \leq P_{\text{max}},$$


Now, we present a randomized solution for the chance constrained AC-OPF (1). Our algorithm employs random samples from the set of the decision variable $x$ and the uncertain parameter $\zeta$. Since the uncertain parameter $\zeta$ is a random variable according to the probability measure $P_Z$, we generate random samples according to $P_Z$. On the other hand, to generate the random samples of $x$, we introduce the uniform measure $P_{X}$ on the set
$$X := \{ x \in \mathbb{R}^{2n_D} : P_{\text{min}} \leq P_g \leq P_{\text{max}}, \quad V_{\text{min}} \leq |V| \leq V_{\text{max}}, \quad i \in \mathcal{G} \}.$$

Algorithm 1 is our randomized algorithm for solving the problem (1). In the algorithm, we randomly sample (i) the active power $P_g$ and voltage magnitude $|V|$, $i \in \mathcal{G}$, according to the probability measure $P_X$; then, we generate random samples of (ii) an uncertain power supply from the set $Z$, according to the probability measure $P_Z$; for each $\zeta$, that is, $P_g$ and $Q_{gi}$, we solve the AC power flow equation, then we check that the voltage is within the prescribed limit. If we select sufficiently large $\ell$ and $k$, then we can expect that this algorithm will find a suboptimal solution. The following theorem is the main result of this paper, and it states relationship between a probabilistic guarantee and the number of random samples.

Theorem 1 For given $\alpha_a \in (0, 1)$, $\alpha_b \in (0, 1)$, $\beta_a \in (0, 1)$, and $\beta_b \in (0, 1)$, select $\ell \in \mathbb{N}$ and $k \in \mathbb{N}$ satisfying
$$\ell \geq \ln \frac{1}{\beta_a} - \ln \frac{1}{1 - \alpha_a}, \quad k \geq \ln \frac{\ell}{\beta_b} - \ln \frac{1}{1 - \alpha_b}.$$

When the algorithm stops with $\hat{x} \neq \text{Null}$, the probability that $\hat{x}$ satisfies
$$P_{Z}(\zeta \in \mathbb{Z} : \text{\exists } y(\zeta) \text{ s.t. } \omega(\hat{x}, y(\zeta), \zeta) = 0, \quad g(\hat{x}, y(\zeta), \zeta) \leq 0) \leq 1 - \alpha_a$$
or
$$P_{X}(x \in X : f(x) < f(\hat{x}), \forall \zeta \in \mathbb{Z} \text{ s.t. } \omega(x, y(\zeta), \zeta) = 0, \quad g(x, y(\zeta), \zeta) \leq 0) \geq \alpha_b$$
is at most $\beta_a + \beta_b$.

(Proof) See the appendix.

Algorithm 1 Randomized Algorithm for Chance Constrained Optimal Power Flow

0: Select $\ell \in \mathbb{N}$ and $k \in \mathbb{N}$. Set $\hat{x} := \text{Null}$. 1: for $\ell := 1$ to $\ell$ do 2: Draw $x^{(\ell)}$ according to the probability measure $P_X$. 3: if $\hat{x} \neq \text{Null}$ and $f(x^{(\ell)}) > f(\hat{x})$, then 4: goto Step 16. 5: end if 6: for $k := 1$ to $k$ do 7: Draw $\zeta^{(k)}$ according to the probability measure $P_Z$. 8: Solve the power flow problem:
$$\text{Find } y_1 \in \mathbb{R}^{2n} \text{ s.t. } \omega(x^{(\ell)}, y_1, \zeta^{(k)}) = 0.$$
9: if $g(x^{(\ell)}, y_1, \zeta^{(k)}) \leq 0$, then 10: goto Step 16. 11: end if 12: end for 13: if $f(x^{(\ell)}) < f(\hat{x})$, then 14: Update the provisional optimal solution as $\hat{x} := x^{(\ell)}$. 15: end if 16: No operation. 17: end for 18: Stop the algorithm with $\hat{x}$ as output.

The theorem says that, when we obtain a suboptimal solution as the output of the algorithm, the event (3) occurs with small probability $\beta_a + \beta_b$. This implies that obtained suboptimal value is close to the optimal value in a probabilistic sense. In addition to this, we consider an event: $\hat{x}$ is not feasible in a probabilistic sense, that is, (2) holds. The theorem states that this event also occurs with small probability $\beta_a + \beta_b$. Note that the sample complexity is independent of the problem size (the dimension of the decision variable). Furthermore, $k$ and $\ell$ are independent of the problem size (the dimension of the decision variable) and polynomials of $1/\alpha_a$, $1/\alpha_b$, $1/\beta_a$, and $1/\beta_b$. That is, if we can solve the power flow analysis within a reasonable computation time, we can easily solve the AC optimal power flow problem.

5. A Numerical Example

In this section, we present a numerical example by using the Japanese power system model (IEEJ EAST 30-machine system model) [23], which is shown in Fig. 1. We utilized the Power
We then applied the uniform probability measure to the follow-
ing set:

\[ \mathcal{Z} := \{ z : (1-r)P_{\text{max}}^{g_{i}} \leq P_{g_{i}} \leq r P_{\text{max}}^{g_{i}}, \quad -r \leq Q_{g_{i}} \leq r \} \]

System Analysis Toolbox (PSAT) [24] for solving the power flow equations at Step 1 of our algorithm.

This is a power system model of the entire eastern region of Japan, which contains Tokyo, Tohoku, and Hokkaido. This model consists of 107 buses, 191 transmission lines, and 30 generators. These generators include 16 thermal electric power plants, 8 hydraulic power plants, 5 nuclear power plants, and 1 renewable plant. The renewable plant is described by the symbol “W” in Fig. 1. All of the generators except for a renewable plant are controllable. It is assumed that for only the thermal electric power there is a cost for fuel when generating electric power. The voltage rating of the thick lines is 500 kV, and that of the thin lines is 275 kV.

The cost function of the controllable generator \( i \in \mathcal{G} \) is quadratic which is given by

\[ f(P_{g_{i}}) := a_{i}P_{g_{i}}^{2} + b_{i}P_{g_{i}} + c_{i}, \]

where \( a_{i} \), \( b_{i} \), and \( c_{i} \) are positive. The coefficients of the cost functions of the thermal plants are given in Table 1. The coefficients of hydraulic and nuclear power plants are \( a = b = c = 0 \). These settings are taken from [23]. Bus 27 was selected as the slack bus. The system power rating and system frequency rating are 1.000 MVA and 50 Hz, respectively.

We set constraints on the reactive power as \(-3.0 \leq Q_{g_{i}} \leq 3.0\) pu for each generator and on the voltage as \(0.8 \leq |V_{i}| \leq 1.2\) pu for each bus. We introduced the uniform probability measure \( P_{X} \) on the set \( X \) as follows:

\[ X := \{ x : 0.585P_{\text{max}}^{g_{i}} \leq P_{g_{i}} \leq 0.65P_{\text{max}}^{g_{i}}, \quad 0.8 \leq |V_{i}| \leq 1.2, \quad i = 1, 2, \ldots, n_{\mathcal{G}} \}. \]

We then applied the uniform probability measure to the following set:

\[ \mathcal{Z} := \{ z : (1-r)P_{\text{max}}^{g_{i}} \leq P_{g_{i}} \leq r P_{\text{max}}^{g_{i}}, \quad -r \leq Q_{g_{i}} \leq r \}. \]
where we set \( P_{\text{max}} = 3.25 \text{ pu} \). We suppose that, from weather forecast, we see that renewable plants could generate its maximum power. However, due to inaccuracy of weather forecast, we solve the chance constrained AC-OPF.

When we gave \( \alpha_a = \beta_a = \alpha_b = \beta_b = 0.01 \), then we selected \( \ell = 1,069 \) and \( \ell = 459 \) based on Theorem 1. By using these parameters, we executed the algorithm for \( r = 0.01 \), and 0.5. Tables 2, 3, 4 lists the generation costs, active power generation, and voltage magnitude. Now, we suppose that the renewable generator is combined gas turbine and its cost coefficients are given by \( a = 0.00166, b = 1.4, \) and \( c = 120 \) [23]. Then, we solved the standard AC optimal power flow via PSAT and we obtained the optimal cost 82,248.7. From Table 2, all of the obtained suboptimal costs are cheaper than this value. Of course, if \( r \) becomes large, optimal cost also becomes large. However, this example implies that its cost for dealing with uncertainty is cheaper than that for operating thermal generator.

On the other hand, we checked the probability that \( g(x, y, \zeta) \leq 0 \) holds by a posteriori analysis when we set \( r = 0.1 \). We employed Chernoff bound [16]. We specified levels \( \alpha = 0.005 \) and \( \beta = 0.01 \). According to these parameters, we selected the number \( N = 105,967 \) of random samples, then we generated \( N \) random samples from \( \mathcal{Z} \). Since the empirical probability was \( p_N = 105,967/105,967 = 1.0 \), we can see that this suboptimal solution satisfies \( P_{\mathcal{Z}}(\omega(\alpha(x, y, \zeta), \zeta) = 0, g(x, y, \zeta) \leq 0) \geq 1.0 - 0.005 = 0.995 > 0.99 \) with probability 0.99.

Furthermore, we investigated fluctuation of \( |V_r|, i \in B\{G \) and \( |Q_p|, i \in G \) with respect to \( \zeta \), when we set \( r = 0.1 \). We generated 10,000 random samples and we solved power flow equations. We lists maximum and minimum of \( |V_r|, i \in B\{G \) and \( |Q_p|, i \in G \) in Tables 5, 6, 7. As a result, we see that fluctuation of these values is small.

When we replaced with generator 4 with a renewable plant. Under the same parameters, \( \alpha_a = \beta_a = \alpha_b = \beta_b = 0.01 \), we executed the algorithm. In Table 8, the generation costs is shown. The generation cost is lower than previous case. However, when we selected \( r = 0.5 \), we could not find any feasible solution. This implies that increasing the number of renewable plants could lead to reduction of generation cost. However, it becomes difficult to maintain reliability of the power system.

### 6. Conclusion

We have proposed a randomized solution for the chance constrained AC optimal power flow problem. Although the problem is nonconvex, the necessary number of random samples is polynomial for a given accuracy and given risk parameters. The efficiency of our algorithm was demonstrated with a numerical example.

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Table 6 Fluctuation of $|V|$ at bus $i \in B\setminus G$ (Part II).

| $|V_{oi}|$ | $|V_{oi}|$ | $|V_{oi}|$ | $|V_{oi}|$ | $|V_{oi}|$ | $|V_{oi}|$ |
|----------|----------|----------|----------|----------|----------|
| 1.0205   | 1.0336   | 1.0941   | 1.0174   | 1.0710   |
| 1.0313   | 1.0404   | 1.0943   | 1.0175   | 1.0711   |
| 1.0074   | 1.0167   | 1.0944   | 1.0176   | 1.0712   |
| 0.9998   | 1.0195   | 1.0945   | 1.0177   |

Table 7 Fluctuation of $Q_{iz}$.

<table>
<thead>
<tr>
<th>$Q_{iz}$</th>
<th>$Q_{iz}(C^{(1)})$</th>
<th>$Q_{iz}(C^{(2)})$</th>
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<td>$q_{11}$</td>
<td>-0.6165</td>
<td>-0.6152</td>
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<tr>
<td>$q_{12}$</td>
<td>-0.3768</td>
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<td>$q_{13}$</td>
<td>1.5991</td>
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<td>$q_{15}$</td>
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<td>$q_{17}$</td>
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<td>$q_{29}$</td>
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Table 8 Generation costs for two renewables case.

<table>
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<tr>
<th>Variation range (%)</th>
<th>$r = 0.0$</th>
<th>$r = 0.1$</th>
<th>$r = 0.5$</th>
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</thead>
<tbody>
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<td>Cost (JPY/h)</td>
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<td>68,282.0</td>
<td>no solution</td>
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</table>

References


Appendix Proof of Theorem 1

Consider the following two events.

$T_{\ell}$: The number of updates reaches $\ell$, and $x(t)$ is not updated $\bar{k}$ consecutive times.

$B_{\ell}$: The candidate $x(t)$ satisfies $x(t)$.
\[
P_Z(\zeta \in \mathbb{Z} : \exists y \in \mathbb{Y} \ s.t. \ \omega(x^{(i)}, y, \zeta) = 0, \ g(x^{(i)}, y, \zeta) \leq 0) < 1 - \alpha_a.
\]

From the definition of \( \tilde{\ell} \) and \( \bar{k} \),
\[
(\mathbb{P}_X^{P_{Z|\{y\}}^\tilde{\ell}} \bigcap_{i=1}^\tilde{\ell} (\mathcal{F}_i \cap \mathcal{B}_i)) \leq (\mathbb{P}_X^{P_{Z|\{y\}}^\bar{k}} \bigcap_{i=1}^\bar{k} (\mathcal{F}_i | \mathcal{B}_i)) < \tilde{\ell}(1 - \alpha_a)^{\bar{k}} \leq \beta_a.
\]

That is, when the algorithm stops with some \( \hat{x} \) as output, the probability that \( \hat{x} \) satisfies
\[
P_Z(\zeta \in \mathbb{Z} : \exists y \in \mathbb{Y} \ s.t. \ g(x^{(i)}, y, \zeta) \leq 0) < 1 - \alpha_a \quad (A.1)
\]
is less than or equal to \( \beta_a \).

Next, we introduce a feasible set of the robust optimal power flow such as \( \mathcal{S} = \{ x \in \mathbb{X} : \forall \zeta, \exists y \in \mathbb{Y} \ s.t. \ \omega(x, y, \zeta) = 0, \ g(x, y, \zeta) \leq 0 \} \) and we consider the following chance-constrained problem:
\[
y_{\alpha_0} := \sup_{x \in \mathbb{X}} f(\hat{x}),
\]
\[
\hat{X} := \{ \hat{x} \in \mathbb{X} : \mathbb{P}_X(x \in \mathbb{X} : f(x) < f(\hat{x}), \ x \in \mathcal{S}) \geq \alpha_b \}.
\]

Now, we assume that the probability that a randomly sampled \( x \) does not belong to the set \( \mathcal{S} \) or that its objective function value is greater than \( y_{\alpha_0} \) is less than \( 1 - \alpha_b \), that is,
\[
P_X(x \in \mathbb{X} : f(x) > y_{\alpha_0}, \ x \notin \mathcal{S}) < 1 - \alpha_b.
\]

In the algorithm, since we generate \( \tilde{\ell} \) random samples \( x^{(1)}, x^{(2)}, \ldots, x^{(\tilde{\ell})} \), the probability that \( f(x^{(i)}) > y_{\alpha_0} \) or \( x^{(i)} \notin \mathcal{S} \) for any \( i = 1, 2, \ldots, \tilde{\ell} \) is less than \( (1 - \alpha_b)^{\tilde{\ell}} \), that is,
\[
\mathbb{P}_X \bigcap_{i=1}^\tilde{\ell} [f(x^{(i)}) > y_{\alpha_0} \text{ or } x^{(i)} \notin \mathcal{S}] < (1 - \alpha_b)^{\tilde{\ell}} \leq \beta_b.
\]

We therefore conclude that
\[
P_X( x : f(x) < f(\hat{x}), \ x \in \mathcal{S}) \geq \alpha_b \quad (A.2)
\]
with at most probability \( \beta_b \).

Two inequalities (A.1) and (A.2) lead to the statement of the theorem.

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