Attitude Control of Two-Wheel Spacecraft Based on Dynamics Model via Hierarchical Linearization

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Abstract: In this paper, we propose an attitude control law for underactuated two-wheel spacecraft under non-zero total angular momentum. Attitude control with non-zero total angular momentum is complicated in the case that the number of reaction wheels equipped on a spacecraft is two. For a spacecraft in this situation, an attitude control law has been proposed based on a kinematics model by Katsuyama et al. [Y. Katsuyama, SICE Annual Conference, pp. 3421-3426, 2013]. However, a dynamics controller is more desirable for a practical system. Thus, in this paper, we expand the controller to a dynamics model. Nevertheless, in the case of dynamics model, the expansion is not straightforward because of the singularity of input transformation. Therefore, we propose to apply the hierarchical linearization technique which separates a system into several subsystems and linearizes the subsystems step by step. Using this method, the input transformation becomes well-defined, and the system is linearized partially. Additionally, the dimension of linearizable state increases compared with the ordinary input-output linearization. Numerical simulation is conducted to show the validity of the proposed controller.

Key Words: attitude control, underactuated spacecraft, non-holonomic system, input-output linearization.

1. Introduction

An attitude control pointing an antenna or a solar paddle at a target direction is the one of important tasks of spacecraft to achieve the mission. To perform the attitude control, the multi actuators are loaded on the spacecraft such as a thruster, a reaction wheel, and a control momentum gyroscope. In particular, the reaction wheel is a widely used actuator because of its simple structure. In this paper, we treat the attitude control of a spacecraft with only the reaction wheels. For attitude control with reaction wheels, a spacecraft employs multiple reaction wheels (four or more wheels for fail-safe) because the degree of freedom of rotation in space is three. However, in the unforgiving space environment, there is a case that reaction wheels are broken by rapid temperature change and stored charge such as the asteroid explorer Hayabusa and the space telescope Kepler. To repair the reaction wheel is too difficult in the space. If usable wheels are two, the attitude control will be difficult since the spacecraft becomes underactuated that has fewer inputs compared to the degree of freedom. If the accuracy of attitude control is deteriorated, a failed spacecraft will be abandoned and a launch of new spacecraft is required to continue the mission. Then, the researches treating the attitude control using only two reaction wheels have been studied [1]–[4]. The spacecraft can be controlled with only two reaction wheels by using these method: time-varying feedback [1], discontinuous feedback [2],[3], switching feedback [4]. However, these methods assume the driftless system. This means that the spacecraft can keep stationary attitude without inputs. In the practical space environment, there exist minute disturbance torques such as the pressures of solar radiation and gravity gradient torque. By the accumulation of the disturbance torque, rotation of the spacecraft occurs. In this paper, the angular momentum which the spacecraft has with zero inputs is called a total angular momentum. A total angular momentum is assumed as a constant because the accumulation of disturbance torques during the attitude control task is small enough.

Since this non-zero total angular momentum is described as a nonlinear drift term, an arbitrary attitude of the spacecraft can not be stabilized with this momentum because the angular momentum has to be cancelled by two reaction wheels. According to references [5] and [6], the spacecraft can be stabilized only at the attitude where the total angular momentum enters in the plane spaned by inputs. In this paper, we assume that a desired attitude satisfies this condition.

Katsuyama et al. have proposed an attitude control law discussing the total angular momentum with only two reaction wheels [7]. This method transforms the kinematics model of the system into a time state control form (TSCF) [8], and then the subsystems are linearized without approximation. A linear controller is designed to the linearized subsystem. Since the kinematics model is simple, the conventional method [7] is designed based on it. However, to apply it to the practical system, it is more desirable to design the controller based on a dynamics model to consider the dynamics of actuators. Thus, in this paper, we try to expand the conventional controller [7] to the dynamics model.

As mentioned above, the conventional method [7] is realized by designing TSCF [8]. Thus, a simple way to design a controller for the dynamics model is to extend the controller for the kinematics model by extracting the essence of TSCF. However, the linearization via the TSCF for the dynamics model is impossible because the regularity of the input transformation for
linearization cannot be satisfied. Therefore, we propose to apply a hierarchical linearization technique [9]. The hierarchical linearization is a method which separates the original system into two subsystems and linearizes each subsystem step by step by regarding the first subsystem as autonomous system. Due to this technique, the input transformation becomes well-defined and the subsystems are linearized. Additionally, the dimension of linearizable states increases compared to the ordinary input-output linearization.

The remainder of the paper contains 5 sections. In Section 2, the dynamics model is derived. The problem to extend the conventional method to the dynamics model is discussed in Section 3. To cope with the problem, the proposed method is introduced in Section 4. In Section 5, the validity of the proposed controller is verified via a numerical simulation. The paper is closed with the concluding remarks.

2. Model of a Spacecraft

In this paper, we adopt following assumptions.

- The spacecraft is a rigid body.
- The total angular momentum is not zero.
- The angular momentum is conserved and known.
- The parameters and states (attitude) of the spacecraft are measured without noise.
- There exists input that keeps the target attitude.

First four simplifications are required to design and apply a controller for the dynamics model. We note that the additional modification might be required to ensure the robustness for neglected measurement noise or deformation of the body when we apply the proposed controller to a practical system. However, the extension to add the robustness is out of the scope of this paper.

Additionally, we suppose the following settings to make clear discussion.

- The three reaction wheels are set along the principal axes of the body, and the one on the Y-axis fails.
- The antenna is attached at Z-axis.

For the other configurations of the antenna and the broken wheel, the following discussion works as well as [5].

system $\Sigma_B = (X_B, Y_B, Z_B)$ along the principal axes of the spacecraft. Z-axis of the inertial coordinates is set such that $-Z$ coincides with a target direction of the antenna, and the antenna is attached along $Z_B$ axis as mentioned above. Thus, the objective of this paper is to stabilize the attitude such that $Z_B$ axis points at $-Z$. In addition, the total angular momentum $H_0$ is fixed in the inertial coordinate system $\Sigma_0$ because of the conservation law assumed above. Next, we introduce the dynamics model of the spacecraft.

The attitude of the spacecraft is represented by Rodrigues parameters [10] defined as follows.

$$p = k \tan \frac{\theta}{2} = [p_1, p_2, p_3]^T,$$  \hspace{1cm} (1)

$$k = [x, y, z]^T, \quad ||k|| = 1,$$  \hspace{1cm} (2)

where $|| \cdot ||$ means the Euclidean norm. Rodrigues parameters can represent the attitude rotated $\theta$ around the rotation axis $k$ and has the singular point of $\theta = \pi$ rad. Now, the rotation matrix consisting of Rodrigues parameters is

$$R(p) = (I_3 + \hat{p})(I_3 - \hat{p})^{-1},$$  \hspace{1cm} (3)

where $I_3$ is the 3x3 identity matrix, $\hat{p}$ is the skew-symmetric matrix of $p$ defined as

$$\hat{p} = \begin{bmatrix} 0 & -p_3 & p_2 \\ p_3 & 0 & -p_1 \\ -p_2 & p_1 & 0 \end{bmatrix}.$$  \hspace{1cm} (4)

Moreover, there is the relation of Rodrigues parameters $p$ and the body angular velocity $\omega_B$ in the following equations:

$$\tilde{p} = P(p)\omega_B,$$  \hspace{1cm} (5)

$$\tilde{p} = P(p)\omega_B + P(p)\dot{\omega}_b,$$  \hspace{1cm} (6)

where $P(p)$ is the Jacobian matrix defined as follows:

$$P(p) = \frac{1}{2} (I_3 + \hat{p} + pp^T).$$  \hspace{1cm} (7)

Let the total angular momentum seen from $\Sigma_0$ be $H_0 = [H_{01}, H_{02}, H_{03}]^T$, and that seen from $\Sigma_B$ be $H_B$. The following relation holds

$$H_0 = RH_B.$$  \hspace{1cm} (8)

The time derivative of $H_0$ is

$$\frac{d}{dt}H_0 = R\dot{\omega}_B H_B + RH_B = O_{3x1},$$  \hspace{1cm} (9)

where $O_{3x1}$ is the zero vector of 3 x 1. This equation is based on the assumption that the angular momentum is conserved.

On the other hand, $H_B$ can be represented as

$$H_B = J_B \omega_B + J_{u1}(\omega_B + v_1 \omega_1) + J_{u2} \omega_B + J_{u3}(\omega_B + v_3 \omega_3)$$

$$= J_B \omega_B + J_{u1}v_1 \omega_1 + J_{u3}v_3 \omega_3,$$  \hspace{1cm} (10)

where $J_B$ is the inertia matrix of the body, $J_{u_i}$, $\omega_B$ and $v_i$ are the inertia matrix, the angular velocity and the direction of the $i$-th reaction wheel, respectively. Moreover, the subscript $i = 1, \ldots, 3$ represents the $X_B$, $Y_B$, $Z_B$ axis respectively, and $J_i$ is the sum of the inertia matrix defined as follows

$$J_i = J_B + J_{u1} + J_{u2} + J_{u3} = \text{diag}(j_{1i}, j_{2i}, j_{3i}).$$  \hspace{1cm} (11)

Then, the time derivative of (10) is calculated by

\[ \text{An image of the spacecraft is shown in Fig. 1. We define the two coordinate systems: the inertial coordinate system } \Sigma_0 = (X, Y, Z) \text{ fixed in inertial space and the body coordinate system } \Sigma_B = (X_B, Y_B, Z_B) \text{ along the principal axes of the spacecraft. Z-axis of the inertial coordinates is set such that } -Z \text{ coincides with a target direction of the antenna, and the antenna is attached along } Z_B \text{ axis as mentioned above. Thus, the objective of this paper is to stabilize the attitude such that } Z_B \text{ axis points at } -Z. \text{ In addition, the total angular momentum } H_0 \text{ is fixed in the inertial coordinate system } \Sigma_0 \text{ because of the conservation law assumed above. Next, we introduce the dynamics model of the spacecraft.} \]
\[ H = J_1 \dot{\omega}_b + J_{w1} \dot{\varphi}_1 \dot{\omega}_1 + J_{w3} \dot{\varphi}_3 \dot{\omega}_3. \]  
(12)

From (12), \( \dot{\omega}_b \) is obtained as
\[ \dot{\omega}_b = -J^{-1} f_{u}(\dot{\omega}_b R^T H_0 + J_{w1} \dot{\varphi}_1 \dot{\omega}_1 + J_{w3} \dot{\varphi}_3 \dot{\omega}_3). \]  
(13)

Let \( u_i \) be the input torque of the \( i \)-th reaction wheel (the input of spacecraft system), and the dynamics of the \( i \)-th reaction wheel is
\[ j_{wi} \dot{\omega}_i = u_i, \]  
(14)

where \( j_{wi} \) is the inertia moment of the rotation direction, and it is the \( i \)-th element of \( J_{wi} \). From (5), (6), (13) and (14), the dynamics model of the spacecraft is
\[ \dot{\rho} = f_d(p, \dot{\rho}, \varphi, \omega) + g_{d1}(\rho)u_1 + g_{d3}(\rho)u_3, \]  
(15)
\[ f_d := Po_h - PJ^{-1} (\dot{\omega}_b R^T H_0), \]  
\[ g_{d1} := -PJ^{-1} \varphi_1, \quad g_{d3} := -PJ^{-1} \varphi_3. \]

Let \( x = [p^T, \dot{p}^T, \varphi, \omega]^T \), the state equation of spacecraft is
\[
\dot{x} = \begin{bmatrix}
    f_d(x) \\
    g_{d1}(x) \\
    g_{d3}(x) \\
    u_1 \\
    u_3
\end{bmatrix} + \begin{bmatrix}
    O_{3 \times 1} \\
    1/(j_{w1}) \\
    0 \\
    0 \\
    1/(j_{w3})
\end{bmatrix} \cdot \begin{bmatrix}
    O_{3 \times 1} \\
    \varphi_1 \\
    \omega_1 \\
    0 \\
    0
\end{bmatrix} \cdot u_3.
\]  
(16)

3. Problem Setting

3.1 Condition for Attitude Control [7]

The control objective of this paper is to point the antenna at the target direction and to keep the attitude. To control the antenna, we use the property of Rodrigues parameters. Since the component of Rodrigues parameters is consisting of \( \tan \theta/2 \) from (1), the rotation angle approaches \( \theta = \pi \) rad as \( |p_1| \) approaches infinity. Thus, if we can satisfy following conditions
\[ z_r = 0, \quad \theta = \pi \text{ rad}, \]  
(17)

the attitude can be rotated \( \pi \) rad around an axis on \( X - Y \) plane such as Fig. 2 since Rodrigues parameters represent the attitude rotated \( \theta = \pi \) around the rotation axis \( k = [x_3, y_3, 0]^T \). Because the antenna is attached at \( Z_b \) axis, the control objective will be achieved by realizing the attitude depicted in Fig. 2.

![Target attitude and its conditions.](image)

Since \( z_r \) is the component of \( p_3 \), to satisfy the above condition, we control Rodrigues parameters as follows.
\[ p_1 \to \infty, \quad p_2 \to \infty, \quad p_3 = 0. \]  
(18)

The purpose is to keep \( z_r = 0 \) but if \( p_3 \) keeps a constant finite value and \( \theta \) approaches \( \pi \), we can achieve \( z_r \to 0 \) from the definition of \( p_3 = z_r \tan \theta/2 \). Therefore, the control objective is to satisfy the condition (18).

3.2 Conventional Method [7]

In the past study, the spacecraft is represented as a kinematics model as follows
\[ \dot{\rho} = f(p) + g_1(p)\varphi_1 + g_3(p)\varphi_3. \]  
(19)

To increase Rodrigues parameters, a time-state control form (TSCF) is designed by using the following coordinate transformation
\[ \tau = p_1, \]  
(20)
\[ \eta_1 = p_1 p_3 + p_2, \]  
(21)
\[ \eta_2 = -\frac{\partial \eta_1}{\partial \tau}. \]  
(22)

TSCF consists of following two dynamics
\[ \frac{d\tau}{dt} = \varrho_1, \]  
(23)
\[ \frac{d\eta_1}{dt} = 0, \quad \frac{d\eta_2}{dt} = 0, \]  
(24)
\[ \varrho_1 = \frac{\partial \eta_1}{\partial \rho} f + \left( \frac{1}{\partial \rho} \frac{\partial \eta_2}{\partial \rho} [g_1, g_3] \right) \begin{bmatrix} \omega_1 \\ \omega_3 \end{bmatrix}, \]  
(25)

where \( \tau \) is a virtual time scale, \( \varrho_1 \) and \( \varrho_3 \) are a virtual input of each subsystem, (23) is called the time scale control part, and (24) is the state control part respectively. In the time scale control part, the virtual time \( p_1 \) is monotonically increases by designing the non-zero constant virtual input \( \varrho_1 \). As the result of stabilization of the state control part, it is reported in [7] that \( p_1 \) will be a constant value, and the condition (18) is satisfied.

3.3 Expansion of Conventional Method to Dynamics Model

The conventional method [7] is based on the kinematics model. This method assumes the radical angular velocity change of the reaction wheel. However, in a practical system, a motion of the reaction wheel is regulated by its own dynamics and cannot realize the jump of the angular velocity. Hence, another controller is required for applying the kinematics based controller. As a simple controller, we have applied a PI controller to the reaction wheel for following the angular velocity calculated in the conventional controller as shown in Fig. 3. However, an attitude control sometimes fails because the state control part will be unstable regardless of the parameters in the PI controller. Therefore, it is required to design the controller based on the dynamics model.

![Block diagram of conventional controller applied to dynamics model with PI controller.](image)
where we adopt the notation using Lie derivative defined as follows:

\[
\frac{\partial h}{\partial x} = \sum_{i=1}^{n} \left( \frac{\partial h}{\partial x_i} \frac{\partial x_i}{\partial x} \right),
\]

where \( h = \begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \end{bmatrix} = \begin{bmatrix} p_1 - C_1 t \\ \eta_1 \end{bmatrix}, \]

\( C_1 = \tilde{\omega}_1 \) (Constant).

From (26), we derive the following dynamics,

\[
\dot{\tilde{h}} = \mathcal{L}_h \dot{h} + \left( \mathcal{L}_{G_1} (\mathcal{L}_p h) \right) u_1 + \left( \mathcal{L}_{G_3} (\mathcal{L}_p h) \right) u_3 = \tilde{u},
\]

where we adopt the notation using Lie derivative defined as follows,

\[
\mathcal{L}_p h = \frac{\partial h}{\partial x} F = \begin{bmatrix} \frac{\partial h}{\partial x_1} F \\ \frac{\partial h}{\partial x_2} F \end{bmatrix},
\]

\[
\mathcal{L}_h \dot{h} = \frac{\partial \dot{h}}{\partial x} F, = \mathcal{L}_p (\mathcal{L}_h^{-1}) \dot{h}.
\]

The control objective will be achieved if the system (28) can be stabilized at zero by applying the linear controller \( \tilde{u} \). However, the decoupling matrix isn’t regular, that is

\[
\det \left( \begin{bmatrix} \mathcal{L}_{G_1} (\mathcal{L}_p h) & \mathcal{L}_{G_3} (\mathcal{L}_p h) \end{bmatrix} \right) = 0.
\]

This means that the ordinary input-output linearization scheme cannot be applied, and then the linear controller \( \tilde{u} \) is not realized by the actual input \( u \). Therefore, in this paper, we propose to apply the hierarchical linearization scheme to design the controller based on the dynamics model.

### 4. Proposed Method

#### 4.1 Hierarchical Linearization

In this section, we explain the proposed linearizing method. It is called the hierarchical linearization. The hierarchical linearization is first introduced in [9], and it can be regarded as a kind of generalization of TSCF. This method separates a system and linearizing a subsystem step by step. Let us focus on a two-input system to explain the hierarchical linearization. The algorithm of the hierarchical linearization is as follows.

- **(a)** Two output functions are chosen for the construction of two subsystems as shown in Fig. 4(a).
- **(b)** Next the effects on the subsystem 1 from the subsystem 2 are decoupled with the input transformation \( u_1 = V_1(\tilde{u}_1, u_2) \) as illustrated in Fig. 4(b).
- **(c)** Regarding subsystem 1 as an autonomous system by applying a linear feedback to subsystem 1.
- **(d)** The subsystem 2 becomes the SISO linear system including autonomous subsystem 1 with input transformation \( u_2 = V_2(\tilde{u}_1, \tilde{u}_2) \).

![Fig. 4 Algorithm of the hierarchical linearization.](image-url)

- **(a)** Choosing two output function \( h_1, h_2 \) for construction of two subsystems
- **(b)** Decoupling the effects on subsystem 1 from \( u_2 \) by an input transformation \( \tilde{u}_1 = V_1(\tilde{u}_1, u_2) \)
- **(c)** Regarding subsystem 1 as autonomous system by applying a linear feedback to subsystem 1
- **(d)** The subsystem 2 becomes the SISO linear system including autonomous subsystem 1 with input transformation \( u_2 = V_2(\tilde{u}_1, \tilde{u}_2) \).

(c) Then, the subsystem 1 can be regarded as the independent autonomous system due to the linear feedback seen in Fig. 4(c).

(d) As depicted in Fig. 4(d), the system from \( \tilde{u}_2 \) to \( h_2 \) becomes the single-input single-output linear system by designing the input transformation \( u_2 = V_2(\tilde{u}_1, \tilde{u}_2) \) canceling the effects from the autonomous subsystem 1.

The actual input is also calculated step by step as follows. First we design linear controller \( \tilde{u}_1 \) and \( \tilde{u}_2 \), and then we calculate \( u_2 = V_2(\tilde{u}_1, \tilde{u}_2) \). Finally, \( u_1 \) is calculated as \( V_1(\tilde{u}_1, u_2) \).

Owing to this step-by-step scheme, the relative degree of \( h_2 \) can be prolonged since the effect of input \( u_1 \) is canceled. Now we remark that the hierarchical linearization can be applied to more than three inputs system by repeating the algorithm.

In the following part, we apply the hierarchical linearization to the spacecraft system with output function defined in (26). We differentiate \( h_1 \) as follows,

\[
\dot{h}_1 = \mathcal{L}_h \dot{h}_1, \quad \ddot{h}_1 = \frac{\partial \dot{h}_1}{\partial x} F \frac{dx}{dt} + \mathcal{L}_h \dot{h}_1 + \left( \mathcal{L}_{G_1} (\mathcal{L}_p h_1) \right) u_1 + \left( \mathcal{L}_{G_3} (\mathcal{L}_p h_1) \right) u_3.
\]

Then, the input transformation

\[
u_1 = (\mathcal{L}_{G_1} (\mathcal{L}_p h_1))^{-1} (\tilde{u}_1 - \mathcal{L}_p h_1 - (\mathcal{L}_{G_3} (\mathcal{L}_p h_1)) u_3)
\]

is applied to (32) to cancel the effect of \( u_3 \). Here, the coefficient of \( u_1 \) in (32) satisfies

\[
\mathcal{L}_{G_1} (\mathcal{L}_p h_1) = -\frac{(1 + p_1^2)}{2 j_{n}} < 0.
\]

Thus, the input transformation is well-defined. From the above, we can obtain the subsystem 1 by linearizing between \( h_1 \) and \( \tilde{u}_1 \) as follows.

\[
\mathcal{L}_{G_1} (\mathcal{L}_p h_1) = \begin{bmatrix} [h_1, \dot{h}_1] T \\ 0 \end{bmatrix}, \quad \mathcal{L}_{G_3} (\mathcal{L}_p h_1) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{h}_1 \\ \tilde{u}_1 \end{bmatrix}.
\]

Now, the output function \( h_1 = p_1 - \mathcal{L}_p h_1 \) converges and \( p_1 \) increases monotonically with slope of \( C_1 \) by designing the following controller:

\[
\ddot{u}_1 = -K_1 \mathcal{L}_{\eta_1}(\mathcal{L}_{\eta_1} h_1),
\]

where \( K_1 \) is a constant matrix of \( 1 \times 2 \). Substituting the input transformation (33) into the system (16), the system of spacecraft becomes

\[
\ddot{x} = F(x) + G_3(x) u_3,
\]

where \( \tilde{F} \) and \( \tilde{G}_3 \) are calculated as follows,

\[
\tilde{F} = F - G_1 (\mathcal{L}_{G_1} (\mathcal{L}_p h_1))^{-1} \mathcal{L}_p^2 h_1 + G_1 (\mathcal{L}_{G_1} (\mathcal{L}_p h_1))^{-1} \tilde{u}_1 (\mathcal{L}_{\eta_1} h_1),
\]

\[
\tilde{G}_3 = G_3 - G_1 (\mathcal{L}_{G_1} (\mathcal{L}_p h_1))^{-1} \mathcal{L}_{G_3} (\mathcal{L}_p h_1).
\]

The whole system is regarded as a single-input single-output system controlled by \( u_3 \) since the virtual input \( \tilde{u}_1 \) has been designed in (37). In this system, \( h_1 \) is uncontrollable because the effect of input \( u_3 \) on \( h_1 \) is cancelled in (33).
We also linearize the subsystem with respect to $h_2$ by differentiating the output function $h_2$ as follows.

$$h_2 = L_3 h_2,$$

$$\dot{h}_2 = L_3 v h_2,$$

$$h_2^{(3)} = L_3 h_2 + \left(L_{G_1} \left(L_3 h_2\right)\right)u_3.$$  

From the above development, by designing the following input transformation

$$u_3 = \left(L_{G_1} \left(L_3 h_2\right)\right)^{-1} \left(\bar{u}_3 - L_3 h_2\right),$$

the linearized subsystem 2 is obtained such as (44)

$$\xi_3 = \left[h_2, \dot{h}_2, \ddot{h}_2\right]^T,$$

$$\frac{d\xi_3}{dt} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xi_3 + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \bar{u}_3.$$.  

The virtual input $\bar{u}_3$ is designed so that the subsystem 2 is exponentially stable:

$$\bar{u}_3 = -K_2 \xi_2,$$

where $K_2$ is a constant matrix of $1 \times 3$. The input transformation (42) has the following singular points:

$$L_{G_1} \left(L_3 h_2\right) = 0.$$  

4.2 Analysis of Linearizable Dimensions

In the linearization of nonlinear system, the number of linearizable dimensions is important. If there remains a nonlinear part, an analysis of stability of the remaining dynamics is required. In general, this analysis is difficult when the remaining nonlinear part has high dimensions. In this section, we analyze the maximum linearizable dimensions of the spacecraft system to show the advantage of the hierarchical linearization.

First, we recall the conditions to calculate the maximum linearizable dimensions for the ordinary input-output linearization from the reference [11]–[13]. Then, since the spacecraft system is (16), the system distributions are

$$\mathcal{G}_1 = \text{span} \left\{G_1, G_3\right\} = \mathcal{G}_1, \quad \text{rank}(\mathcal{G}_1) = 2,$$

$$\mathcal{G}_2 = \mathcal{G}_1 + \text{ad}_f \mathcal{G}_1, \quad \text{rank}(\mathcal{G}_2) = 4,$$

$$\mathcal{G}_3 = \mathcal{G}_2 + \text{ad}_f G_1, \quad \text{ad}_f G_1, \quad \text{rank}(\mathcal{G}_3) = 5,$$

$$\mathcal{G}_4 = \mathcal{G}_3 + \text{ad}_f \mathcal{G}_3, \quad \text{rank}(\mathcal{G}_4) = 5.$$  

Here $\mathcal{G}_i$ is the distribution defined as follows.

$$\mathcal{G}_i = \text{span} \left\{g_1, \cdots, g_m\right\},$$

$$\text{ad}_f \mathcal{G} = \text{span} \left\{\text{ad}_f g \mid g \in \mathcal{G}\right\}.$$  

In addition, $\text{ad}_f g = [f, g]$ represents the Lie bracket defined as follows:

$$[f, g] = \frac{\partial g}{\partial x} f - \frac{\partial f}{\partial x} g,$$

$$\text{ad}_f^2 g = [f, \text{ad}_f g].$$

and $\mathcal{G}$ is an involutive closure such as

$$\mathcal{G} = \mathcal{G} \Leftrightarrow [g_1, g_2] \in \mathcal{G} \quad \forall g_1, g_2 \in \mathcal{G}.$$  

The dimension of the spacecraft system is 8 but the maximum dimensions of $\mathcal{G}_i$ is 5 because the spacecraft has 3-dimensional constraints known as the conservation law of the angular momentum (9). Then, the dimension of the reachable state is spanned by $\mathcal{G}_2$ of (47). This means that there exists a function whose relative degree is less than or equal to 2, and a function with relative degree 3 does not exist. The linearizable dimension of the ordinary input-output linearization is equal to the relative degree. Moreover, the maximum linearizable dimension of the spacecraft system in the sense of the ordinary input-output linearization is 4.

On the other hand, as shown in Section 4.1, we achieved 5 dimensional linearization using the proposed method. The key idea to control the target system is the divergence of $p_1$ as explained in Section 3.1. This divergence is realized in the first layer, and the remaining task is to stabilize the other states. Hence, we discuss only the second layer system described in (38). In this case, we also borrow the analysis tool for the ordinary input-output linearization. The system distributions of the second layer system are

$$\mathcal{G}_1 = \text{span} \left\{G_3\right\} = \mathcal{G}_1, \quad \text{rank}(\mathcal{G}_1) = 1,$$

$$\mathcal{G}_2 = \mathcal{G}_1 + \text{ad}_f \mathcal{G}_1 = \mathcal{G}_2, \quad \text{rank}(\mathcal{G}_2) = 2,$$

$$\mathcal{G}_3 = \mathcal{G}_2 + \text{ad}_f \mathcal{G}_2, \quad \text{rank}(\mathcal{G}_3) = 3.$$  

This means that the maximum linearizable dimension is 3 since $\text{rank}(\mathcal{G}_1)$ is 3 and $\mathcal{G}_2$ is involutive. Hence, the maximum linearizable dimensions of the hierarchical linearization is 5 because the 2 dimensions can be linearized in the first layer such as (36). Therefore, the maximum linearizable dimension is increased with the hierarchical linearization compared to the ordinary input-output linearization. The fact supports the result of Section 4.1. Moreover, we can say that it was reasonable to choose the output function as (26) by following the idea explained in Section 3.1. With the idea in Section 3.1, the other choice of the first layer is $p_2 - C_{2} t$. In this case the result is the same as the above.

5. Numerical Simulation

In this section, we show that the attitude control based on the dynamics model can be achieved by the hierarchical linearization via a numerical simulation. The parameters of the simulation are listed in Table 1. Physical parameters are set based on the small size satellite MUSES-C. For the detailed parameters of the reaction wheel, refer to [14].

The result of the simulation is shown in Fig. 5. (a) is the Rodrigues parameters, (b) is the enlarged graph of Rodrigues parameters, (c) is time derivative of Rodrigues parameters, (d)
is the angular velocity of the reaction wheels, (e) is the input to the spacecraft, (f) is the state of the subsystem 1, (g) is the state of the subsystem 2, (h) is the error between the antenna direction and the target direction. To show the detail, the figures (d), (e) and (h) depict first 1500 seconds.

The simulation result depicted in Fig. 5(a)-(b) shows that Rodrigues parameters $p_1$ and $p_2$ decrease and $p_3$ approaches a constant value. As shown in Fig. 5(c), $p_1$ decreases with the slope $C_1 = -1$. The validity of these features are guaranteed by the result that the states $\xi_1$ and $\xi_2$ of the subsystem 1 and 2 approach zero as shown in Fig. 5(f)-(g). As the result of decrease of $p_1$ and a finite constant value of $p_3$, the rotation axis becomes $z_r = 0$, and the rotation angle becomes $\pi$ rad. This means that the antenna on $Z_B$ axis points the target direction. This is also confirmed by the convergence of the error angle between the antenna and the target direction in Fig. 5(h).

By the hierarchical linearization, the number of the controlled states is 5, but all states are stable since the spacecraft satisfies the conservation law of the angular momentum (9) as discussed in Section 4.2. Figure 5(d) and (e) shows that the input torque and the angular velocity change smoothly. Moreover the rotation speed of the reaction wheels is less than 5000 rpm, and this value is feasible by a commonly used reaction wheel. Therefore, the proposed method works well for the attitude control of a two-reaction-wheel spacecraft.

At the end of this section, we point out the importance of the translation of the state to a quaternion to avoid the computational problem of the value of infinity when we apply the proposed controller to a practical system. An attitude representation via quaternion is well-known as singularity free one. Hence, the instability from the singularity of Rodrigues parameter is resolved. On the other hand, the desired state is a singular point for the proposed controller. However, thanks to the numerical error, the state will never arrive at the desired state exactly. As the result, the proposed controller can continue to calculate the input and to keep the target posture with enough
accuracy due to the usage of the quaternion as the state. As a more conservative approach, we can truncate the input after the state approaches the desired state with a satisfactory accuracy. Indeed, the absolute value of the input $u$ becomes smaller than $10^{-10}$ at $t = 30000$ in this simulation, and after that the input can be set to zero.

6. Conclusion

In this paper, we developed the dynamics based attitude controller for an underactuated spacecraft using two reaction wheels under the existence of the total angular momentum. For the underactuated spacecraft, the conventional attitude controller [7] has been designed for the kinematics model. The extension of the controller to the dynamics model is not straightforward because of the ill-defined input transformation. The hierarchical linearization scheme overcame this problem and realized the linearization of the reachable states. Effectiveness of the proposed method was explained via the analysis of the linearizable dimension and a numerical simulation.

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