Energy-Efficient Power Assist Control with Periodic Disturbance Observer and Its Experimental Verification Using an Electric Bicycle

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Abstract: This paper addresses the problem of energy-efficient power assist control for quasiperiodic motions. The simplest assist method would be to apply additional torque in proportion to the instantaneous value of torque generated by a user. In our previous study, it was shown that energy efficiency improves by flattening the torque pattern. To cope with the frequency fluctuation of our motion, we introduce a periodic disturbance observer with a frequency estimator and suppress the pulsation of the human torque based on a disturbance observer framework. The effectiveness of the proposed method is evaluated through numerical simulations and experiments with an actual electric bicycle being pedaled by a human.

Key Words: energy efficiency, power assist, periodic motion, disturbance observer.

1. Introduction

It is well understood that the aging of the population is a critical problem for the current generation. Improving of the quality of life of elderly people is an urgent issue related to this global trend. Power assist technology is one of the promising solutions for several aging-related problems. Various studies related to power assist have been conducted from the viewpoint of control engineering, e.g., electric wheelchairs in welfare field [1],[2] and overhead cranes in industries [3].

An electric bicycle (Fig. 1) is a popular application of power assist technology [4]. Because the bicycles are battery powered, several studies have considered the effective use of power resource [5],[6]. To achieve a long operation time, energy efficiency must be optimized for such battery-powered devices. To tackle this issue, we focus on the periodicity of human persistent tasks. In our previous study, an optimal power assist method for periodic motions was developed based on a frequency shaping by a feedback controller1. The basic principle for an energy-efficient operation is to suppress the velocity deviations caused by fluctuations in the pedaling torque due to the muscular skeletal structure of our body [7]. A practical problem is that the pedaling period of a human is not exactly constant. We addressed this problem by employing adaptive notch filters [8],[9]. Although this method works for artificial quasiperiodic signals with relatively small fluctuations, it fails to deal with actual human pedaling because real pedaling data contains signals with a wide range of frequencies.

Fig. 1 Electric bicycle.

A suppression scheme for periodic disturbances with fluctuating frequencies was proposed in [10]. This method is based on online frequency estimation and state estimation by periodic disturbance observer. In this study, we regard the pedaling torque generated by a human as a quasiperiodic disturbance and apply the aforementioned technique to the power assist control. In this approach, the bias and the vibrational components of the pedaling torque are estimated separately. Not to waste the energy supplied by a human, vibrations are suppressed while the bias is amplified based on the result of an estimation.

This paper is structured as follows: Section 2 introduces a drive train model of a bicycle and energy efficiency of power assist control for it. Then a new assist method is summarized in Section 3. The results of numerical simulations are presented in Section 4. Section 5 shows the results of experiments with an actual electric bicycle (being pedaled by a machine and a human). Finally, conclusions are given in Section 6.

In this paper, we use the following notations: Let \( \mathbf{R} \) be the set of real numbers and \( \mathbf{R}^n \) be the set of \( n \)-dimensional real vectors, respectively. Unless otherwise noted, all signals are real and scalar. \( \lfloor x \rfloor \) denotes the largest integer less than or equal to a real number \( x \). \( \mathbf{I} \) and \( \mathbf{0} \) denote an identity matrix and a

1 Note that if we drive the motor at a constant angular velocity for an optimal efficiency, it is no longer regarded as a bicycle legally and it violates the road traffic law of Japan.
zero matrix of an appropriate dimensions, respectively. \( \text{He}(X) \) is a shorthand notation for \( X + X^T \). * denotes an abbreviated off-diagonal block in a symmetrical matrix.

2. Energy Efficiency of Power Assist for Almost-Periodic Motions

2.1 Modeling of the Drive Train

A schematic diagram of the drive train of our experimental bicycle is shown in Fig. 2. Let us denote the human pedaling torque, the motor assist torque and the rotation angle of the rear wheel as \( \tau_h \), \( \tau_m \) and \( \theta \), respectively. Let \( J_d \) be the total moment of inertia for all rotating elements. We assume that the total travel resistance is represented by the viscous friction with coefficient \( D_d \). Thus the transfer function from \( \tau_h + \tau_m \) to \( \theta \) is given by

\[
G_p(s) = \frac{1/J_d}{s(s + D_d/J_d)},
\]

and the velocity of the bicycle \( v \) is given by

\[
v = r_w \dot{\theta},
\]

where \( r_w \) is the radius of the rear wheel. The dynamics of the assist motor is approximated by a first order lag. Let \( k_m \) and \( T_m \) represent the corresponding gain and time constant, respectively. Then the transfer function from the reference signal to \( \tau_m \) is given as

\[
M(s) = \frac{k_m}{T_m s + 1}.
\]

2.2 Energy Efficiency

We determine the assist torque \( \tau_m \) according to the human torque and the velocity, namely,

\[
\tau_m = C(\tau_h, v).
\]

If the function above (the feedback controller) is given by a transfer function matrix, one can obtain the following closed-loop transfer function \( G_c(s) \) from \( \theta \) to \( v \) as

\[
v = G_c(s) \tau_h.
\]

The block diagram of closed-loop transfer function \( G_c(s) \) is shown in Fig. 3. From the principle of frequency response, the following optimality condition is derived [7]. We suppose that the pedaling torque \( \tau_h \) is a band limited \( T \)-periodic signal with frequencies in the range \( [0, \omega_0] \) and the DC gain \( G_c(\omega) \) of the closed-loop system is the same for all feedback controllers. Then the optimal energy efficiency is achieved when \( G_c(\omega) \) satisfies the following condition:

\[
G_c(\omega_k) = 0, \quad \omega_k = \frac{2nk}{T}, \quad k = 1, 2, 3, \ldots, N, \quad N = \left\lfloor \frac{\omega_0 T}{2\pi} \right\rfloor.
\]

Let us briefly explain this condition. Due to the assumption that the total travel resistance is represented by the viscous friction at the rear wheel, the energy consumption within a period is computed as

\[
E := \int_0^T \tau(t)v(t)dt = \int_0^T D_d \nu^2(t)dt.
\]

For simplicity, consider the case with single frequency component. Then the steady state velocity should be given by \( v(t) = a \sin(2\pi/T)t + b \) and the corresponding \( E \) in (7) is calculated as \( E = D_d(a^2/2 + b^2)T \). Then it is concluded that the energy consumption is minimized when we achieve a flat velocity pattern (\( a = 0 \)).

2.3 Pedaling force Proportional Control

In most power assist applications, additional force is generated in proportion to the instantaneous value of the human force. This is also true for commercial electric power-assisted bicycles. This control method is called the pedaling force proportional control (PPC). The block diagram of PPC with an assist rate \( \delta > 0 \) is shown in Fig. 4. The working principle of PPC is described as follows. The pedaling torque is measured by the torque sensor attached to the crank shaft. The measured value is amplified by the factor \( \delta \). Then the corresponding current reference is sent to the driver of the assist motor.

The pedaling torque of bicycles varies with the crank angle depending on the mechanical structure of our lower limb. An example of a measured time series of human pedaling torque is shown in Fig. 5. The pulsation of the pedaling torque is amplified by the PPC and results in a poor energy efficiency.

![Fig. 2 Schematic diagram of the drive train.](image)

![Fig. 3 Block diagram of \( G_c(s) \).](image)

![Fig. 4 Block diagram of PPC.](image)

![Fig. 5 Measured human pedaling torque.](image)
3. Assisting Control System with Periodic Disturbance Observer

3.1 Overview of the Proposed System

Current pedaling frequency \( \omega \) is estimated based on the noisy measurement signal of the pedaling torque. The estimate \( \hat{\omega} \) is used to construct the periodic disturbance observer [10],[11]. This observer estimates the human pedaling torque less contaminated by the noise. The vibrational component of the human pedaling torque is suppressed by adding a corresponding command to the motor driver to cancel the disturbance effect. The block diagram of the proposed assist control system is illustrated in Fig. 6. The transfer functions \( C_d(s) \) and \( C_a(s) \) denote the observer dynamics from \( u \) and \( y \) to the estimated state \( \hat{\dot{x}} \). They are subject to change according to the estimated frequency \( \hat{\omega} \). The feedback gain for the estimated state \( \hat{\dot{x}} \) is denoted by \( K \).

3.2 Frequency Estimation [12]

We consider the following quasiperiodic function \( d(t) \) with varying frequency

\[
d(t) = a \sin (\omega(t) t + \phi),
\]

where \( a, \phi \) and \( \omega(t) \) denote the fixed amplitude, the phase and the time-varying frequency, respectively. Assume that all of \( a, \phi \) and \( \omega(t) \) are unknown.

For the estimation of the frequency \( \omega(t) \), the method described in [12] is employed. Let \( \hat{\omega}(t) \) and \( \chi(t) \) denote the estimation of \( \omega(t) \) and the internal state of the frequency estimator, respectively. Then the estimator is given by the following coupled equations:

\[
\begin{align*}
\dot{\chi}(t) &= 2\zeta \hat{\omega}(t) \chi(t) + \hat{\omega} \chi(t) + \hat{\omega} \gamma(t) d(t), \\
\dot{\gamma}(t) &= 2\zeta \hat{\omega}(t) \gamma(t) - \hat{\omega} \gamma(t) d(t), \\
\hat{\omega}(t) &= \gamma(t) \chi(t) + \gamma(t) d(t),
\end{align*}
\]

where \( \zeta \) and \( \gamma \) represent the damping and adaptation speed coefficients, respectively. If the estimated frequency \( \hat{\omega}(t) \) is constant, then the Laplace transform of (9) and (10) yields

\[
\nu(s) = \frac{s^2 \hat{\omega} \gamma (s^2 + 2\zeta s + \dot{\omega}) \hat{\omega} \gamma d(s)}{s^2 + 2\zeta s + \dot{\omega} \gamma d(s)}.
\]

This is a notch filter that blocks signals with the estimated frequency \( \hat{\omega} \) in \( d(s) \). Thus, one can interpret the adaptation law above as updating the estimation \( \hat{\omega} \) until the output of this filter becomes small (\( \hat{\omega} \rightarrow \omega \)). The parameter \( \gamma(t) \) is determined by

\[
\gamma(t) = \frac{e}{[1 + N(\chi^2(t) + \hat{\omega}^2(t))] [1 + \mu \hat{\omega}(t)]^\mu}
\]

where \( e, N, \mu, \) and \( \alpha \) are design parameters.

3.3 Periodic Disturbance Observer for Varying Frequency

[10],[11]

We consider the following SISO system with the input disturbance \( d(t) \)

\[
\begin{align*}
\dot{x}_p(t) &= A_p x_p(t) + b_p (u(t) + d(t)), \\
y(t) &= c_p x_p(t),
\end{align*}
\]

where \( x_p(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^m \) and \( y(t) \in \mathbb{R}^m \) denote the state, the control input and the measurement output, respectively. We assume that the pair \((A_p, b_p)\) is controllable and the pair \((c_p, A_p)\) is observable. Since the coefficient matrix for \( u(t) \) and \( d(t) \) is the same, \( d(t) \) satisfies the matching condition [13]. Suppose that \( d(t) \) is given by

\[
\begin{align*}
d(t) &= a_0 + a_1 \sin (\omega t + \phi) \\
\end{align*}
\]

where \( \omega \) is slowly varying in time. Then it can be verified that the state-space model

\[
\begin{align*}
\dot{\eta}(t) &= \omega A_p \eta(t), \\
d(t) &= c_p \eta(t),
\end{align*}
\]

with

\[
A_d = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}, \quad c_d = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}
\]

is a generator of the quasiperiodic disturbance signal \( d(t) \) in (15). Note that from the structure of \( A_d \), first and the second elements of \( \eta \) correspond to the bias and sinusoidal components of the estimated disturbance, respectively. Let \( x(t) = [x_p^T(t), \eta^T(t)]^T \in \mathbb{R}^{n+p}. \) Then the augmented system describing the plant with the disturbance signal generator is given by

\[
\begin{align*}
\dot{x}(t) &= A_u x(t) + b_u u(t), \\
y(t) &= c_u x(t),
\end{align*}
\]

where

\[
A_u = \begin{bmatrix} A_p & b_p c_d \\ 0 & \omega A_d \end{bmatrix}, \quad b_u = \begin{bmatrix} b_p \\ 0 \end{bmatrix}, \quad c_u = \begin{bmatrix} c_p & 0 \end{bmatrix}
\]

The observability of the pair \((c_u, A_u)\) is guaranteed if both conditions

1. the pair \((c_p, A_p)\) is observable,
2. the triple \((c_p, A_p, b_p)\) has no transmission zeros on the imaginary axis,

are satisfied. Suppose that the pair \((c_u, A_u)\) is observable and \( L \) is a stabilizing output feedback gain. Then the estimate \( \hat{d}(t) \) of \( d(t) \) is given by the observer

\[
\begin{align*}
\dot{\hat{x}}(t) &= (A_u - L c_u) \hat{x}(t) + b_u u(t) + Ly(t),
\end{align*}
\]

and

\[
\begin{align*}
\hat{d}(t) &= \hat{c}_d \hat{x}(t),
\end{align*}
\]

with \( \hat{c}_d = [0, c_d] \).

From (17), the matrix \( A_u \) is affine in terms of \( \omega \). Thus, one can employ the gain scheduling control strategy to determine the observer gain \( L \). Suppose that the range of \( \omega \) is known to be \([\omega_1, \omega_2]\). Then choose the scheduling parameter as...
\[ q = \frac{\omega_2 - \omega}{\omega_2 - \omega_1}. \quad (20) \]

Let \( A_a \) (\( i = 1, 2 \)) be the matrix \( A_a \) with \( \omega = \omega_1 \). Then, for intermediate values of \( \omega \), \( A_a \) is described as the following polytopic form

\[
A_a = qa_a + (1 - q)A_{a2}. 
\]

Determine the matrices \( L_i \) (\( i = 1, 2 \)) by solving the following Linear Matrix Inequalities (LMIs)

\[
\begin{align*}
PA_{a1} + A_{a1}^T P - M(c_a - c_{a1}^1)M_1^T &< 0, \\
PA_{a2} + A_{a2}^T P - M(c_a - c_{a2}^1)M_2^T &< 0,
\end{align*}
\]

and letting \( L_i = P^{-1}M_i \) (\( i = 1, 2 \)). For intermediate values of \( \omega \), choose the observer gain \( L \) as

\[
L = qa_L + (1 - q)L_2.
\]

From (21) and the definition of \( L_i \),

\[
P(A_{a1} - Lc_a) + (A_{a2} - Lc_a)^T P < 0 \quad (i = 1, 2)
\]

is derived. Then the convex combination of (23) is described as

\[
P(A_a - Lc_a) + (A_a - Lc_a)^T P < 0.
\]

Thus the observer is stable for all \( q \in [0, 1] \). Note that in our case, the estimate \( \hat{\omega} \) is used in place of \( \omega \).

\section{Numerical Simulations}

\subsection{System Identification and Design Procedure}

The dynamics of the drive train and the assist motor of our electric bicycle discussed in Section 2.1 are identified from the experimental data. The least square solution yields

\[
G_p(s) = \frac{1.701}{s(s + 1.544)}, \quad M(s) = \frac{3.985}{4.500 \times 10^{-3} s + 1}.
\]

Since the time constant of \( M(s) \) is sufficiently small, we model \( M(s) \) by a constant as

\[
M(s) = 3.985.
\]

We regard \( \tau_h \) invoked by the pedaling action as the quasiperiodic disturbance \( d \). Then \( \tau_m \) and \( \theta \) correspond to \( u \) and \( y \) described in Section 3. Since the simple sum of \( \tau_h \) and the motor torque \( \tau_m \) goes into the drive train, our system satisfies the matching condition. By taking a minimal realization, observability of

\[
G_p(s) = c_p(sI - A_p)^{-1}b_p
\]

is automatically guaranteed. From the LMI solutions, \( L \) is determined by (22). Then, the transfer function blocks \( C_y(s) \) and \( C_y(s) \) related to the observer are given by

\[
C_u(s) = (sI - \tilde{A})^{-1}b_u, \quad C_y(s) = (sI - \tilde{A})^{-1}L,
\]

where \( \tilde{A} = A_a - Lc_a \). To adjust the response of the observer, the pole assignment and \( H_2 \) performance conditions are added to the LMIs. The center and the radius of the specified disk are denoted by \( p_1 \) and \( p_r \), respectively. By solving the following LMIs, we determine \( L_1 \) and \( L_2 \).
5. Experiments

5.1 Experimental Setup

Generally, human pedaling patterns are different for each trial, and it makes the comparison of experimental results difficult. To avoid this situation, we developed the pedaling machine shown in Fig. 10. We use this pedaling machine first to verify the effectiveness of the proposed method. The pedaling torque is regulated by the current control of the motor driver. In every experiment, the same time series data of measured human torque is used as the reference signal for the pedaling machine. The experimental bicycle is mounted on a cycle trainer that mimics the road resistance. The rotation angle of the rear wheel is measured by a rotary encoder attached to the wheel hub. The velocity of the bicycle is estimated by an observer. To deal with the effect of the noise on the actual data, the parameters for the frequency estimator is tuned as

\[ \epsilon = 30, \quad N = 100, \quad \zeta = 0.5, \quad \mu = 1, \quad \alpha = 1.\]

To unify the average velocity of the bicycle for the proposed method and PPC, we chose \( k_{dc} = 0.2\). The continuous-time frequency estimation algorithm is discretized with the sampling time \( T_s = 0.01 \text{s} \) via the Runge-Kutta method. The same sampling time is used for the implementation of the observer and feedback power assist controller. To compensate the Coulomb friction, a bias signal with 0.5 Nm is added to the assist torque reference. The block diagram of the experimental system is illustrated in Fig. 11.

5.2 Experimental Results

In the experiments, the pattern of the reference signal for the pedaling machine is changed every 10 s. The result of the pedaling torque estimation is plotted in Fig. 12. Figure 13 (a) shows the estimated frequency while Fig. 13 (b) is a level plot illustrating the result of the frequency analysis via a short-time Fourier transform. As mentioned earlier, the bias component of the pedaling torque is removed by a high-pass filter. One can verify that the estimated frequency is located around the peak of the numerically computed power spectrum. Based on this estimation, the periodic disturbance observer estimates the pedaling torque as in Fig. 12. However, it is rather ambiguous whether the estimate is precise enough. Therefore the energy
efficiency should be evaluated how much the velocity deviation is reduced. Then, the results of the power assist control are shown in Fig. 14. The velocity deviation of the proposed method is smaller than that of PPC. It is similar to the behavior of the numerical simulation. The rotation distances of the rear wheel during the both experiments are shown in Fig. 15. This figure verifies that the average velocities of the bicycle in each case are almost the same, and it is also true for the pedaling speeds (because the gear ratio is fixed). Since the pedaling torque is identical, one can conclude that there is no difference in the amount of energy supplied by the pedaling machine in both cases. Therefore, the method consuming less electrical energy is more efficient in achieving the same moving speed. The voltage drop of the battery during the experiments is shown in Fig. 16. As expected, the suppression of velocity pulsations reduces the voltage drop. Thus, the energy efficiency of the proposed assist control method is better than that of PPC and promises longer operation time with the same battery under the same load condition.

When we use the pedaling machine, the interaction between the human and the bicycle is ignored. Next we proceed to a realistic situation that the pedaling torque is generated by a human. As a guideline, the pedaling person watches a reference movie during the experiment (Fig. 17). In this case, the reference ped-
The pedaling pattern is changed every 10 s. The bicycle velocities are compared in Fig. 18. The rotation distance of the rear wheel and the voltage drop of the battery during the experiments are depicted in Fig. 19 and Fig. 20, respectively. We observe that the proposed method can suppress the velocity fluctuation and reduce the voltage drop well. They reveal similar tendencies as in the case of pedaling by the machine. From the results, the effectiveness of the proposed method is verified.

Let us evaluate the proposed method from a man-machine coordination perspective. The assist torque is increased when we are unable to generate enough torque. Therefore the proposed method can give a comfortable and supported feeling to the pedaling person at the expense of slight loss of direct maneuvering feel.

Fig. 18 Velocity of the bicycle (pedaling by a human).

Fig. 19 Rotation distance of the rear wheel (pedaling by a human).

Fig. 20 Voltage drop (pedaling by a human).

6. Conclusions
This paper developed an estimation-based power assist control method for quasiperiodic motions. Especially, we focus on the pedaling of bicycles. To improve the energy efficiency, the proposed method suppresses the pulsations in the pedaling torque and amplifies the bias component under the situation with varying pedaling frequency. The velocity pulsation is reduced successfully as demonstrated by numerical simulations. Then, we validated the effectiveness of our method through experiments with pedaling by the machine and a human. It is verified that the energy efficiency of the proposed method is actually improved compared to that of PPC by monitoring the voltage drop of the battery.

In the current setting, the estimation of \( \omega \) should be precise enough. However, under more realistic situations, it is desired to tolerate a mismatch between \( \omega \) and \( \hat{\omega} \). Recent studies related to this issue [15],[16] will improve the robustness of the proposed method. This is a problem that will be addressed next.

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References


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