Aircraft Gust Alleviation Preview Control with a Discrete-Time LPV Model

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Abstract: This paper deals with a gust alleviation (GA) system using gain-scheduled (GS) discrete-time preview control. The key points to improve control performance of this system are in both modeling accuracy of the linear parameter-varying (LPV) system with a small number of scheduling parameters and design of the advanced GS controller. In this study, a discrete-time LPV model of the aircraft with a small number of essential scheduling parameters is proposed through a series of approximations based on understanding of flight dynamics. A GS controller is designed with the extended linear matrix inequality (LMI) while constructing a smaller convex hull to the LPV model. The simulation result shows that the proposed control system effectively attenuates aircraft vertical acceleration in turbulence and it is robust against cruising speed changes.

Key Words: GS control, LPV model, gust alleviation, LIDAR, preview control.

1. Introduction

In Japan, in the decade from 2004 to 2013, the percentage of accidents related to turbulence is about 50% among all aircraft accidents [1, pp. 246–247]. Especially, vertical acceleration of aircraft caused by turbulence may result in serious injuries of passengers. Thus, it is very important to alleviate vertical acceleration of aircraft for safer and more reliable air transportation. For this demand, a gust alleviation (GA) system using discrete-time preview control assumed to utilize prior information of turbulence measured by the light detection and ranging (LIDAR) system has been developed (Fig. 1). The basic idea of the discrete-time preview control is that an extended state equation is applied to the control performance condition, which enables us to improve control performance [2],[3].

The robust control theory is well established for linear systems, which has been used for GA control in order to cope with cruising speed changes [4]. However, the linear robust controller may be inapplicable for GA control since real nonlinear processes such as aircraft dynamics have a wide operating range, which leads to conservatism of control design. For this reason, much attention has been paid on gain-scheduled (GS) control via linear parameter-varying (LPV) control theory. GS control is one of the most common approaches for nonlinear systems which have a wide operating range, and it is a technique to improve control performance by changing gain values of controllers according to different operating conditions.

In order to apply GS control to a GA system, modeling of LPV systems of aircraft is an important task since aircraft dynamics are complex functions of various parameters. For the past few decades, many researchers have proposed LPV models of aircraft [5]–[8]. However, most researches have been studied for developing an accurate continuous-time LPV model, which means that they may not be suitable for a discrete-time GS controller via linear matrix inequalities (LMIs) for a discrete-time preview control. The difficulty of obtaining a discrete-time LPV model for a GS controller via LMIs is summarized as the following three problems. The first of which is that too many scheduling parameters may make LMIs infeasible. Secondly, too few scheduling parameters make the LPV model different from the actual aircraft model. Finally, inappropriate scheduling parameters for designing GS controllers lead to the conservatism of control design, even if they satisfy the LMI conditions and accuracy of an LPV model.

The key point to obtain an accurate LPV model with a small number of parameters is identifying the essential scheduling parameters for operating conditions. In the case of aircraft dynamics, they are considered to be both the velocity of the aircraft and the atmospheric density that yield the dynamic pressure, which mainly changes as the flight condition changes. Therefore, in this study, a discrete-time LPV model represented by essential scheduling parameters is proposed through a series of approximations of the elements of the state-space matrices based on understanding of flight dynamics, while only largely varying elements in state-space matrices were regarded as scheduling parameters in [5]. In addition, by constructing a smaller convex hull to the LPV model, it is shown that the proposed control system more effectively attenuates aircraft acceleration in turbulence than previous studies [4],[5].

The rest of this paper is organized as follows. Section 2 describes the LPV model of the aircraft and discrete-time preview control. Section 3 describes GS controller design based on LPV
control theory by using extended LMIs. Section 4 gives a numerical simulation, and Section 5 concludes this paper.

The symbol $*$ denotes the symmetric block, and $M > 0$ ($M < 0$) means that $M$ is positive (negative) definite. The symbol $H_{\infty}$ is the closed-loop transfer function from $w$ to $z$. Finally, $\| \cdot \|_2$ denotes the $H_2$ norm.

2. Aircraft Model and Discrete-Time Preview Control

2.1 The Linearized Equation of Motion for Small Perturbations

In this paper, we deal with a large aircraft equipped with four jet engines and consider only the longitudinal motion (Fig. 2). In this case, the velocity of the aircraft $V$ consists of $U$, the $X$-direction velocity, and $W$, the $Z$-direction one, as follows:

$$ V = \sqrt{U^2 + W^2}. $$

(1)

If the aircraft experiences small perturbations about a trim condition, the linearized longitudinal equation of motion is described as follows [9]:

$$ \dot{x}(t) = A_x x(t) + B_x \delta_e(t) + E_x w_g(t), $$

(2)

$$ A_x = \begin{bmatrix} X_u & X_w & -W_0 & -g \cos \theta_0 & X_b \\ Z_u & Z_w & U_0 + Z_0 & -g \sin \theta_0 & Z_b \\ M_u & M_w & M_0 & 0 & M_b \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{\tau_e} \end{bmatrix}, $$

$$ B_x = \begin{bmatrix} 0 \\ 0 \\ E_c \\ 0 \\ \frac{1}{\tau_e} \end{bmatrix}, $$

$$ E_c = \begin{bmatrix} X_c \\ Z_c \\ \delta_c(t) \end{bmatrix}, $$

where $x(t)$, $\delta_e(t)$, and $w_g(t)$ are the state vector, the elevator deflection command, and the wind velocity in the $Z$-direction, respectively. Symbols $g$, $T_a$, $\theta_0$, $U_0$, and $W_0$ are the acceleration due to gravity, the actuator time constant approximated as the first-order lag, the steady equilibrium pitch angle, and velocities in the $X$- and $Z$-directions, respectively. Moreover, $U_0$ and $W_0$ are given by

$$ U_0 = V_0 \cos \sigma_0, \quad W_0 = V_0 \sin \sigma_0, $$

(3)

where $V_0$ and $\sigma_0$ are the equilibrium velocity and the equilibrium angle of attack, respectively. The state vector $x(t)$ is given by

$$ x(t) = \begin{bmatrix} u(t) \\ w(t) \\ q(t) \\ \theta(t) \\ \delta_e(t) \end{bmatrix}^T, $$

(4)

where $u(t)$, $w(t)$, $q(t)$, $\theta(t)$, and $\delta_e(t)$ are the velocity perturbations in the $X$- and $Z$-directions, the pitch rate, the pitch angle, and the elevator deflection, respectively.

Fig. 2 Body-fixed coordinate system of a large aircraft.

2.2 Estimation of Aerodynamic Derivatives

Expressions for all the dimensional aerodynamic derivatives appearing in Eq. (2) are summarized in Table 1. They are given by non-dimensional aerodynamic derivatives, which are summarized in Table 2, where we assume that $C_{\mu a} \approx 0$, $C_{\mu v} \approx 0$ [10, p. 79], and $C_{\mu q} \approx 0$ [10, p. 87].

If it is assumed that the aircraft is in a level flight, the lift coefficient $C_L$ is approximated as follows [10, p. 69]:

$$ C_L \approx \frac{2mg}{\rho U_0^2 V^2}. $$

(5)

The total drag coefficient is given by

$$ C_D = C_{DP} + \frac{C_L^2}{\pi eAR}, $$

(6)

where $C_{DP}$, $e$, and $AR$ are the parasite drag coefficient, the airplane efficiency factor, and the aspect ratio, respectively.

When the flight is in subsonic velocities, Mach effects may be neglected. Thus $\partial C_D/\partial M \approx 0$, where $M$ denotes the Mach number [10, p. 78]. For a jet-powered aircraft, we have $\partial T/\partial V = 0$ [10, p. 79]. Then, the non-dimensional derivative $C_{\mu u}$ is represented as follows:

$$ C_{\mu u} = -2(C_D + C_L \tan \theta_0). $$

(7)

The change in lift coefficient due to the elevator deflection is given by

$$ \frac{\partial C_L}{\partial \delta_e} = \frac{S_f}{S} (C_{\mu e})_{\text{full}} = \frac{S_f}{S} \alpha_l \tau, $$

(8)

where $\alpha_l$, $S$, $S_f$, and $\tau$ are the vehicle lift curve slope, the wing reference area, the tailplane reference area, and the tail efficiency factor, respectively [10, p. 84].

After substituting Eqs. (5) – (8) into Table 2 and Table 2 into Table 1, we have estimated the dimensional aerodynamic derivatives, which are summarized in Table 3.

2.3 Discrete-Time LPV Model

As shown in Table 3, the continuous-time LPV model is represented as follows:

$$ \dot{x}(t) = A_x x(t) + B_x \delta_e(t) + E_x w_g(t), $$

(9)

$$ A_x = \begin{bmatrix} a_{11}(pV, \rho_V^{-1} V^{-3}) & a_{12}(V) & a_{13}(V) \text{ const.} & a_{15}(pV^2) \\ a_{21}(V^{-1}) & a_{22}(pV, V^{-1}) & a_{23}(V) \text{ const.} & a_{25}(pV^2) \\ 0 & a_{32}(pV) & a_{33}(pV) & 0 & a_{35}(pV^2) \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{\tau_e} \end{bmatrix}, $$

$$ B_x = \begin{bmatrix} 0 \\ 0 \\ e_2(V^{-1}) \end{bmatrix}, $$

$$ E_c = \begin{bmatrix} e_1(V) \\ e_2(pV) \\ e_3(pV) \\ 0 \\ 0 \end{bmatrix}, $$

where $a_{11}(pV, \rho_V^{-1} V^{-3})$ means the element $a_{11}$ affinely depends on $pV$ and $\rho_V^{-1} V^{-3}$, which are scheduling parameters. The same is true with the other elements.

The discrete-time equivalent via a zero-order hold of sampling period $T$ is given by

$^1$ Almost similar to Table 3.2 in p. 70 of Ref. [10].

$^2$ Almost similar to Table 4.1 in p. 88 of Ref. [10].
TABLE 1 Dimensional aerodynamic derivatives.

| $X_p$ | $\rho U_0 S \left( C_{ar} + 2 C_L \tan \theta_0 \right)$ |
| $X_q$ | $\rho U_0 S \left( C_{ar} + 2 C_L \tan \alpha \tan \theta_0 \right)$ |
| $X_a$ | $\rho U_0 S C_{ar}$ |
| $Z_0$ | $\rho U_0 S \left( C_{ar} - 2 C_L \right) = -\frac{\rho U_0 S}{m} C_L$ |
| $Z_q$ | $\rho U_0 S \left( C_{ar} - 2 C_L \tan \alpha \right)$ |
| $Z_a$ | $\rho U_0 S \left( C_{ar} - 2 C_L \tan \theta_0 \right)$ |

The relation between coefficients $[\bullet \; \bullet]$ and $[\bullet \; \bullet \_]$ is given by $[\bullet \; \bullet \_] = [\bullet \; \bullet] / U_0$.

TABLE 2 Non-dimensional aerodynamic derivatives.

| $C_{sw}$ | $\frac{2 \alpha }{\rho U_0 S m} - 2(C_D + C_L \tan \theta_0) - \frac{M_0}{\rho A} \frac{\partial C_D}{\partial M}$ |
| $C_{ar}$ | $C_0 \left( 1 - 2 C_{ar} \frac{\rho U_0 S}{\rho A} \right)$ |
| $C_{st}$ | $\frac{\partial C_L}{\partial \alpha_0} \left[ \{ 10, pp. 77-78 \} \right]$ Eqns. (4.6), (4.9) |
| $C_{sw}$ | $-\frac{M_0}{\rho A} \frac{\partial C_L}{\partial M} \times 0$ |
| $C_{sa}$ | $-(C_{sa} + C_{sw}) = -C_{sw}$ [10, p. 80] |
| $C_{cd}$ | $\frac{\partial C_T}{\partial \alpha_0} \left[ \{ 10, pp. 77-78 \} \right]$ Eqns. (4.7), (4.9) |
| $C_{sw}$ | $C_{sw}(h_l - h_c)$ |
| $C_{sw}$ | $-2 \psi \frac{I_p}{m} \alpha_0, \quad V_{sw} = \left( \frac{1}{\rho S} \right) \left[ \{ 10, p. 85 \} \right]$ |

In the case of simply approximating the exponential part with the first order of the Taylor series, we have

\[
\exp(A_t) \approx I + A_t T, \quad (11)
\]

where we assume that $A_t$ is constant for a small control period of $T$. Then, Eq. (10) is approximated as follows.

\[
A_d \approx I + A_t T, \quad \theta_d = T \theta_c, \quad (12)
\]

Thus, the discrete-time LPV model in the case of the simple approximation is represented as follows:

\[
x(t + k) = A_d x(k) + B_d \theta_d(k) + E_d \theta_c(k) \quad (13)
\]

where $A_d, B_d, \text{ and } E_d$ are the discrete-time coefficient matrices. Note that Eq. (10) has the matrix exponential and the integral, which means that the dependence of the scheduling parameters on the coefficient matrices of the continuous-time LPV model may not be succeeded by using Eq. (10). The dependence structure of the continuous-time LPV model is succeeded by the discrete-time LPV model via the approximation of the exponential part with the first order of the Taylor series. However, the accuracy of discretization may not be enough for designing a GS controller based on the LPV control theory. In order to solve this problem, we approximate the dimensional aerodynamic derivative part and the actuator part with the different accuracy of discretization.
\[ a_{ss} = 1 - \frac{T}{T_a} + \frac{T^2}{2T_a^2} - \frac{T^3}{6T_a^3} + \cdots \]
\[ = \sum_{n=0}^{\infty} \frac{1}{n!} \left(-\frac{T}{T_a}\right)^n \quad (\text{const.}) \]  
(14)

The same is true with the element \( b_5 \) of \( B_d \):

\[ b_5 = 1 - \sum_{n=0}^{\infty} \frac{1}{n!} \left(-\frac{T}{T_a}\right)^n \quad (\text{const.}) \]  
(15)

After deducting the discrete-time equivalent of the continuous-time plant with some approximation from Eq. (12), the variation of the diagonal components may be neglected.$^3$

In addition, the variable elements depending on parameters \( V^{-1} \) and \( V^3 \) are much smaller or more negligible than those depending on parameters \( V \) and \( V^2 \). In this way, we have the following LPV model:

\[ x(k+1) = A_d(V, V^2) x(k) + B_d \delta_e(k) + E_d(V) w_g(k), \quad (16) \]

\[
A_d(V, V^2) = \begin{bmatrix} \text{const.} & \text{const.} & a_{13}(V) & a_{15}(V^2) \\ \text{const.} & a_{23}(V) & a_{25}(V^2) \\ 0 & a_{32}(V) & 0 & a_{35}(V^2) \\ 0 & 0 & T & 1 & 0 \\ 0 & 0 & 0 & 0 & a_{ss} \end{bmatrix},
\]

\[
B_d = \begin{bmatrix} \text{const.} \\ \text{const.} \\ \text{const.} \\ \text{const.} \\ \text{const.} \end{bmatrix}, \quad E_d(V) = \begin{bmatrix} E_1(V) \\ 0 \\ 0 \\ 0 \end{bmatrix},
\]

which only and affinely depends on parameters \( \rho, V, \) and \( V^2 \).

Note that we assumed the element \( e_2 \) of \( E_d \) is constant similar to the continuous-time linearized model in Eq. (2), where \( E_2 \) behaves similarly to the second column of \( A_c \). Regarding \( B_d \) except for \( b_5 \), more accurate Taylor expansion yields some values other than 0. Then we use the constant values of the mid point of the operating range.

If it is assumed that the aircraft is in a level flight, the atmospheric density \( \rho \) is constant. Thus, we have the following LPV model:

\[ x(k+1) = A_d(V, V^2) x(k) + B_d \delta_e(k) + E_d(V) w_g(k), \quad (17) \]

\[
A_d(V, V^2) = \begin{bmatrix} \text{const.} & \text{const.} & a_{13}(V) & a_{15}(V^2) \\ \text{const.} & a_{23}(V) & a_{25}(V^2) \\ 0 & a_{32}(V) & 0 & a_{35}(V^2) \\ 0 & 0 & T & 1 & 0 \\ 0 & 0 & 0 & 0 & a_{ss} \end{bmatrix},
\]

\[
B_d = \begin{bmatrix} \text{const.} \\ \text{const.} \\ \text{const.} \\ \text{const.} \\ \text{const.} \end{bmatrix}, \quad E_d(V) = \begin{bmatrix} E_1(V) \\ 0 \\ 0 \\ 0 \end{bmatrix},
\]

which only and affinely depends on parameters \( V \) and \( V^2 \). This LPV model is used for designing a GS controller.

$^3$ E.g., in the case of the element \( a_{11} = 1 + T a_{11} \), the variation due to the parameter \( a_{11}(V, \rho^{-1}(V^2)) \) can be neglected since \( T a_{11} << 1 \). The same is true with \( a_{22} \) and \( a_{33} \).

### 2.4 The Accuracy of the Proposed LPV Model

In this section, we plot the pole-zero map for confirming the accuracy of the proposed LPV model. The LPV model and the vertical acceleration of the aircraft \( a_e(k) \) are represented as

\[ x(k+1) = A_d(V, V^2) x(k) + B_d \delta_e(k) + E_d(V) w_g(k), \quad (18) \]

\[ a_e(k) = C_d x(k) + D_d \delta_e(k) + F_d w_g(k), \quad (19) \]

where \( a_e(k) \) is defined as upward positive. Symbols \( C_d, D_d, \) and \( F_d \) are set to the fixed values at \( M = 0.74 \) being the same as in [5].

The linearized discrete-time model at a certain trim point \( j \) is represented as follows:

\[ x(k+1) = A_{lon}^{(j)} x(k) + B_{lon}^{(j)} \delta_e(k) + E_{lon}^{(j)} w_g(k), \quad (20) \]

\[ a_e(k) = C_{lon}^{(j)} x(k) + D_{lon}^{(j)} \delta_e(k) + F_{lon}^{(j)} w_g(k), \quad (21) \]

where the trim conditions are given by the steady level flights of the Boeing 747 at an altitude of 30,000ft for the initial Mach numbers 0.6, 0.74, and 0.9.

Figure 3 shows comparison between the linearized model and the proposed LPV model in pole-zero maps of the transfer function from \( \delta_e(k) \) to \( a_e(k) \) for the three cruising speed described above. It shows that the proposed LPV model accurately represents the nonlinearity of the aircraft dynamics by only two scheduling parameters \( V \) and \( V^2 \).

### 2.5 Discrete-Time Preview Control

The basic idea of the discrete-time preview control is that an extended state equation is applied to the control performance condition such as \( H_2 \) performance. The extended state equation consists of both state equations of the state vector and the wind velocity data obtained as a prior information. The controller

![Comparison between the linearized model and the proposed LPV model in pole-zero maps for the three cruising speed.](image)
is given in the form of a state feedback. Variations can be expressed with only two parameters given by the velocity of the aircraft.

A wind velocity information is described as follows:

\[ x_w(k+1) = A_w x_w(k) + B_w w_g(k+1), \]  

The extended vector is given by

\[ x_e(k) = \begin{bmatrix} w_g(k) \\ w_g(k+1) \ldots w_g(k+h) \end{bmatrix}^T. \]  

The controlled output is described by

\[ z(k) = Q^2 a(k) \]  

so as to minimize the weighted quadratic sum of the vertical acceleration \( a(k) \) and the elevator command \( \delta_e(k) \).

From the above, the generalized plant is described as follows:

\[ x_g(k+1) = A_{g} x_g(k) + B_{g} u(k) + B_{w} w(k), \]

\[ z(k) = C x_g(k) + D u(k), \]

\[ A_{g} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \]

\[ B_{g} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}, \]

where \( h \) denotes the preview length, which may affect the resulting control performance, and

\[ x_e(k) = \begin{bmatrix} w_g(k) \\ w_g(k+1) \ldots w_g(k+h) \end{bmatrix}^T. \]

The extended vector is given by

\[ x_e(k) = \begin{bmatrix} x^T(k) \\ x^T_e(k) \end{bmatrix}^T. \]

The controlled output is described by

\[ z(k) = \begin{bmatrix} Q^2 a(k) \\ R^2 \delta_e(k) \end{bmatrix} \]

so as to minimize the weighted quadratic sum of the vertical acceleration \( a(k) \) and the elevator command \( \delta_e(k) \).

From the above, the generalized plant is described as follows:

\[ x_g(k+1) = A_{g} x_g(k) + B_{g} u(k) + B_{w} w(k), \]

\[ z(k) = C x_g(k) + D u(k), \]

\[ A_{g} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \]

\[ B_{g} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}, \]

\[ C = \begin{bmatrix} Q^2 C \delta_c(k) \\ Q^2 F \delta \end{bmatrix}, \]

where \( u(k) := \delta_e(k), \ w(k) := w_g(k+h+1) \).

and \( A_{g} \) is described in the following matrix polytope:

\[ A_{g} = \sum_{i=1}^{N} \xi_i A_i, \quad \sum_{i=1}^{N} \xi_i = 1, \quad \xi_i \geq 0, \]

where \( N \) is the number of the vertices. In general, a disturbance matrix \( E_{d} \) in Eq. (17) is assumed to be constant for the simplicity of designing a GS controller. However, the discrete-time preview control enables us to handle the scheduling parameters of \( E_{d} \) easily, since the extended matrix \( A_{g} \), which is used for designing and constructing the GS controller, includes not only \( A_d \) but also \( E_{d} \).

For this LPV model, we consider the following state-feedback controller:

\[ u(k) = K x_e(k). \]
hold for each \( i \), where the matrices \( X_i \) and \( L_i \) and the symmetric matrices \( P_i \) and \( W_i \) are variables.

Using the extreme controller \( K_i \), the generalized plant in each vertex \( i = 1, \ldots, N \), is described as follows:

\[
\begin{align*}
    x_a(k+1) &= (A_i + B_i K_i)x_a(k) + B_i w(k), \\
    z(k) &= (C + D K_i)x_a(k).
\end{align*}
\]

The GS controller is constructed by a convex combination of the extreme controllers:

\[
K(\lambda) = \sum_{i=1}^{N} \xi_i(\lambda) K_i, \quad (38)
\]

Here, the extreme controllers \( K_i \) may not meet the coalition LMIs condition \([12]\). Thus, the extreme controllers \( K_i \) need to be verified that the LMIs

\[
\begin{bmatrix}
    P_t (A_i + B_i K_i) G \\
    + G G^T - P_t
\end{bmatrix} > 0 \quad (39)
\]

are feasible for all vertices \( i = 1, \ldots, N \), which ensures the overall stability.

In Eq. (39), the matrix \( G \) and the symmetric matrices \( P_t \) are variables. If it is infeasible, some alternative ways are available. See Ref.[12] for the details.

### 4. Simulation

#### 4.1 Simulation Condition

The simulation is carried out in Matlab/Simulink environment around the trim conditions. The simulation condition is the same as in [4] and [5]. At an altitude of 30,000 ft, the range of Mach number is in \([0.6, 0.9]\) and the atmospheric density \( \rho = 0.45 \text{ kg/m}^3 \). The sampling and control period \( T = 100 \text{ ms} \), while the simulation sampling interval of the aircraft longitudinal motion is 10 ms. The weight matrices \( Q \) and \( R \) for the performance evaluation are set to 1 and 100, respectively. The reason why we more focus on reducing the magnitude of the control input is taking the limit of the elevator deflection into account. The preview length \( h \) is set to 11 (that is 1.1 s), which is fixed for any flight speed. Although the preview length can be increased as the flight speed is decreased, we fix it for simplicity while setting it to the value of the maximum speed. Figure 5 shows the time history of the assumed turbulence. This data was obtained by an actual observation while defined as upward positive \([13]\). The peak of this turbulence exists at around 40 s.

#### 4.2 Simulation of GS Control

Figures 6 to 8 show the time histories of the aircraft acceleration \( a_z \) in their upper figures and those of the commanded input \( \delta_{ec} \) in their lower ones at each initial Mach number of \( M = 0.6, 0.74, \) and \( 0.9 \). The data is magnified into 30 s to 60 s. The chain line shows the behavior of the conventional method of the Ref.[4], the dashed line the Ref.[5], and the solid line the proposed LPV model constructed by the triangle 1-3-5. Figure 9 shows the \( H_2 \) cost value of four methods in the cases of the initial Mach numbers of \( M = 0.6, 0.74, \) and \( 0.9 \). In the LPV model
in Ref. [5], only three large-variable elements were considered as independent scheduling parameters. On the other hand, in the proposed LPV model, all the variable elements are considered as functions of parameters $V$ and $V^2$. This fact suggests that the control performance is improved by using the proposed LPV model.

Figures 6 to 8 indicate that the gust alleviation preview control is sensitive to control inputs and relatively large inputs are required in order to attenuate the vertical acceleration. Especially, in the low speed range, larger inputs are needed since the effectiveness of the elevator is reduced as the dynamic pressure is reduced. This means that improving the $H_2$ cost value in the low speed range is more difficult than in the high speed range, which can be confirmed from Fig. 9.

4.3 Comparison between the Rectangle 1-2-3-4 and the Triangle 1-3-5

Figures 10 to 12 show the time histories of the aircraft acceleration $\ddot{z}$ in their upper figures and those of the commanded input $\delta_{ec}$ in their lower ones at each initial Mach number of $M = 0.6, 0.74, \text{ and } 0.9$. The dashed line shows the behavior of the rectangle 1-2-3-4 and the solid line the triangle 1-3-5. As shown in Fig. 9, the control performance of the triangle 1-3-5 at $M = 0.6$ and 0.74 is better than that of the rectangle 1-2-3-4. In our control design procedure which is known as the post-guaranteed LMIs [12], the extreme controllers $K_i$ are given by solving the LMIs at each vertex independently, which means that the extreme controllers $K_i$ of the rectangle and the triangle are the same as each other at the vertices numbered 1 and 3, respectively. Thus, the difference of the triangle from the rectangle is simply due to the introduction of the extreme controller at the vertex numbered 5 instead of those at the vertices numbered 2 and 3. This fact suggests that the control performance is significantly improved by constructing the tightest convex hull. On the other hand, the control performance is almost not improved at $M = 0.9$. This is explained as follows. The GS controller is designed for the range of Mach number [0.6, 0.9], but the airspeed of the aircraft increases from 272.8 m/s to about 280 m/s in turbulence, that is, the airspeed exceeds the range. In this case, we treat $K(\rho) = K_3$ in the simulation, which means that the same gain is used between the rectangle and the triangle. Thus, the $H_2$ costs and the behaviors of the rectangle and the triangle are the same at $M = 0.9$.

The deflection limit for the elevator of this aircraft is $-23$ deg to $17$ deg. From the time histories of the commanded input in Figs. 6 to 8 and 10 to 12, it is noted that this constraint has been satisfied in all the cases.

5. Conclusion

In this paper, we have obtained a discrete-time LPV model for a discrete-time GS controller via LMIs for a discrete-time
preview control to improve the control performance. The key point to achieve our goal is in both modeling accuracy of the LPV model with a small number of scheduling parameters and design of the advanced GS controller. In order to obtain such an LPV model, identifying the essential scheduling parameters for operating conditions is very important task. Through a series of approximations of the continuous-time aircraft model, we have identified $\rho$, $V$, and $V^2$ as such essential scheduling parameters. These parameters are related to the dynamic pressure which mainly changes as the flight condition changes.

The second contribution of this paper is to have established a discrete-time LPV model while keeping the discretization accuracy and succeeding the dependence structure of the continuous-time LPV model. The discrete-time equivalent via a zero-order hold has been obtained by approximating the dimensional aerodynamic derivative part and the actuator part with the different accuracy of discretization. The accuracy and the validity of the proposed discrete-time LPV model has been verified in the pole-zero map and in the simulation of the gust alleviation preview control.

The simulation result shows that the proposed control system effectively attenuates the aircraft vertical acceleration in turbulence and it is robust against cruising speed changes. By applying GS control and constructing the tightest convex hull to this LPV model, we confirmed that the proposed method has improved the control performance, compared with the conventional methods of Refs.[4] and [5]. Our future work is considering more realistic flight conditions and validating the proposed control system through some nonlinear simulations.

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**References**


