Chance-Constrained Optimization for Torque Tracking Control with Improving Fuel Economy in Spark-Ignition Engines

Xun Shen* and Tielong Shen*

Abstract: This paper proposes a control scheme based on chance-constrained optimization for spark-ignition engines to ensure transient torque tracking performance with improving thermal efficiency under chance-constraint for combustion phase. Firstly, the optimal equilibrium operating points are obtained by solving a chance-constrained optimization problem offline based on scenario approach. Then, linear quadratic regulator is applied to control the engine operate at the optimal equilibrium operating points under certain torque demand and engine speed. The proposed method is experimentally validated on a commercial gasoline engine, and the results demonstrate the method’s performance.

Key Words: chance-constrained optimization, spark-ignition engines, torque tracking control.

1. Introduction

A spark-ignition (SI) engine is a sophisticated system and control design for torque control as well as improving the fuel economy is a difficult task that has attracted considerable attention from the research community. To address engine control issues, many researches have been conducted various methods. In [1], a linear quadratic regulator (LQR) controller is proposed for engine speed and torque tracking control by using a simplified transfer function as an engine model. A robust controller based on the proportional integral (PI) and LQR control schemes is designed for torque control with a Smith predictor for tackling the delayed system as presented in [2]. For improving fuel economy, the solutions proposed in [3] and [4] are adjusting spark advance (SA) timing in real-time based on the extremum seeking method. Moreover, since the engine system is a physical plant with the limits on actuators, model predictive control (MPC) has also been investigated in many literature, such as torque tracking control [5], idle speed control [6], and indicated mean effective pressure (IMEP) tracking control [7].

However, the stochastic property of combustion phenomenon makes the engine control issues challenging. The cyclic variations in air-fuel mixture’s temperature before ignition and the air fuel ratio causes the variations in combustion processes even under the invariable control inputs [8]. The indicators for combustion process, such as combustion phase and thermal efficiency, are always with uncertainties. Thus, to deal with the two issues, achieving torque tracking control and fuel economy improvement, stochastic optimization framework is a feasible way to handle the uncertain parameters. Compared to deterministic optimization problems, due to the existence of uncertain parameters, it is more intractable on handling with the constraints in stochastic optimization problems since the constraints are infinite as the sample space has infinite samples which means that the uncertain parameters have infinite possible realization values [9]. Thus, the problem might be NP-hard [10]. Even the problem is solvable, the obtained solution which satisfies all the constraints is generally too conservative. The current state of the art for attacking a stochastic optimization problem is to loose the constraints of the original problem by reformulating a chance-constraint optimization (CCO) problem and solve it via the scenario approach [11].

In this paper, a control scheme based on chance-constraint optimization is proposed for SI engines to ensure transient torque tracking performance with improving thermal efficiency under chance-constraint for combustion phase. The chance-constrained optimization problem is solved offline to obtain the optimal equilibrium operating points based on the scenario approach. Then, an LQR controller is designed to achieve the optimal equilibrium operating point under certain torque demand and engine speed. Experimental validations are conducted finally to evaluate the proposed controller.

The rest of this paper is organized as follows. Section 2 gives problem description after a brief introduction for the torque generation mechanism and combustion process in a gasoline engine. Then, Section 3 firstly presents the general structure of control scheme. Also, the design for optimal operating points map and LQR are introduced. In Section 4, experimental results are demonstrated. Finally, Section 5 concludes this paper.

2. Background and Problem Description

2.1 Physical Background

As illustrated in Fig. 1, for the torque generation in a typical four-stroke gasoline engine, the fresh air is charged from the manifold into the cylinder and mixed with fuel. The fuel-air mixture is ignited, and the explosive power from the combustion will push the piston motion and create the torque for rotating the crankshaft. Two main factors effect the process of torque generation: the air charging dynamics in the intake manifold and the spark advance (SA) since torque generation relates to energy amount of fuel-air mixture and the state of combustion. The torque model is as follows:

\[ T_e(p_m, S_A, \omega) = h_1 p_m + h_2 S_A + \frac{h_3}{\omega} + h_4, \]  

\( T_e \) is the engine torque, \( p_m \) is the manifold pressure, \( S_A \) is the spark advance, and \( \omega \) is the engine speed. The terms \( h_1, h_2, h_3, h_4 \) are constants determined by the engine design.
where $p_{in}$ denotes the intake manifold pressure and $\omega$ is engine speed. The air charging dynamics in the intake manifold can be represented as [12]

$$\dot{p}_m = \frac{R_m T_m}{V_m} (\dot{m}_i - \dot{m}_o),$$

which is derived from the ideal gas law with the isothermal assumption. Moreover, $R_m$, $T_m$, and $V_m$ denote the ideal gas constant, the air temperature in the intake manifold, and the volume of intake manifold, respectively. Additionally, the air mass flow rates going into and out the manifold are denoted as $\dot{m}_i$ and $\dot{m}_o$. Thus, $p_m$ reflects the air charging dynamics and implies the amount of air charged into the cylinder, which also implies the fuel amount since the air-fuel ratio is controlled to constant stoichiometric proportion. Consequently, the amount of potential energy from combustion is decided. Moreover, $\dot{m}_i$ and $\dot{m}_o$ can be expressed as

$$\dot{m}_i = \gamma_1 \gamma,$$
$$\dot{m}_o = \gamma_2 p_m \omega,$$

where $\gamma$ is air-flow velocity, and $\gamma_1$ and $\gamma_2$ are constant decided by the structure of the engine.

### 2.2 Combustion Process in Gasoline Engine

The crank angle CA50 where 50% of fuel burnt is usually used as an indicator of the combustion phase. CA50 attracts so much attention because combustion phase affects thermal efficiency $\eta$ which is an important performance indicator of combustion quality. The CA50 for the highest thermal efficiency and stable combustion is among 8 to 12 [13]. Thus, the CA50 is always constrained into the interval from 8 to 12.

CA50 is directly affected by SA [14]. The effect of SA on combustion is shown in Fig. 2, which presents data from experiments implemented on the four-cylinder gasoline engine targeted in this research. In an SI engine, a complete cycle consists of intake, compression, expansion, and exhaust, totally four processes. Figure 2(a) (b), represents pressure profile without ignition, two pressure profiles with SA = 23 before top dead center (BTDC), the pressure profiles with SA = 29 BTDC. The corresponding heat release ratio profiles are shown in Fig. 2(c). Clearly, the combustion process will be retarded as the SA of the cycle is decreased, namely, CA50, the crank angle after top dead center (ATDC) when the burnt percent reach 50% is larger. However, due to the uncertainty of the combustion phenomenon, the effect of SA on the combustion process is not deterministic even at a fixed SA value [15], also as shown in Fig. 2, although the SA is the same, the curves are different due to the uncertainty in combustion process. A plot of a typical cycle-by-cycle history of CA50 recorded under certain operating condition (engine speed as 1200 rpm, manifold pressure as 56 kPa and SA as 15° BTDC) is shown in Fig. 3. The corresponding calculated histogram of CA50 measured data and fitted normal distribution is plotted in b) of Fig. 3. Moreover, as shown in Fig. 4, the causality of SA to the mean value of CA50 can be regarded as a linear regression. Thus, the SA-CA50 causality can be expressed as a linear regression coupled with a normally distributed random noise as follows [16]:

$$CA50 = a(p_{in})SA + b(p_{in}) + \mu, \mu \in N(0, \sigma^2).$$

Here, $a(p_{in})$ and $b(p_{in})$ are related to manifold pressure $p_{in}$ since the CA50’s mean value varies even with the same SA when $p_{in}$ changes as in Fig. 4. To keep the same CA50’s mean value, the adjustment of SA is necessary.

### 2.3 Problem Description

Figure 5 shows the surf map of thermal efficiency while Fig. 6 shows the contour map of torque and CA50. The purpose of the study is to fulfill the demanded torque with highest thermal efficiency under the chance constraint of CA50. Namely, when the demanded torque is decided, the feasible points of SA and $p_{in}$ will be a line as the lines with dark color in Fig. 6 under which the generated torque is the same. Besides, not all the points on a line with the same torque is feasible since CA50 should be constrained in the interval [6],[15]. The control design problem considered in this paper is to find a feedback torque tracking control law with high combustion quality in the sense of rejecting the stochastic variation of combustion phase by constraint of probability of CA50 hanging out a pre-specified range.
3. Control Design

3.1 General Structure of Control Scheme

Figure 7 illustrates the general framework of the proposed control strategy, which is essentially a two stage framework: set-point generation and on-line feedback control. The sensors capture the signals of intake manifold pressure $p_m$, engine speed $\omega$, and cylinder pressure. The engine torque $T_e$ cannot be measured directly in real-time, and the on-line estimation is thus provided by a calibrated map chart based on $\omega$ and $p_m$.

The formulation of the map is described as $T_e = C(\omega)p_m$, which is a linear regression of $p_m$ with the coefficient $C$ depending on $\omega$. CA50 and thermal efficiency $\eta$ are estimated from the measured cylinder pressure cycle by cycle. Besides, $T_e$, $\gamma$, $p_m$, $\omega$, CA50, and $\eta$ are taken as feedbacks to the proposed controller. The proposed controller consists with three blocks. Firstly, the set-point generation block outputs the optimal equilibrium point $(p_m^*, S A^*)$ by solving a stationary optimization with chance-constraint. The optimal spark advance timing $S A^*$ can be achieved directly to operate ignition timing as the command. While, for $p_m$, the classical linear quadratic regulator (LQR) is designed to accomplish the tracking control and generates demanded air-flow velocity $\gamma_d$ which is realized by a PI controller with throttle angle $\phi$ as control input.

3.2 Stationary Optimization with Chance-Constraint

3.2.1 Chance-constrained optimization

Chance-constrained optimization (CCO), also known as chance-constrained programming, has been pioneered by Charnes and Cooper [17] and developed by them and others to deal with programming under uncertainty. In [11], a general framework named “scenario approach” is proposed to solve CCO problems and apply it to robust control design. The general CCO problem can be formulated as

$$\mathcal{P}_1 : \min_{u \in \mathcal{U}} f(u)$$

s.t.

$$\Pr[\delta \in \Delta : g(u, \delta) \leq 0] \geq \alpha.$$  \hspace{1cm} (6)

Here, $u$ denotes the control variable vector, and $\mathcal{U} \subset \mathbb{R}^n$ is the admissible set for $u$, moreover, $\delta$ is an uncertain variable with sample space $\Delta$ and $\forall \delta \in \Delta$, $g(u, \delta) \leq 0$ is convex. An exact numerical solution of $\mathcal{P}_1$ is in general hopeless to be solved
Here, $C_{\text{min}} = 8$, $C_{\text{max}} = 12$, and $\alpha = 0.7$. The noise $\delta_{\text{CASO}} \in \Omega_{\text{CASO}}$ is the random noise of CASO where $\Omega_{\text{CASO}}$ represents the sample space of $\delta_{\text{CASO}}$. The constraint for CASO is chance-constraint. The problem $\mathcal{P}$ is solved based on the scenario approach mentioned in section 3.2.1 by choosing $N = 60$ which have confidence that the optimal solution of the scenario problem satisfies (7) with probability no smaller than 0.99. (Here, $n_u$ is 2. Then, $N_{\text{gen}}(\alpha, \beta) = N_{\text{gen}}(0.7, 0.01)$ is 59.9961.)

The problem is changed to a deterministic problem as follows:

$$
\mathcal{P} : \min_{J_0} J_0
$$

s.t.

$$
T_d(p_{\text{m}}, S_A, \omega) - T_d = 0,
$$

$$
P_{\text{min}} < p_m < P_{\text{max}},
$$

$$
S_{\text{A_{min}}} < S_A < S_{\text{A_{max}}},
$$

$$
P_{\text{C50}(p_m, S_A, \omega, \omega, \delta_{\text{CASO}})} - C_{\text{max}} \leq 0,
$$

$$
P_{\text{C50}(p_m, S_A, \omega, \omega, \delta_{\text{CASO}})} - C_{\text{min}} \leq 0
$$

$$
\forall i = [1, \ldots, N], \delta_{\text{CASO}} \in \Omega_{\text{CASO}}.
$$

The new deterministic problem can be solved by the ‘minsearch’ function in MATLAB.

3.3 LQ Tracking Controller Design

To achieve torque tracking, a linear quadratic regulator (LQR) is designed based on the linear dynamic discrete-time model:

$$
p_m(k + 1) = A p_m(k) + B y(k) / \omega(k),
$$

which is discretized by linear interpolation from (2), and the output is given as

$$
T_e(k) = C(\omega)p_m(k).
$$

The definitions of $A$ and $B$ are the parameters in the state space equation, $C(\omega)$ is the coefficient which is related to engine speed $\omega$ for estimation torque from manifold pressure $p_m$. The efficient $C(\omega)$ is calibrated as a map of $\omega$. Define $e = p_m - p_m^*$, $v = \gamma - \gamma^*$, then the error dynamics can be obtained:

$$
e(k + 1) = Ae(k) + Bv(k) / \omega(k),
$$

$$
\gamma(k + 1) = \gamma(k) + T_e(k) - T_d(k).
$$

Thus, the control problem is formulated as follows:

$$
\mathcal{P}_3 : \min_{J_0} J_0
$$

s.t.

$$
x_v(k + 1) = A_v x_v(k) + B_v v(k) / \omega(k)
$$

$$
+ B_0(T_e(k) - C(\omega)p_m^*(k)),
$$

where $Q$ and $R$ are the weighting coefficients, and other vectors and matrices are as follows:
According to the LQR optimal control theory, the final control law can be derived:
\[
\gamma_d(k) = -K(\omega)x_e(k) + \gamma'(k)
\] (33)
where \(K(\cdot) = -R^{-1}B_0^TP\), and here \(P\) is the solution of an algebraic Ricatti equation. It should be pointed out the proposed control scheme, while the closed-loop stability can be examined by checking whether the eigenvalues of \(A_e - B_eK\) are within in the unit circle.

4. Experimental Validation

4.1 Experimental Setup

The validating experiments were implemented on a state-of-the-art engine test bench as shown in Fig. 8 to evaluate the proposed control scheme. In this test bench, an L-4 type gasoline engine is equipped, and it is coupled with a low-inertia production electrical control unit (ECU) and an prototype controller, dSPACE1006, to realize the full control of the gasoline engine. The dSPACE1006 system can take over the function of engine control by sending control signals to ECU through the bypass interface.

4.2 Model Validation

In this study, the adopted model parameters are identified based on the experimental data. The accuracy of the model is firstly investigated in this section as the model precision is critical factor to the control performance. The parameters in (23) are \(A = 0.7045\) and \(B = 0.3771\). The parameters in (1)

\[
x_e = \begin{bmatrix} e \\ z \end{bmatrix},
\]
\[
A_e = \begin{bmatrix} A & 0 \\ T_zC(\omega) & 1 \end{bmatrix},
\]
\[
B_e = \begin{bmatrix} B \\ 0 \end{bmatrix},
\]
\[
B_0 = \begin{bmatrix} 0 \\ -T_s \end{bmatrix}.
\]

are \(h_1 = 12.53, h_2 = -0.116, h_3 = -12.28,\) and \(h_3 = -18.683\). Figure 9 illustrates a comparison result between the models (1), (5), (23) and the experimental data. It can be observed that the model has accurate dynamical behavior. Notice that, for thermal efficiency, there is a modeling error since we use the simple model only considering the linear regression part of \(p_m\) and the quadratic regression part of \(SA\). While, the influence of the error does not matter since the error along with \(p_m\) and \(SA\) can be almost regarded as a constant.

As mentioned above, the optimal equilibrium points corresponding to the engine operating condition which minimize the thermal efficiency can be obtained by offline solving the CCO problem \(\mathcal{P}\). The results of the optimal intake manifold pressure \(p_m\), corresponding air-flow velocity \(\gamma\) and spark advance \((SA)\) are shown in Fig. 10 a), b) and c). As a comparison, another map is obtained by solving the stationary optimization (SO):
\[
\mathcal{P}_{SO} : \min_{\{p_m, SA\}} (T_s(p_m, SA, \omega) - T_d)^2
\]
\[\text{s.t.}\]
\[
P_{\min} < p_m < P_{\max},
\]
\[
SA_{\min} < SA < SA_{\max},
\]
where \(SA_{\max}\) is calibrated as the boundary where there is no knock event and much more retarded than \(SA_{\max}\) in (15). The results are shown in Fig. 10 d), e) and f).

4.3 Experimental Analysis

To fully evaluate the effectiveness of the proposed control schemes, two experiments were conducted to compare the two methods: CCO and SO. The experiments were operated in the speed control mode of dynamo. In case 1, the engine speed follows a sinusoidal change from the 500th cycle to the 1500th cycle. While, in case 2, the engine speed follows a trapezoid change in the same range. The torque demand command consists of step, ramp, and sinusoidal components in order to verify the transient tracking performance. The same weighting coefficients have been chosen for both controllers in LQR feedback control. The control was cyclicly implemented. The overall control effects of the both controllers are compared in Fig. 11 and Fig. 12, respectively.

It can be obviously seen that both controllers can achieve the good tracking performance with the similar transient control performance. Compared to the SO based strategy, the CCO

<table>
<thead>
<tr>
<th>Engine system</th>
<th>Detail</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of cylinders</td>
<td>4-cylinder</td>
</tr>
<tr>
<td>Arrangement</td>
<td>L-type</td>
</tr>
<tr>
<td>Ignition system</td>
<td>DIS</td>
</tr>
<tr>
<td>Cylinder bore</td>
<td>80.5 mm</td>
</tr>
<tr>
<td>Stroke</td>
<td>88.3 mm</td>
</tr>
<tr>
<td>Crank radius</td>
<td>44.15 mm</td>
</tr>
<tr>
<td>Cylinder clearance volume</td>
<td>37.4375 ml</td>
</tr>
<tr>
<td>Cylinder Maximal volume</td>
<td>486.6875 ml</td>
</tr>
<tr>
<td>Compression ratio</td>
<td>13:1</td>
</tr>
<tr>
<td>Displacement</td>
<td>1797 ml</td>
</tr>
<tr>
<td>Maximum speed</td>
<td>6000 rpm</td>
</tr>
<tr>
<td>Maximum output</td>
<td>72 kW at 5200 rpm</td>
</tr>
<tr>
<td>Maximum torque</td>
<td>142 Nm at 3600 rpm</td>
</tr>
</tbody>
</table>

Table 1 Technical data of the proposed gasoline engine.
based strategy takes more advanced SA which improves thermal efficiency by 5.245% and 5.366% respectively in case 1 and case 2. On the other hand, for the CCO based strategy, the rate of $CA_{50} > 12$ is 12.81% and 12.61%, the rate of $CA_{50} < 8$ is 18.09% and 18.58%. Totally, the probability that $CA_{50}$ is between $C_{\min}$ and $C_{\max}$ \( \Pr \{CA_{50}(P_m, SA, \omega) \in [C_{\min}, C_{\max}] \} \) is 69.1% and 68.9%. While, for SO based strategy, the rate of $CA_{50} > 12$ is 45.36% and 57.14%, the rate of $CA_{50} < 8$ is 0.3% and 0.3%. Totally, the probability that $CA_{50}$ is between $C_{\min}$ and $C_{\max}$ \( \Pr \{CA_{50}(P_m, SA, \omega) \in [C_{\min}, C_{\max}] \} \) is 54.34% and 42.56%. These exhibit that the CCO based strategy can ensure tracking performance as well as gaining thermal efficiency improvement under chance-constraint of $CA_{50}$.

Obviously, from the result, the difference of SA between CCO and SO is that SA from CCO is advanced. But, the advanced magnitude is decided by the proposed CCO method which cannot be decided arbitrarily. The CCO method essentially proposes a reasonable way to advance SA which gains better fuel economy efficiency with acceptable risk of knock.

5. Conclusion

In this paper, the engine torque tracking with fuel economy optimization based on chance-constrained optimization is proposed. The distinctive features of the control scheme are concluded as follows.

Firstly, the set-point maps are designed by solving chance-constrained optimization problems through the scenario approach under different engine speeds and torque demands. The optimization problems are formulated to maximize the thermal efficiency under the equation constraint for torque and chance-constraint for $CA_{50}$.

Secondly, the linear quadratic regulator is designed to accomplish the engine operating on the set-points as well as torque.
tracking control.

In validating experiments, the proposed method was compared to another method whose only difference is the design of the set-point maps. In the SO based method, the set-point maps are designed without considering the thermal efficiency and CA50 constraint. The experimental results show the good transient tracking performances for both methods. While, the proposed method accomplished better thermal efficiency and satisfied CA50 constraint better than the SO based method. This work presented in this paper provides the practical example for chance-constrained optimization in SI engine control.

References

