A Control Structure for Unilateral System with Communication Rate Constraint

Hiroshi OKAJIMA *, Yuki MINAMI **, and Nobutomo MATSUNAGA *

Abstract: This study proposes a design method for unilateral control systems with communication rate constraints. In the case where it is necessary to control under a communication rate constraint, the effect of the quantization noise should be minimized using effective signal quantization methods. One of the effective methods for signal quantization is a feedback-type dynamic quantizer. We previously proposed a design method for a dynamic quantizer under communication rate constraints. In this paper, a unilateral control structure is proposed to minimize the effect of the quantization error. The design method for the quantizer is applied to the proposed structure. The effectiveness of the proposed system with the designed quantizer is assessed via numerical examples.

Key Words: unilateral control, networked control systems, signal quantization, quantizer.

1. Introduction

Networked control systems (NCSs) have been studied by several researchers [1]–[3]. Control signals need to be appropriately compressed using quantizers to satisfy the limited communication rate [4], [5]. Performance degradation is caused by the quantization because plants are controlled by compressed (quantized) signals.

Feedback-type dynamic quantizers are known to play an important role in minimizing performance degradation [6], [7]. These quantizers consist of a filter and a static quantizer and use previous quantization error values to generate the quantizer output. Such quantization methods are widely used in signal processing [6], such as AD-DA conversion systems and data compressors for music audio signals. Recently, feedback-type dynamic quantization methods have been studied in control engineering fields [7]–[10]. In the previous researches, one of key points is to choose an appropriate filter in dynamic quantizer to minimize the performance degradation. An optimal dynamic quantizer based on an \( \ell_\infty \) cost function was proposed in [7]. In case minimum-phase plants are given, the optimal quantizer was analytically expressed as a function of the plant parameters.

When we want to use the dynamic quantizers for NCSs, the output level number of dynamic quantizers needs to satisfy the limited communication rate. Thus, we have to design appropriately not only the filter but also quantization intervals in the static quantizer to satisfy the communication rate constraint. We have studied dynamic quantizers for the systems with communication rate constraints [11]–[13]. An analysis method of the quantization intervals that satisfy the communication rate constraint is addressed in [11]. Based on the analysis method design methods of the dynamic quantizers considering the admissible communication rate are presented in [12],[13]. The dynamic quantizers proposed by these researches are effective for various input signals which satisfy a designated signal range.

In this paper, we propose a control structure for unilateral control systems including the dynamic quantizer to overcome the adverse effect of the communication rate constraint. Unilateral control systems are a kind of networked control systems and have one communication channel. Unilateral control systems are useful for constructing the systems under communication rate constraints. We assume that the reference signal is given. We design not only the dynamic quantizer but also the filters in the control structure. Using the proposed control structure, the adverse effect of the quantization noise can be reduced. The effectiveness of the proposed structure is illustrated via numerical examples.

This paper is an extended version of our conference paper presented at the 6th IFAC Workshop on Distributed Estimation and Control in Networked Systems paper [14]. Simultaneous design of the quantizer and filter is discussed in detail.

In the remainder of the paper, the set of \( n \times m \) real matrices is denoted as \( \mathbb{R}^{n \times m} \). A notation \( \mathbb{R}_+ \) is the set of positive real numbers and \( I \) is the identity matrix. For a matrix \( H \), \( H^T \) and \( \rho(H) \) denote its transpose and spectral radius, respectively. A notation \( \|X\| \) represents the infinity norm of a vector \( X = [x_1, x_2, \ldots, x_n] \), namely, \( \|X\| = \sup_{\|x\|} \|x\| \) holds.

2. Problem Formulation

2.1 Control Systems with a Communication Channel

A single input single output discrete-time plant \( P \) is defined as

\[
P \left\{ \begin{array}{l}
x_p(t+1) = A_p x_p(t) + B_p u_p(t), \\
y_p(t) = C_p x_p(t),
\end{array} \right.
\]

(1)

where \( x_p \in \mathbb{R}^{n_p \times 1} \) is the state, \( u_p \in \mathbb{R} \) is the control input, and \( y_p \in \mathbb{R} \) is the plant output. In addition, \( A_p \in \mathbb{R}^{n_p \times n_p} \), \( B_p \in \mathbb{R}^{n_p \times 1} \) and \( C_p \in \mathbb{R}^{1 \times n_p} \) are coefficient matrices of \( P \), and \( x_p(0) \) is the initial state.

* Graduate School of Science and Technology, Kumamoto University, Kumamoto 860-8555, Japan.
** Graduate School of Engineering, Osaka University, 2-1 Yamadaoka, Suita, Osaka 565-0871, Japan.
E-mail: okajima@cs.kumamoto-u.ac.jp
(Received May 17, 2017)
(Revised January 12, 2018)
A desired control system is shown in Fig. 1. Figure 1 includes a feedforward controller $C_1$ and a feedback controller $C_2$. The controllers $C_i (i = 1, 2)$ are defined by the following state space realization:

$$
C_i \begin{cases}
    x_i(t+1) = A_i x_i(t) + B_i u_i(t), \\
    y_i(t) = C_i x_i(t) + D_i u_i(t),
\end{cases}
$$

(2)

where $x_i \in \mathbb{R}^{n_i \times 1}$ are the state, $u_i \in \mathbb{R}$ are the input of $C_i$ and $y_i \in \mathbb{R}$ are the output of $C_i$. Matrices $A_i \in \mathbb{R}^{n_i \times n_i}$, $B_i \in \mathbb{R}^{n_i \times 1}$, $D_i \in \mathbb{R}^{1 \times n_i}$ and $C_i \in \mathbb{R}^{1 \times n_{oi}}$ are given constant matrices of $C_i$ and $x_i(0)$ is the initial state. We assume that $C_1$ is stable and that $C_2$ stabilizes the system in Fig. 1.

Figure 2 shows the structure of a control system equipped with a communication channel, in which $u$ is a reference signal and $y$ is the plant output from Eq. (1). Symbols $K_P$ and $K_R$ denote controllers and $Q$ is a quantizer. The proposed structure for unilateral systems is proposed based on the feedforward system structure with pre-filter [15]. To provide more insight into the design problem, the control structure of $K_P$ is restricted as shown in Fig. 3. Then, the design problem is reduced to a problem of $K_P$ and $Q$.

The quantizer $Q$ transforms the high-resolution reference signal $u$ into a lower resolution signal $\tilde{u}$. It is assumed that there is no delay or no packet-loss in the communication channel. The controller $K_R$ and the quantizer $Q$ should be designed to minimize the difference between the desired system in Fig. 1 and the system in Fig. 2 in terms of the input-output relationship.

The number of quantization levels $N$ depends on the communication rate of the channel. When $M$ bits of data are transmitted through the channel over the sampling period of the control system, $N$ needs to satisfy the following inequality:

$$N \leq 2^M.
$$

(3)

$N$ is assumed to be even in this study.

![Fig. 1 Desired control system.](image)

![Fig. 2 Unilateral control system with quantizer Q.](image)

The reference signal $u$ is assumed to be given in a range $t = 0, \ldots, T$, where $T$ is the terminal value of $t$. It is assumed that $u$ is constrained by the upper and lower boundaries, resulting in the signal range $U = [u_{\min}, u_{\max}]$. Therefore, the reference signal $u$ is assumed to satisfy the following relationship:

$$u(t) \in U \ \forall t \in [0, \ldots, T].
$$

(4)

The reference signal $u(t)$ is assumed to be given as a specific signal in this paper. In many actual control systems, the frequency characteristics of the signal is roughly determined. Therefore, our design problem can be applied to control systems with various reference signals when we adequately select the test signal $u(t)$.

### 2.2 Dynamic Quantizer Form

A feedback-type dynamic quantizer is defined as follows:

$$Q \begin{cases}
    \xi(t+1) = A \xi(t) - B \tilde{u}(t) + B \tilde{u}(t), \\
    \tilde{u}(t) = Q_{st}\{C \xi(t) + \tilde{u}(t)\},
\end{cases}
$$

(5)

where $Q_{st}$ is a saturated uniform static quantizer, and $\tilde{u}$ and $\tilde{u}$ are the input and output of the quantizer, respectively. Furthermore, $A \in \mathbb{R}^{8 \times 8}$, $B \in \mathbb{R}^{8 \times 1}$ and $C \in \mathbb{R}^{1 \times 8}$ are constant matrices of the feedback-type dynamic quantizer $Q$. The initial state is assumed to be $\xi(0) = 0$. The number of quantization levels is given as $N$ in $Q_{st}$. The quantizer $Q_{st}$ is defined using a quantization interval $d \in \mathbb{R}$, and a center point $h \in \mathbb{R}$. The length of the intervals are the same for the input and output axes. Figure 4 shows an example of $Q_{st}$ (solid line, $N = 8$, mid-riser-type uniform static quantizer). The controllable canonical form is assumed for matrices $A$, $B$, and $C$. For example, $A$, $B$, and $C$ are given as follows in case $n_q = 2$:

$$
\begin{pmatrix}
    A \\ C
\end{pmatrix} = 
\begin{pmatrix}
    0 & 1 \\ \phi_2 & \phi_3
\end{pmatrix}
\begin{pmatrix}
    0 \\ 1
\end{pmatrix}
$$

Then, $\eta = [\phi_1, \phi_2, \phi_3, \phi_4]$ is a design parameter vector for the dynamic quantizer. In general, the design parameter vector is given by the form $\eta = [\phi_1, \ldots, \phi_{2n_q}]$.

### 2.3 Control Objective

In this paper, we attempt to design $K_R$ and $Q$ such that $y_q$ in Fig. 2 approximates $y$ in Fig. 1. Consequently, we evaluate the
error signal $e = y_q - y$, and consider the following evaluation function:

$$E(Q, K_R) = ||Y_q - Y||,$$  \hspace{1cm} (6)

where $Y = \{y(1), y(2), \ldots, y(T + 1)\}$ and $Y_q = \{y_q(1), y_q(2), \ldots, y_q(T + 1)\}$ are the output time series. Because $E(Q, K_R)$ produces the maximum value for $e(t)$, $y_q$ is expected to be similar to $y$ when $E(Q, K_R)$ becomes smaller. In existing dynamic quantizer designs [7]–[9], $E(Q)$ with fixed $K_R$ is used as a performance index for these quantizers.

The objective of this study is to design the filter $K_R$ and the quantizer $Q$ on the basis of the performance index Eq. (6).

3. Idea for Designing $K_R$ in Unilateral Control System

In this paper, $K_R$ and $Q$ are designed to minimize $E(Q, K_R)$ in Eq. (6). This study considers the case that both $Q$ and $K_R$ are design parameters. When $K_R$ is given, the output of $K_R$ can be regarded as a reference signal $u$. Then, the following control system $\bar{G}$ is obtained:

$$\bar{G} = \frac{PC_2}{1 + PC_2} C_1 K_R^{-1}. \hspace{1cm} (7)$$

The quantizer $Q$ can be designed if $\bar{G}$ and the input $\bar{u} = K_R u$ are given. Therefore, the objective is to find $K_R$ which minimizes $E(Q, K_R, K_R)$. Figure 5 shows the proposed structure and its evaluation system. We can see that Fig. 5 is similar to the system shown in Fig. 6 which is shown in [13]. If we can find the signal range of $\bar{u}$, we can design $Q$ by the design method in [13] with the range of $\bar{u}, N$, and $G$.

Here, $K_R$ should be designed to minimize the effect of quantization noise. When $K_R$ is designed as a high-pass filter, $\bar{G}$ has low-pass characteristics, which are effective in minimizing the quantization noise. The form of $K_R$ is assumed to be given as follows:

$$K_R(z) = \frac{r_1 z^{-u} + r_2 z^{-2u-1} + \cdots + r_{2u}}{z^{2u} + r_{u+1} z^{-2u+1} + \cdots + r_{2u+1}}, \hspace{1cm} (8)$$

where $\zeta = [r_1, \ldots, r_{2u}]$ is a design parameter vector of the filter $K_R$. The vector $\zeta$ should be given to satisfy $K_R$ as a stable and minimum-phase transfer function.

The signal $\bar{u} := K_R u$ needs to satisfy the following conditions:

$$\bar{u} \in [\bar{u}_{\min}, \bar{u}_{\max}], \hspace{1cm} (9)$$

$$\bar{u}_{\max} - \bar{u}_{\min} \leq u_{\max} - u_{\min}. \hspace{1cm} (10)$$

Values $\bar{u}_{\min}$ and $\bar{u}_{\max}$ are the minimum and maximum values of the signal $\bar{u}(t)$. By using $\bar{u}_{\min}$ and $\bar{u}_{\max}$, $h$ can be defined by

$$h = \frac{\bar{u}_{\max} + \bar{u}_{\min}}{2}. \hspace{1cm} (11)$$

A coefficient $r_0$ in Eq. (8) should be selected to satisfy Eq. (10). First, assume that $\zeta$ is given. Then, we define $K_R$ by Eq. (8) with $r_0 = 1$ and $\zeta$. By calculating $\bar{u} = K_R \bar{u}$, we can obtain the maximum and minimum values of $\bar{u}$. These values are defined as $\bar{u}_{\max}$ and $\bar{u}_{\min}$, respectively. Then, $r_0$ is set as follows:

$$r_0 = \frac{u_{\max} - u_{\min}}{\bar{u}_{\max} - \bar{u}_{\min}}. \hspace{1cm} (12)$$

We can regard $r_0$ as a function of $\zeta$. By using $K_R$ with Eq. (12), $\bar{u}_{\max} - \bar{u}_{\min} = u_{\max} - u_{\min}$ holds. Therefore, the constraint in Eq. (10) is satisfied by the designed $K_R$.

4. Design of Dynamic Quantizer under Communication Rate Constraint

4.1 Condition to Satisfy Communication Rate Constraint

In our previous studies, $Q$ was designed to minimize $E(Q)$ under the communication rate constraint [13]. The following inequalities hold for any $u(t)$:

$$-Nd/2 + h \leq CE(t) + u(t) \leq Nd/2 + h. \hspace{1cm} (13)$$

The range of $C\hat{e}(t) + u(t)$ depends not only on $u(t)$ but also on the filter parameters $A$, $B$, $C$, and $d$. If $d$ is given as a large number, Eq. (13) can be satisfied, but the control performance is worse. However, Eq. (13) cannot be satisfied when $d$ is too small. Analysis of the smallest $d$ for the matrices $A$, $B$, and $C$ has been discussed in [16].

First, assuming that the quantiser parameter vector $\eta$ is given, an analytical method is developed for the communication rate constraints along with a derivation of the minimum quantization interval. The problem of satisfying Eq. (13) is formulated as follows [13]:

[Problem] Consider the following state equation:

$$x_\xi(t + 1) = (A + BC)x_\xi(t) + Bu(t), \hspace{1cm} (14)$$

$$y_\xi(t) = Cx_\xi(t), \hspace{1cm} \|w\| \leq 1, \hspace{1cm} (15)$$

where $x_\xi(0) = 0$. Find $\psi(> 0)$ that satisfies the inequality condition

$$-\psi \leq y_\xi(t) \leq \psi \forall x_\xi(t) \in \Xi, \hspace{1cm} (16)$$

where $\Xi$ is the reachable set of $x_\xi$ for $w$. □

Moreover, using the optimal solution $\psi_{opt}$ of the minimization problem of $\psi$ under the condition given in [Problem], the minimum quantization interval $d_{opt}$ is obtained using the following theorem [13]:

[Theorem] Using the optimal solution $\psi_{opt}$, the minimum quantization interval $d_{opt}$ and $h_{opt}$ are expressed as

$$d_{opt} = \frac{u_{\max} - u_{\min}}{N - \psi_{opt}}, \hspace{1cm} h_{opt} = \frac{u_{\max} + u_{\min}}{2}. \hspace{1cm} (17)$$

If $N - \psi_{opt} \leq 0$, no $d$ can satisfy the condition regarding the permissible number of quantization levels. □
The value $\psi_{opt}$ which characterizes the signal amplitude caused by the dynamic quantization is obtained using the quantizer parameter vector $\eta$. If $\psi_{opt}$ is small, $d^{opt}$ is also set to a small value. In contrast, a large $d^{opt}$ is chosen when $\psi_{opt}$ is large. In particular, when $N - \psi_{opt} \leq 0$, the communication rate constraint is not satisfied for any $d$. In such a case, $\eta$ needs to be re-designed. The amplitude of $\psi_{opt}$ is regarded as an index of the usability of the dynamic quantizer for signal communication, providing valuable information about the construction of NCS.

4.2 Dynamic Quantizer Design

Based on this analysis method, the design problem of $Q$ has been addressed in [13] as shown in Fig. 6. The design problem corresponds to minimizing $E(Q)$ in (6) for the case of $K_R = 1$:

$$E(Q) = \sup_{u \in U} |Y_q - Y|.$$  (18)

The parameters of $Q$ are designed for a given system $G = PC_1C_2/(1 + PC_2)$. The design method is given as follows. Control performance depends on $N$ and $G(z)$. Based on the evaluation system in Fig. 6, design methods of $Q$ using [Theorem] has been provided in [13].

In this section, the dynamic quantizers are designed using a two-step design method [13]. An iterative design method based on an invarient set analysis [11],[12] and a particle swarm optimization method (PSO) [17] are used together to obtain a quantizer for minimizing Eq. (18). PSO is a type of optimization method based on swarm behavior and requires many particles to represent the candidate for the quantizer parameters. We denote the positions of particles in PSO as $p_i$ and the velocities, which are used in the PSO algorithm, as $\Delta p_i$. The number of particles in the PSO algorithm is defined as $m$. In the standard PSO design, the initial positions and velocities of particles are randomly provided. In contrast, the quantizer parameters are designed using an iterative design method as the first step in the proposed design method. Thus the quantizers obtained via the iterative method are used as part of the initial positions of particles in the PSO algorithm. It is expected that the dynamic quantizers, which achieve good performance, can be obtained using the two-step design method.

5. Simultaneous Design of $K_R$ and $Q$

This paper studies the design methods of $K_R$ and $Q$. The PSO algorithm is a flexible method to design parameters, and it is easy to extend the design method of [13] to simultaneously design $K_R$ and $Q$. In the PSO algorithm, a fixed value is obtained and the evaluation function value is calculated to update the new parameters. Vectors $p_i = \{\eta_i, \xi_i\}, i = 1, \ldots, m$ are candidates of the solution in the PSO method. The PSO algorithm is solved using the similar manner of [13]. The number of design parameters are larger than the case of [13] because not only $Q$ but also $K_R$ should be designed.

In this section, a concrete design procedure to determine the design parameters $p$, which minimize (18), is presented. At first, we describe the conventional particle swarm optimization algorithm which is a kind of the optimization method based on the swarm behavior [17]. The following minimization problem is considered in this section.

[Problem 2] Find $p_{opt}$ satisfying the following conditions:

$$p_{opt} = \arg \min_{p \in \mathbb{R}^n} E(p)$$  (19)

subject to $\psi_{opt}(p) < N, Q$ is stable,  (20)

where $E: \mathbb{R}^n \rightarrow \mathbb{R}$ is the objective function and $p = \{\eta, \xi\}$ is the design variable vector. The inequality $\psi_{opt}(p) < N$ is a design condition stated in [Theorem]. The value $\psi_{opt}(p)$ can be calculated by the recursive method in [13]. The quantizer $Q$ is stable if and only if the eigenvalues of the matrix $A + BC$ are inside the unit circle in the complex plane. We find the solution of Problem 2 by using the following PSO method.

Multiple particles $p_1, \ldots, p_m$ are used in the PSO algorithm. To solve (19) under the constraints (20), the following objective function $E_j(p)$ is assumed to be given:

$$E_j = \left\{ \begin{array}{ll} E(p) & (\psi_{opt}(p) < N, Q \text{ is stable}), \\ E_{pen} + E(p) & (\text{otherwise}). \end{array} \right.$$  (21)

The penalty $E_{pen}$ is the larger positive value compared to the value of $E(p)$ which is an acceptable solution. The position and the velocity of the $i$th particle are denoted as $p_i = \{\eta_i, \xi_i\}$ and $\Delta p_i = \{\Delta \eta_i, \Delta \xi_i\}$, respectively. The particle $p_i$ is updated based on the following update laws:

$$p_{i+1}^{k+1} = p_i^k + \Delta p_i^{k+1},$$  (21)

$$\Delta p_i^{k+1} = \omega_\Delta \Delta p_i^k + \omega_\gamma \text{rand}^k_i (p_{gbest}^k - p_i^k) + \omega_\varepsilon \text{rand}^k_i (p_{best}^k - p_i^k)$$  (22)

The value $k$ denotes the iteration number, and its initial value is $k = 0$. Values $\omega_\Delta, \omega_\gamma, \omega_\varepsilon$ and $\text{rand}_i^k$ are selected in the range $[0,1]$. In (22), $p_{gbest}^k$ means the personal best solution which is determined by the following statement:

$$p_{gbest}^k := \arg \min_{p \in \mathbb{R}^{n+1}} E_j(p),$$  (23)

The vector $p_{gbest}^k$ means the global best solution which is determined by the following statement:

$$p_{gbest}^k := \arg \min_{p \in \mathbb{R}^{n+1}} E_j(p).$$  (24)

The PSO algorithm for the dynamic quantizer design is given as the following steps:

PSO algorithm

- **Step2-1:** Set $k = 0$. Select initial parameters $p_i^0$ and its parameter velocities $\Delta p_i^0$ randomly for $i = 1, \ldots, m$. Then, evaluate the corresponding objective function value at each initial parameter $p_i^0$. If $K_R$ with $\xi_i^0$ in $p_i^0$ does not become a stable and minimum-phase transfer function, we re-select $\xi_i^0$.

- **Step2-2:** Select numbers $s_i, \ldots, s_f$ from $i = 1, \ldots, m$. Update $\eta_i^k, j = 1, \ldots, f$ using iterative method in [13]. Where $f$ is defined as a small number rather than $m$.

- **Step2-3:** Update $p_{gbest}^k$ and $p_{gbest}^k$ by (23) and (24), respectively. Then, apply the update laws (21), (22) for all particles, and go to Step2-4. If $K_R$ with $\xi_i^{k+1}$ does not become a stable and the minimum-phase transfer function, $\xi_i^{k+1} = \xi_i^k + \gamma \Delta \xi_i^k$ is used to update $\xi_i^{k+1}$. A parameter $\gamma (0 \leq \gamma < 1)$ is selected to satisfy the stability condition and the minimum-phase condition for the transfer function based on the stability criterion of the discrete time systems [18].


The discrete time transfer function \( \text{KR} \) is given as a high pass transfer function as shown in Fig. 10. The modified signal \( \tilde{u} \) with Eq. (27), satisfies \( \tilde{u} \) ∈ \([13]\) using the method in \([13]\). Figure 9 is the reference and quantized output from \( Q_{\text{prev}} \). The quantizer’s output \( u_q \) takes only two values, and it can satisfy communication rate constraint.

In contrast, \( K_R \) and \( Q \) are derived by using the proposed PSO algorithm in Section 5. The condition parameters \( m \) and \( k_{\text{max}} \) for the PSO algorithm are given as 500 and 200, respectively. The parameters in update law \( \omega_0 = 0.9 \) and \( \omega_1 = \omega_2 = 1 \) are selected for the PSO algorithm. The transfer function of the designed controller \( K_R \) by the proposed method is given as follows:

\[
\text{Step2-4: Evaluate all position } p^i_k \text{ by (21). Set } k = k+1 \text{ and go to Step2-3 if } k < k_{\text{max}}. \text{ Else, update } p^i_{\text{best},j} \text{ and } p^i_{\text{best},j}. \]

The vector \( p^i_{\text{best},j} \) is the designed parameter. For the given \( p^i_{\text{best},j} \), \( \psi^{opt} \) is calculated based on \([13]\) and the quantization interval \( d^{opt} \) is derived using \( \psi^{opt} \).

The cost value \( \zeta \) of two values, and it can satisfy communication rate constraint. For the PSO algorithm are given as 500 and 200, respectively. A quantizer \( Q_{\text{prev}} \) is used as a reference and quantization interval \( d^{opt} \) is derived using \( \psi^{opt} \).

Because \( \zeta^i_{k+1} \) satisfies the conditions, there exists at least one solution for \( \zeta^i_{k+1} \) when we choose \( \gamma = 0 \). The parameter \( \gamma \) is selected to satisfy Eq. (25) and Eq. (26).

The PSO algorithm is simple and makes no assumption about the vector \( \text{KR} \) is given as follows:

\[
\begin{align*}
-1 &\leq r_2 \leq 1, -1 - r_2 \leq r_1 \leq 1 + r_2, \\
-1 &\leq r_4 \leq 1, -1 - r_4 \leq r_3 \leq 1 + r_4.
\end{align*}
\]

Because \( \zeta^i_{k+1} \) satisfies the conditions, there exists at least one solution for \( \zeta^i_{k+1} \) when we choose \( \gamma = 0 \). The parameter \( \gamma \) is selected to satisfy Eq. (25) and Eq. (26).

The effective of the proposed method is evaluated by illustrating numerical examples. The input signal \( u(t) \) is given by the following equation:

\[
u(t) = 0.6 \sin(0.025t) - 0.25 \sin(0.055t) + 0.15 \sin(0.2t).
\]

(27)

The signal \( u(t) \) satisfies \( u(t) \in U = [-1, 1] \) for all \( t \). The terminal value is given as \( T = 300 \). The communication rate constraint is characterized by \( N = 2 \). Plant parameters are defined as

\[
G(z) = \frac{0.3535z - 0.3362}{z^2 - 1.724z + 0.7408}.
\]

(28)

The discrete time transfer function \( G \) is derived using \( G(s) = (4s + 2)/(s^2 + 3s + 2) \) with a sampling time \( \Delta = 0.1 \). As a target for comparison, A quantizer \( Q_{\text{prev}} \) is designed for \( G \) using the method in \([13]\), which corresponds to the case of \( K_R = 1 \). \( Q_{\text{prev}} \) is given as follows:

\[
Q_{\text{prev}} = \begin{pmatrix} A_{\text{prev}} & B_{\text{prev}} & C_{\text{prev}} & D_{\text{prev}} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 1 \\ -0.4004 & 1.2710 & 0.3855 & -0.7669 & 1.637 \end{pmatrix}.
\]

The cost value \( E(Q_{\text{prev}}) = 0.3114 \) is obtained for \( Q_{\text{prev}} \). Simulation results of \([13]\) are shown in Figs. 7–9. Figure 7 shows the outputs \( y \) and \( y_q \) illustrated in Fig. 6. Figure 8 shows error between \( y \) and \( y_q \). We can see that the maximum absolute value of \( e(t) \) is smaller than \( E(Q_{\text{prev}}) \). Figure 9 is the reference and quantized output from \( Q_{\text{prev}} \). The quantizer’s output \( u_q \) takes only two values, and it can satisfy communication rate constraint.

In contrast, \( K_R \) and \( Q \) are derived by using the proposed PSO algorithm in Section 5. The condition parameters \( m \) and \( k_{\text{max}} \) for the PSO algorithm are given as 500 and 200, respectively. The parameters in update law \( \omega_0 = 0.9 \) and \( \omega_1 = \omega_2 = 1 \) are selected for the PSO algorithm. The transfer function of the designed controller \( K_R \) by the proposed method is given as follows:

\[
K_R = \frac{27.39z^2 - 19.93z - 6.628}{z^2 + 0.4631z + 0.5872}.
\]

(29)

The poles and zeros of \( K_R \) are \(-0.2315 \pm 0.7304i \) and \( 0.9756, -0.2480 \), respectively, and we can see that \( K_R \) is given as a stable and minimum-phase transfer function. The optimal controller \( K_R \) is obtained as a high pass transfer function as shown in Fig. 10. The modified signal \( \tilde{u} \), which is the output of \( K_R \) with Eq. (27), satisfies \( \tilde{u} \in U \) for all \( t \). The transfer function \( G \) is given using \( K_R \) as follows:

\[
G = GK_R^{-1} = \frac{0.3535z^2 - 0.1725z^2 + 0.05186z - 0.1974}{27.39z^2 - 67.14z^2 + 48.01z^2 - 3.341z - 4.91}.
\]

A dynamic quantizer for the proposed method \( Q_{\text{proposed}} \) is given as follows:
Fig. 10 Gain diagram of $K_R$.

Fig. 11 $G$ and $G K_R^{-1}$.

Fig. 12 Outputs $y$ and $y_q$ using proposed method.

$$Q_{proposed} = \begin{pmatrix} A & B \\ C & d \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0.2308 & 0.7306 & 1 \\ -0.2355 & -0.9709 & 1.9604 \end{pmatrix}.$$ 

By using obtained $K_R$ and $Q$, $E(Q_{proposed}) = 0.0951$ is obtained. We can find that $E(Q_{proposed})$ is smaller than $E(Q_{prev})$. We can see from Fig. 11 that the high frequency part of the quantization noise, caused by $Q_{proposed}$, is removed effectively by $G K_R^{-1}$ (solid line) because the gain of $G K_R^{-1}$ is smaller than that of $G$ in the high frequency domain.

The simulation results for designed $K_R$ and $Q_{proposed}$ are shown in Figs. 12–15. In Fig. 12, $y(t)$ is the desired output (thin line) and $y_q(t)$ is the actual output using $Q_{proposed}$ (thick line). Both outputs are similar. The error signal is shown in Fig. 13 and satisfies $E(Q_{proposed})$. Figure 14 shows the output of the quantizer $Q_{proposed}$ and indicates that the communication rate constraint is satisfied by the designed $Q_{proposed}$ and $K_R$. Figure 15 shows the filtered signal $\tilde{u}$. We can find that the error signal of the proposed method is drastically improved compared to the case with [13]. The reason why the error signal is drastically minimized by the proposed method is that the signal-to-noise (S/N) ratio in the high frequency part can be improved by $K_R$. It is expected that the improvement rate from [13] depends on the frequency characteristics of the reference signal.

7. Conclusion

Here we propose a structure for unilateral control systems under communication rate constraints. Using the performance index, $Q$ and $K_R$ are designed on the basis of the communication rate constraint. The effectiveness of the proposed control structure and design method is illustrated via numerical examples.

In this paper, we assume a design method for a specific refer-
ence signal $u(t)$. Our method can be extended to the case where a set of reference signals $u_i(t), i = 1, \ldots, n$ are given when $r_0$ is well designed by setting an adequate margin in $K_R$.

This work was supported by JSPS KAKENHI Grant-in-Aid for Scientific Research (C) 16K06419.

References


---

**Hiroshi OKAJIMA** (Member)

He received his M.E. and Ph.D. degrees from Osaka University, Japan, in 2004 and 2007, respectively. He is presently an associate professor of Kumamoto University, Japan. His research interests include tracking control, analysis of non-minimum-phase systems, and data quantization for networked systems. He is a member of ISCIIE and IEEE.

**Yuki MINAMI** (Member)

He received the M.S. and Ph.D. degrees in informatics from Kyoto University, Uji, Japan, in 2007 and 2009, respectively. From 2008 to 2009, he was a research fellow of the Japan Society for the Promotion of Science at Kyoto University. He is currently an Associate Professor in Osaka University, Suita, Japan. His current research interests include quantized control and control of mechanical systems. He is a member of JSME, ISCIIE and IEEE.

**Nobutomo MATSUNAGA** (Member)

He received M.D. and Ph.D. degrees from Kumamoto University, Japan, in 1987 and 1993, respectively. He joined OMRON (Corp.) in 1987. Since 2002, he has been with the Department of Computer Science and Electrical Engineering, Kumamoto University, where he is a Professor. His research interests include thermal process control, automotive control, and human-machine system design. He is a member of IEEJ and IEEE.