Generalized Minimum Variance Control for MIMO System with Multiple Sampling Periods

Satoshi BESSHO*, Masayoshi DOI**, and Yasuchika MORI*

Abstract: This paper proposes a new control technique for Multi-Inputs Multi-Outputs (MIMO) systems with multiple time delays. The control design method is the Generalized Minimum Variance Control (GMVC). As for MIMO systems, it is assumed that time constants are different greatly each other. In this case, setting a suitable sampling period in every control loop, we can reduce computational complexity very much keeping the effective performance.

Key Words: generalized minimum variance control, MIMO system, multi sampling periods, modified z transform.

1. Introduction

There are a lot of MIMO systems in the chemical plants of the manufacturing, which contain various time delays [1]. The Self Tuning Control (STC) is used in the field of the process control, and GMVC is one of the design methods in STC [2]. The GMVC achieves good control performances in the case where time delays exist, and can handle unstable poles at the origin [3]. Furthermore, GMVC rejects model errors and load disturbances with pre-compensation which is based on the internal model principle. Generally, MIMO systems contain various time delays, but the conventional GMVC can be applied to MIMO systems with only one constant time delay. This study proposes an extension of GMVC, which can be applied to the plant with the various time delays. The proposed GMVC reduces a computational stress and maintains the effective control performance.

2. Problem Statement

The plant model is stable and the n-inputs and n-outputs system [4]. This system expresses as the following transfer function.

\[
G(s) = \begin{bmatrix}
G_{11}(s)e^{-L_{11}s} & \cdots & G_{1n}(s)e^{-L_{1n}s} \\
\vdots & \ddots & \vdots \\
G_{n1}(s)e^{-L_{n1}s} & \cdots & G_{nn}(s)e^{-L_{nn}s}
\end{bmatrix}
\]

(1)

the control system design of MIMO system will be examined. It is a characteristic that time delays are different from each other.

3. Control System Design

First of all, the MIMO process model is given by equation (2), this is used as the modified z transform, and this discrete-time system is called CARMA model. In equation (2), for re-moving of the offset against the load disturbance, the model includes an integrator.

\[
\Delta A(q^{-1})y(k) = q^{-j \text{ave}} QB(q^{-1})u(k) + \Delta C(q^{-1})\xi(k)
\]

(2)

\[
A(q^{-1}) = I + a_1q^{-1} + a_2q^{-2} + \cdots + a_nq^{-n_t}
\]

(3)

\[
B(q^{-1}) = b_0 + b_1q^{-1} + b_2q^{-2} + \cdots + b_nq^{-n_b}
\]

(4)

\[
C(q^{-1}) = I + c_1q^{-1} + c_2q^{-2} + \cdots + c_nq^{-n_c}
\]

(5)

\[
q^{-j} = \begin{bmatrix}
q^{-j_{11}} & \cdots & q^{-j_{1n}} \\
\vdots & \ddots & \vdots \\
q^{-j_{n1}} & \cdots & q^{-j_{nn}}
\end{bmatrix}
\]

(6)

where \(y(k)\) is output, \(u(k)\) is input, \(\xi(k)\) is white noise, and \(A(q^{-1}), B(q^{-1}), C(q^{-1})\) are the matrixes in terms of the backwards shift operator \(q^{-1}\). \(q^{-j\text{ave}}\) is described as the smallest time delay element \(q^{-1}\), such as a scalar. \(j\) expresses the number of the steps in the time delay, and it is step in the time delay +1.

The cost function expresses as

\[
J = E[h^T(k + j_{\text{min}})h(k + j_{\text{min}})]
\]

(7)

\[
h(k + j_{\text{min}}) = P(q^{-1})y(k + j_{\text{min}}) + S(q^{-1})Au(k) - R(q^{-1})w(k + j_{\text{min}})
\]

(8)

where \(w(k)\) is a reference signal, \(P(q^{-1}), S(q^{-1}), R(q^{-1})\) are polynomial matrices in the cost function (7). These matrices are parameters for improving the closed-loop performance by adjusting these parameters

\[
P(q^{-1}) = I + P_1q^{-1} + P_2q^{-2} + \cdots + P_{n_p}q^{-n_p}
\]

(9)

\[
S(q^{-1}) = S_0 + S_1q^{-1} + S_2q^{-2} + \cdots + S_{n_s}q^{-n_s}
\]

(10)

\[
R(q^{-1}) = R_0 + R_1q^{-1} + R_2q^{-2} + \cdots + R_{n_r}q^{-n_r}
\]

(11)
where $y(k+j)$ and $w(k+j)$ are controlled variable and manipulating variable $j$ step ahead. In order to obtain a one step predictor of $y(k+1)$, consider the following identity. Equation (12) is a Diophantine equation [5].

$$C(q^{-1})P(q^{-1}) = E(q^{-1})A(q^{-1}) + q^{-ju}QF(q^{-1})$$

(12)

$$E(q^{-1}) = I + E_1q^{-1} + E_2q^{-2} + \cdots + E_nq^{-n}$$

(13)

$$F(q^{-1}) = F_0 + F_1q^{-1} + F_2q^{-2} + \cdots + F_nq^{-n}$$

(14)

The control low is obtained by minimizing the cost function (7). The closed-loop equation of GMVC derives due to equation (15).

$$u(k) = \{B(q^{-1})E(q^{-1}) + C(q^{-1})S(q^{-1})\}^{-1} \times \{C(q^{-1})R(q^{-1})w(k + j_{\text{min}}) - QF(q^{-1})y(k)\}$$

(15)

$$y(k) = T^{-1}(q^{-1})R(q^{-1})w(k) + T^{-1}(q^{-1})\Xi(q^{-1})\xi(k)$$

(16)

$$T'(q^{-1}) = P(q^{-1}) + \Delta S(q^{-1})B^{-1}(q^{-1})A(q^{-1})$$

(17)

$$\Xi(q^{-1}) = \Delta[B(q^{-1})E(q^{-1}) + C(q^{-1})S(q^{-1})]$$

(18)

Figure 1 shows the closed-loop system of GMVC.

![Fig. 1 Block diagram of survo GMVC for MIMO system.](image)

### 4. Adjusting Method of Sampling Periods for Matching Time Delay

The numerical analysis of control system is necessary to consider time function and frequency function. Generally, the characteristic of the discrete time system is shown as a frequency function. The length of time delay in the plant does not match the multiples of the sampling period. This study proposes a method, which can be handled the time delay accurately by increasing one step.

#### 4.1 Modified z Transform

The time delays ($L_1, \ldots, L_n$) are not every sampling interval. As a result, two kind of input of $u(k-1)$ and $u(k)$ is generated in the section from $kT_s$ to $(k+1)T_s$, and it is reflected in the further output of the passage of time delay [6]. There is modified z transform as a method for expressing the value of the output in such a situation. Modified z transform is a calculation method separately for $x(kT_s) \rightarrow x(kT_s+L_1)$ and $x(kT_s+L_1) \rightarrow x(kT_s+T_s)$ as for state transition. The continuous-time system expresses as

$$\dot{x}(t) = Fx(t) + bu(t - L_1)$$

(19)

$$y(t) = Cx(t)$$

(20)

where $L_1$ is the time delay, $T_s (T_s = L_1/j, j = 1, 2, 3, \ldots)$ is the sampling interval. When time delay is integral multiples at the sampling period, this system expresses due to the following discrete time system.

$$x(k+1) = A x(k) + z^{-j}Bu(k)$$

(21)

$$y(k) = Cx(k)$$

(22)

$$A = e^{F \bar{T}_s}$$

(23)

$$B = Q_0$$

(24)

$$Q_0 = \int_{0}^{\bar{T}_s} e^{F \bar{T}_s}d\eta$$

(25)

However, when time delay cannot be divided by integral multiples at the sampling period, it is expressed by the following system. Time delay $\bar{L}_1$ is expressed as follows.

$$L_1 = NT_s + \bar{L}_1$$

(26)

$0 < \bar{L}_1 < T_s$

- **Section** $x(kT_s) \rightarrow x(kT_s + L_1)$

$$x(k + \bar{L}_1) = e^{F \bar{T}_s}x(k) + \int_{\bar{T}_s}^{\bar{T}_s + \bar{L}_1} e^{F(\tau - \bar{T}_s)} B \eta \cdot u(k - 1)$$

(27)

- **Section** $x(kT_s + L_1) \rightarrow x(kT_s + T_s)$

$$x(k + 1) = e^{F(\bar{T}_s + \bar{L}_1)}x(k + \bar{L}_1) + \int_{\bar{T}_s + \bar{L}_1}^{\bar{T}_s + T_s} e^{F(\tau - \bar{T}_s - \bar{L}_1)} B \eta \cdot u(k)$$

(28)

when these two equations are arranged, the state transition from $x(k)$ to $x(k+1)$ is obtained.

$$x(k+1) = e^{F \bar{T}_s}x(k) + Q_1u(k) + Q_2u(k - 1)$$

(29)

$$Q_1 = \int_{0}^{\bar{T}_s} e^{F \bar{T}_s}d\eta$$

(30)

$$Q_2 = \int_{\bar{T}_s}^{\bar{T}_s + \bar{L}_1} e^{F \bar{T}_s}d\eta$$

(31)

#### 4.2 Generalized Minimum Variance Control for MIMO That is Applied Modified z Transform

Generally MIMO system includes the various time delays. The proper sampling period can handle each time delay. Figure 2 shows a block diagram of closed-loop control system for
5. Introduction of Multiple Sampling Periods

In this section, it is introduce the different sampling period that is this study theme [7]. In the conventional law, a common sampling period is used for the entire faction. That purpose, the computational complexity of the computer increases. By the suggestion technique, setting suitable sampling period in every control loop, as a result we can reduce a lot of computational complexity keeping the effective performance. It is thought when temperature with large time constant and pressure with small time constant are controlled at the same time as such an example.

It is assumed that a control object of 2-inputs 2-outputs system is given as an example of the control object of MIMO.

\[ G(s) = \begin{bmatrix} G_{11}(s)e^{-L_{11}T} & G_{12}(s)e^{-L_{12}T} \\ G_{21}(s)e^{-L_{21}T} & G_{22}(s)e^{-L_{22}T} \end{bmatrix} \]  

\[ (31) \]

6. Numerical Example

In this section, control performance of proposed method is confirmed by simulation.

6.1 Simulation Model

In simulation, it is used the distillation column is shown in Fig. 5 [7]. The simulation model is 2-inputs 2-outputs system column model in [8].

\[ Y_1(s) = G_{11}(s)e^{-L_{11}T}U_1(s) + G_{12}(s)e^{-L_{12}T}U_2(s) \]  

\[ (32) \]

\[ Y_2(s) = G_{21}(s)e^{-L_{21}T}U_1(s) + G_{22}(s)e^{-L_{22}T}U_2(s) \]  

\[ (33) \]

\[ G_{11}(s)e^{-L_{11}T} \text{ and } G_{12}(s)e^{-L_{12}T} \text{ are considered to be disintegration in } T_{11}, G_{21}(s)e^{-L_{21}T} \text{ and } G_{22}(s)e^{-L_{22}T} \text{ are in } T_{22}. \]

The controller’s composition expresses as follows.

\[ U_1(s) = C_{11}(s)E_1(s) + C_{12}(s)E_2(s) \]  

\[ (34) \]

\[ U_2(s) = C_{21}(s)E_1(s) + C_{22}(s)E_2(s) \]  

\[ (35) \]

for each output like this, when it is controlled the temperature control that a time constant is long and the pressure control that a time constant is short at the same time are thought about as an example of it. Figure 4 shows the block diagram of 2-inputs 2-outputs system control object in [8].

\[ T_{s1} \]

\[ T_{s2} \]
system. The purpose of the distillation column is to extract the methanol of the decided density from the mixture liquid of water and the methanol.

\[ Y_1(s) : \text{Density of methanol with high purity obtained by condensing steam collected in top management of the distillation column with cooling water. [%]} \]

\[ Y_2(s) : \text{Density of methanol with low purity obtained by warming liquids collected in bottom part of the distillation column with boiler. [%]} \]

\[ U_1(s) : \text{Flowing quantity rate of cooling water injected into the distillation column. [lb/min]} \]

\[ U_2(s) : \text{Flowing quantity rate of steam injected into boiler. [lb/min]} \]

The plant model is the following transfer function matrix.

\[
G(s) = \begin{pmatrix}
6.4 & -4.725 \\
66.8s + 1 & 84.0s + 1 \\
3.3 & -4.85 \\
10.9s + 1 & 14.4s + 1
\end{pmatrix}
\]

6.2 Simulation Condition

(Set points)

\[ w_1: \text{The step size 80 in the time t=700[min]} \]

\[ w_2: \text{The step size 30 in the time t=40[min]} \]

(Disturbance)

\[ D: \text{The step size 5 applies } U_1(s) \text{ in the time } t=1500[min] \]

(Noise)

The noise is disregarded.

6.3 Setting of Sampling Periods

Two kinds of sampling periods of the conventional method and the proposed method are set as follows.

Conventional method: \(T_1=5[\text{min}],\ T_2=5[\text{min}]\)

Proposed method: \(T_1=25[\text{min}],\ T_2=5[\text{min}]\)

The time delay is expresses at one sampling period in the conventional method [9]. A different sampling period in \(T_1\) and \(T_2\) is set to five times in the proposed method at the sampling period to \(G_{11}\) and \(G_{12}\) that the time constant is long. Moreover, either method uses modified \(z\) transform.

6.4 Simulation Results

Case 1) Designing conventional GMVC

CARMA model is as follows.

\[
\begin{pmatrix}
A_1(q^{-1}) & 0 \\
0 & A_2(q^{-1})
\end{pmatrix} y(k) = q^{-1} \begin{pmatrix} B_{11}(q^{-1}) & B_{12}(q^{-1}) \\ B_{21}(q^{-1}) & B_{22}(q^{-1}) \end{pmatrix} u(k) + I \xi
\]

\[ A_1(q^{-1}) = 1 - 1.8287q^{-1} + 0.8360q^{-2} \]

\[ A_2(q^{-1}) = 1 - 1.3387q^{-1} + 0.4467q^{-2} \]

\[ B_{11}(q^{-1}) = 0.4911 - 0.3369q^{-1} - 0.1079q^{-2} \]

\[ B_{12}(q^{-1}) = -0.0744 + 0.2188q^{-1} + 0.2589q^{-2} \]

\[ B_{21}(q^{-1}) = 0.2893 + 0.7204q^{-1} - 0.6535q^{-2} \]

\[ B_{22}(q^{-1}) = -0.9121 + 0.0659q^{-1} + 0.3228q^{-2} \]

\[ E(q^{-1}), F(q^{-1}) \] of this time seems to become next.

\[
E(q^{-1}) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}
\]

\[
F(q^{-1}) = \begin{pmatrix} F_{11} & 0 \\ 0 & F_{22} \end{pmatrix}
\]

\[ F_{11} = 2.4287 - 2.6647q^{-1} + 0.8360q^{-2} \]

\[ F_{22} = 1.8387 - 1.7854q^{-1} + 0.4467q^{-2} \]

the weighting factors are set as follows.

\[
P(q^{-1}) = \begin{pmatrix} 1 & 0 \\ 0 & 1 - 0.5q^{-1} \end{pmatrix}
\]

\[
R(q^{-1}) = \begin{pmatrix} 0.6 & 0 \\ 0 & 0.5 \end{pmatrix}
\]

\[
S(q^{-1}) = \begin{pmatrix} 50 & 0 \\ 0 & 0 \end{pmatrix}
\]

Figure 6(a) shows step responses, Fig. 6(b) shows control inputs of conventional GMVC, respectively. In the part that the output crosses, there is hardly the influence of interfering it.

Case 2) Designing proposed GMVC

CARMA model is as follows.

\[
\begin{pmatrix}
A_1(q^{-1}) & 0 \\
0 & A_2(q^{-1})
\end{pmatrix} y(k) = q^{-1} \begin{pmatrix} B_{11}(q^{-1}) & B_{12}(q^{-1}) \\ B_{21}(q^{-1}) & B_{22}(q^{-1}) \end{pmatrix} u(k) + I \xi
\]

\[ A_1(q^{-1}) = 1 - 1.2796q^{-1} + 0.4083q^{-2} \]

\[ A_2(q^{-1}) = 1 - 1.3387q^{-1} + 0.4467q^{-2} \]

\[ B_{11}(q^{-1}) = 2.4460 - 1.5597q^{-1} - 0.0527q^{-2} \]

\[ B_{12}(q^{-1}) = -1.3394 + 0.6049q^{-1} + 0.1265q^{-2} \]

\[ B_{21}(q^{-1}) = 0.2893q^{-1} + 0.7204q^{-2} - 0.6535q^{-3} \]

\[ B_{22}(q^{-1}) = -0.9121q^{-1} + 0.0659q^{-2} + 0.3228q^{-3} \]
Fig. 6 Responses of closed-loop system of conventional GMVC.

\[ E(q^{-1}), F(q^{-1}) \] of this time seems to become next.

\[
E(q^{-1}) = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}
\] (44)

\[
F(q^{-1}) = \begin{bmatrix} F_{11} & 0 \\ 0 & F_{22} \end{bmatrix}
\] (45)

\[
F_{11} = 1.7796 - 1.6879q^{-1} + 0.4083q^{-2}
\]

\[
F_{22} = 1.5887 - 1.7854q^{-1} + 0.3228q^{-2}
\]

the weighting factors are set as follows.

\[
P(q^{-1}) = \begin{bmatrix} 1 - 0.5q^{-1} & 0 \\ 0 & 1 - 0.75q^{-1} \end{bmatrix}
\] (46)

\[
R(q^{-1}) = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.25 \end{bmatrix}
\] (47)

\[
S(q^{-1}) = \begin{bmatrix} 25 & 0 \\ 0 & 0 \end{bmatrix}
\] (48)

Figure 7 (a) shows step responses, Fig. 7 (b) shows control inputs of proposed GMVC, respectively. Compared with Fig. 6, in the part that the output crosses, there is influence of interfering it a little, but the other parts are performance same as a method in before.

6.5 Computational Complexity

The next expression shows a reduction of computational complexity.

\[
\frac{1}{2} \left( \frac{5}{25} \right) + \frac{1}{2} \left( \frac{5}{25} \right) = \frac{3}{5} = 60\%
\] (49)

because one at the sampling period is increased by five times, it is possible that 40% is reduced the computational complexity.

7. Conclusion

This study proposed a method of GMVC for the MIMO systems with various time delays. The proposed method can reduce the amount of the calculation to maintain the control performance. The reduction was achieved by adjusting the sampling periods for the various time delays. In other words, the proposed method improves the complexity of GMVC for MIMO systems with various time delays. The time delays in the conventional GMVC should be determined according to the characteristic of the fastest process because the GMVC controller was designed by one common sampling period. However, there were some faulty responses due to the computational complexity. Therefore, the method in Section 5 was proposed to allow GMVC to have multiple sampling periods. The utility has been confirmed in Section 6. When we set a sampling period every each output, the effective performance is provided as follows.

1. By the proposed method, the achieved performance is almost the same as that by the conventional law compared with law conventionally.

2. The computational complexity is largely reduced.

As future problems, it is necessary to examine the case of two degree of freedom and robustness.

References


Satoshi BESSHO
He received his B.S. degree from Tokyo Metropolitan University, Japan, in 2008. He is currently a graduate student of the Department of Electronic System, Tokyo Metropolitan Institute of Technology.

Masayoshi DOI (Member)
He received his B.S. degree from the Toyohashi University of Technology, Japan, in 1994, M.S. degree from National Defense Academy, Japan, in 1999 and Ph. D. degree from Keio University, Japan, in 2003. In 2010, he joined the Hiroshima Institute of Technology where he is currently an Associate Professor of the Department of Intelligent Mechanical Engineering. His research interests include predictive control design. He is a member of IEEJ and RSJ.

Yasuchika MORI (Member)
He received the B.S., M.S., and Ph. D. degrees in electrical engineering from Waseda University in 1976, 1978, and 1981, respectively. He is a Professor in the Faculty of System Design, Tokyo Metropolitan University. His current research interests include adaptive control, time delay control, sliding mode control, and robot control.