Fictitious Reference Tuning of the Feed-Forward Controller in a Two-Degree-of-Freedom Control System

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Abstract: In this paper, we provide a new effective tuning method to obtain the optimal parameter of the feed-forward controller in a two-degree-of-freedom (2DOF) control system for the purpose of achieving the desired response without using a mathematical model of a plant. The first author proposed “fictitious reference iterative tuning” (which is abbreviated to FRIT) as an effective method for the tuning of parameters of a controller by using only one-shot experimental data instead of using mathematical models of a plant. Here, we extend FRIT to the tuning of the feed-forward controller in a 2DOF control system and develop the off-line computation so as to be done by the least squares method. For these purposes, we introduce a new cost function consisting of the fictitious reference and the actual data (since we do not have to do iterative computation in the least squares method, we call the proposed method here as “fictitious reference tuning”). Since the new cost function is quadratically-parameterized, it is possible to analyze how far the obtained parameter is apart from the desired one. Thus, we then derive a pre-filter which is applied to the actual data so as to guarantee that the obtained parameter is close to the desired one. We also show that the proposed method is applicable to the case in which the initial experiment is performed in the conventional 1DOF control system. Finally, we illustrate experimental results in order to show the utility and the validity of the proposed method.

Key Words: fictitious reference iterative tuning, two-degree-of-freedom control, controller parameter tuning, unfalsified control.

1. Introduction

The actual input/output data of a plant includes fruitful information on the dynamics of a plant, so it is expected that control system synthesis based on the direct use of the data provides high-performance controllers. In fact, such a direct approach attracts attentions from the practical view points, and thus several approaches have been proposed in the literatures [1]–[11]. In the case in which the structure of a controller is fixed (e.g., PID controllers [12]), such a direct approach is regarded as the parameter tuning of a controller based on the direct use of the data, e.g., [13]. As one of these tuning methods, iterative feedback tuning (IFT) was proposed and studied in [14]. IFT updates the tunable parameter of an implemented controller so as to minimize a cost function, e.g., the sum of squared errors between the desired reference signal and the actual output. This minimization is done by performing nonlinear optimization in which Hessian and Jacobian consist of the experimental data. This also implies that many experiments must be iteratively done in order to update the parameter of a controller so as to achieve the desired property. Thus the use of IFT spends considerable expense and time.

On the other hand, virtual reference feedback tuning (VRFT) was proposed in [15] and [16] as an effective tuning method in the sense that it requires only one-shot experimental data for achieving the desired response. The cost function for the parameter tuning in VRFT is described by the sum of the squared errors between the initial input and the virtual one computed by using the virtual reference and the initial output. Since the cost function in VRFT differs from the original cost function, VRFT uses the pre-filter applied to the initial data so as to guarantee that the parameter can be obtained as close to the desired one as possible. As another approach to the parameter tuning with only one-shot experimental data, the first author proposed fictitious reference iterative tuning (FRIT) in [17] and [18]. The cost function in FRIT is described by the sum of the squared errors between the initial output and the output of the desired transfer function with respect to the fictitious reference signal. One of the fundamental difference between FRIT and VRFT is that the cost function to be minimized in FRIT focuses on the output while that in VRFT focuses on the input. This implies that FRIT is more intuitively understandable than VRFT from the practical points of view. Another fundamental difference is that pre-filter in FRIT requires only the desired transfer function while VRFT requires not only it but also its sensitivity function. This implies that the pre-filter required for the optimality in FRIT is simpler than that in VRFT.

By the way, it is well-known that the two-degree-of-freedom (2DOF) control structure achieves not only a desired feedback property but also a desired tracking output by using the feed-forward controller, e.g., [19],[20] and so on. If the dynamics of a plant is completely known, setting the feed-forward controller as the serial connection of the inverse of the transfer function of a plant and the desired transfer function yields

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1 The fictitious reference signal was introduced in the unfalsified control framework [10] for the sake of choosing a desired controller from the set of the candidate controllers.
the desired output completely. In the case in which we have no mathematical model, we have to construct the feed-forward controller by using the data. In regards to this point, VRFT in [16] and FRIT in [18] were also extended in 2DOF control systems. In VRFT [16], similarly to the one-degree-of-freedom (1DOF) controller case, the pre-filter to guarantee the optimality was proposed by using the power spectrum density of the data, which implies that we should execute “identification” of an appropriate pre-filter. In FRIT [18], the optimal parameter should be obtained by using off-line non-linear optimization, which implies that the computation for the optimization might lead to a local minimum. In this regard, although the least squares method for FRIT are discussed in [21] and [22], the control structures addressed in those references are one-degree-of-freedom control systems.

From these backgrounds, this paper provides a new effective tuning method to obtain the optimal parameter of the feed-forward controller in a 2DOF control system for the purpose of achieving the desired response without using a mathematical model of a plant. Here, we extend FRIT to the tuning of the feed-forward controller in the 2DOF control system and develop the off-line computation so as to be done by the least squares method. For these purposes, we introduce a new cost function consisting of the fictitious reference and the actual data (since we do not have to do iterative computation in the least squares method, we call the proposed method here as “fictitious reference tuning”). Since the new cost function is quadratically-parameterized, it is possible not only to obtain the global minimum but also to analyze how far the obtained parameter is apart from the desired one. Thus, we then derive the pre-filter which is applied to the actual data so as to guarantee the optimal parameter. Moreover, we explain that the pre-filters in our method do not require the data, which implies that we do not have to identify the pre-filter. This is a remarkable advantage over VRFT for the 2DOF control system [16]. Finally, we illustrate experimental results in order to show the utility and the validity of our proposed method.

This paper is organized as follows. In Section 2, we give some required preliminaries, assumptions, and the problem formulation we consider here. In Section 3, we provide the main result, i.e., a new method for the optimal parameter tuning of the feed-forward controller in the 2DOF control systems using fictitious reference. We also derive a pre-filter so as to guarantee the optimality in the proposed method. Moreover, we show that our proposed method is applicable to the case in which the initial experiment is performed in the 1DOF control system. In Section 4, we show experimental results in order to validate our results. In Section 5, we give some concluding remarks.

2. Preliminaries

2.1 Notations

Let \( \mathbb{R} \) and \( \mathbb{R}^n \) denote the set of real numbers and that of real vectors of size \( n \), respectively. Let \( z \) denote the \( z \)-operator. Let \( \mathbb{R}(z) \) denote the set of real coefficient rational functions with the indeterminate \( z \). Let \( \mathbb{R}(z)^n \) denote the set of the vectors of size \( n \) whose elements are included in \( \mathbb{R}(z) \). If \( w \) is the discrete time signal, \( w(t) \) denote the value at the time \( t \). For a discrete time signal \( w = \{w_0, w_1, \ldots, w_k, \ldots\} \), its \( z \)-transformed representation is described by

\[
\mathbf{w}(z) = \sum_{k=0}^{\infty} w_k z^{-k}.
\]

Let \( G(z) \in \mathbb{R}(z) \) denote a discrete time transfer function of a linear time-invariant system. Then the output of \( G(z) \) with respect to the input time series whose \( z \) transformed representation is \( u(z) \) is described by \( G(z)u(z) \). For a \( w(z) \), we use the norm defined by

\[
\|w(z)\|_N = \left( \sum_{k=0}^{N} w_k^2 \right)^{1/2}.
\]

which was introduced in [2] and [11]. For \( \mathbf{w}(z) = \sum_{k=0}^{N} w_k z^{-k} \), let \( \mathbf{w}(z) \) denote

\[
\mathbf{w}(z) := \begin{bmatrix} w_0 & w_1 & \cdots & w_{N-1} \end{bmatrix}^T \in \mathbb{R}^N.
\]

2.2 Assumptions and the Problem Formulation

Consider the 2DOF control system illustrated in Fig. 1. We assume that a plant, which is denoted by \( P(z) \), is a linear, time-invariant, single-input and single-output system. We also assume that \( P(z) \) has no unstable zeros and its relative degree is known. The feedback controller, which is denoted by \( C_{rb}(z) \), is assumed to be implemented so as to “stabilize” the closed loop. In this paper, we assume that a feed-forward controller \( C_{ff}(z) \) is linearly parameterized with respect to a tunable parameter vector

\[
\rho := \begin{bmatrix} \rho(1) & \rho(2) & \cdots & \rho(n) \end{bmatrix}^T \in \mathbb{R}^n.
\]

as

\[
C_{ff}(\rho, z) = \alpha(z)^T \rho,
\]

where \( \alpha(z) \) is a rational function vector determined by the designer

\[
\alpha(z) := \begin{bmatrix} \alpha_1(z) & \alpha_2(z) & \cdots & \alpha_n(z) \end{bmatrix}^T \in \mathbb{R}(z)^n.
\]

The input and the output of the plant in the 2DOF control system with \( C_{ff}(z) \) are denoted by \( u(z) \) and \( y(z) \), respectively, to represent that they depend on the parameter \( \rho \). Let \( T(\rho) \) be the transfer function from the reference \( r \) to the output \( y \). From Fig. 1, we see

\[
T(\rho) = \frac{P(z)\left(C_{ff}(\rho, z) + T_d(z)C_{rb}(z)\right)}{1 + P(z)C_{rb}(z)},
\]

namely,

\[
y(\rho, z) = T(\rho, z)r(z).
\]

Then, the problem in this paper is formulated as follows.
Problem 1 Perform a one-shot experiment by using $C_{ff}(p_0, z)$ with the initial parameter $p_0$ in the 2DOF control system illustrated in Fig. 1, and obtain the initial data $u(p_0, z)$ and $y(p_0, z)$. Assume that we are given a desired closed loop transfer function from $r$ to $y$ as $T_d(z)$, that is,

$$y_d(z) = T_d(z)r(z)$$

is the desired output. Then, find a parameter $p$ so as to minimize the cost function

$$J(p) := ||y(p, z) - T_d(z)r(z)||_N^2$$

with only $u(p_0, z)$ and $y(p_0, z)$.

2.3 Fictitious Reference Iterative Tuning (FRIT) Before going to the main result, we give a brief review on FRIT [17],[18]. Consider the 2DOF control system illustrated in Fig. 1. By using the initial data $(u(p_0, z), y(p_0, z))$, define the “fictitious reference” signal $\tilde{r}(p, z)$ as

$$\tilde{r}(p, z) =\frac{u(p_0, z) + C_{fb}(z)y(p_0, z)}{C_{ff}(p, z) + T_d(z)C_{fb}(z)}$$

When we apply $\tilde{r}(p, z)$ to (5), we have

$$T(p, z)\tilde{r}(p, z) = \frac{P(z)(C_{ff}(p, z) + T_d(z)C_{fb}(z))}{1 + P(z)C_{fb}(z)} u(p_0, z) + C_{fb}(z)y(p_0, z) \times \frac{P(z)u(p_0, z) + P(z)C_{fb}(z)y(p_0, z)}{1 + P(z)C_{fb}(z)}$$

From the trivial relation

$$y(p_0, z) = P(z)u(p_0, z)$$

and (5), we also see that (10) can be written

$$T(p, z)\tilde{r}(p, z) = y(p_0, z).$$

Hence, the fictitious reference $\tilde{r}(p, z)$ is the reference input that produces $y(p_0, z)$ as the output of the 2DOF control system with $C_{ff}(p, z)$. Here, we define the error signal described by

$$e_F(p, z) = y(p_0)|_{[-1, 1]} - T_d(q)\tilde{r}(p)|_{[-1, 1]}$$

and also introduce the cost function

$$J_F(p) := ||y(p_0, z) - T_d(z)\tilde{r}(p, z)||_N^2.$$  

3. Main Results

3.1 Fictitious Reference Tuning by the Least Squares Method We modify the error signal $e_F(p, z)$ used in (13) as

$$e_{FM}(p, z) := \left(C_{ff}(p, z) + T_d(z)C_{fb}(z)\right)y(p_0, z) - T_d(z)u(p_0, z) + C_{fb}(z)y(p_0, z)$$

$$= C_{ff}(p, z)y(p_0, z) - T_d(z)u(p_0, z)$$

and define the new cost function described by

$$J_{FM}(p) := ||e_{FM}(p, z)||_N^2 = \frac{1}{N^2}\text{vec}\left[e_{FM}(p, z)\right]_N^T\text{vec}[e_{FM}(p, z)]_N.$$  

From Eq.(4), $C_{ff}(p, z)y(p_0, z)$ is written as

$$C_{ff}(p, z)y(p_0, z) = \sum_{i=1}^{n} a_i(p)\rho(i)$$

and thus we obtain

$$\text{vec}\left[C_{ff}(p, z)y(p_0, z)\right]_N = \text{vec}\left[\sum_{i=1}^{n} a_i(p)\rho(i)\right]_N$$

$$= \text{vec}[^T\rho] \Theta$$

where

$$\Theta := [\theta_1 \cdots \theta_n] \in \mathbb{R}^{2n}$$

$$\theta_i := \text{vec}[a_i(p)] \in \mathbb{R}^n.$$  

Hence, together with

$$\text{vec}[e_{FM}(p, z)]_N = \text{vec}\left[C_{ff}(p, z)y(p_0, z)\right]_N - \text{vec}\left[T_d(z)u(p_0, z)\right]_N,$$

we see that $J_{FM}(p)$ in (15) is quadratically parameterized by $\rho$. This implies that the optimal parameter $\rho_{FM}^*$ minimizing $J_{FM}(p)$ can be obtained by using the least squares method as

$$\rho_{FM}^* = (\Theta^T\Theta)^{-1}\Theta^T\text{vec}[T_d(z)u(p_0, z)]_N.$$  

3.2 The Optimality of the Parameter In the previous subsection, we have introduced the new cost function $J_{FM}(p)$ so as to be performed by the least squares method for the purpose of obtaining the “optimal” parameter. On the other hand, the purpose for achieving the desired output is to minimize $J(p)$ described by (8). Thus, it is required to provide a strategy that makes the optimal parameter of $J_{FM}(p)$ close to the desired one of $J(p)$. In order to do this, we introduce the prefilter, say $F(q)$, to be applied to the initial data $u(p_0)$ and $y(p_0)$, which were also taken in VRFT [15],[16] and FRIT [21]. However, differently from VRFT or the previous FRIT, the filter used here consists only known transfer functions without the initial data. This feature is one of the main advantages of our result.

First, for a given $T_d(z)$, we consider the ideal feed-forward controller $C_{ff}^*(q)$ achieving the desired tracking property completely. It satisfies
Since \( P(z) \) is unknown, so is \( C_{ff}^d(z) \). In addition, it may be a non-proper transfer function. However, since \( C_{ff}^d(z) \) is used for only the derivation of the filter and not implemented, there is no need for addressing these points. By using \( C_{ff}^d(z) \) with (5) and (19), we rewrite the cost function (8) as

\[
J(p) = \frac{P(z)(C_{ff}^d(z) + T_d(z)C_{fb}(z))}{1 + P(z)C_{fb}(z)}. \tag{19}
\]

namely,

\[
C_{ff}^d(z) = T_d(z)P(z)^{-1}. \tag{20}
\]

To rewrite (19) as

\[
J(p) = \|T(p)\alpha(z) - T_d(z)\alpha(z)\|_N^2
\]

we modify (15) as

\[
J_{FM}^F(p) = \left\| \frac{C_{ff}^d(p,z) - C_{ff}^d(z)}{1 + P(z)C_{fb}(z)} F(z) y(p_0,z) \right\|_N^2. \tag{21}
\]

By using \( u(p_0,z) = P(z)^{-1}y(p_0,z) \) and (20), the cost function (22) can be rewritten as

\[
J_{FM}^F(p) = \left\| \frac{C_{ff}^d(p,z) - C_{ff}^d(z)}{1 + P(z)C_{fb}(z)} F(z) y(p_0,z) \right\|_N^2. \tag{23}
\]

Here, if the pre-filter \( F(z) \) satisfies

\[
F(z) y(p_0,z) = \frac{P(z)}{1 + P(z)C_{fb}(z)} r(z), \tag{24}
\]

then the cost functions (21) and (23) are equivalent. Thus, all we have to do is to obtain the pre-filter \( F(z) \) satisfying (24). In order to obtain such a pre-filter without using \( P(z) \), we rewrite (24) as follows. First, the right-hand side of (24) can be rewritten as

\[
F(z) y(p_0,z) = \frac{1}{C_{ff}^d(p_0,z) + T_d(z)C_{fb}(z)} \times \frac{P(z)(C_{ff}(p_0,z) + T_d(z)C_{fb}(z))}{1 + P(z)C_{fb}(z)} r(z). \tag{25}
\]

At the same time, by using (5) and (6) for the initial parameter \( p_0 \), we see that

\[
F(z) y(p_0,z) = \frac{1}{C_{ff}(p_0,z) + T_d(z)C_{fb}(z)} y(p_0,z). \tag{26}
\]

Hence, by taking the pre-filter as

\[
F(z) = \frac{1}{C_{ff}(p_0,z) + T_d(z)C_{fb}(z)}. \tag{28}
\]

it is possible to guarantee that the optimal parameter for the cost function (21) and that for (23) are the same. Summing up these discussions, we can obtain the following theorem on the optimality of our proposed method.

\textbf{Theorem 1} Assume that the initial experiment is performed in the 2DOF control system with \( C_{ff}(p_0,z) \) and obtain \( u(p_0,z) \) and \( y(p_0,z) \). By using \( C_{ff}(p_0,z) \), \( C_{fb}(z) \) and \( T_d(z) \), make the pre-filter \( F(z) \) described by (28). Then, the identity

\[
J(p) = J_{FM}^F(p) \tag{29}
\]

holds, i.e., the minimizations of the cost function (8) and (22) are equivalent.

Our proposed method is summarized as follows.

1. Perform a one-shot experiment and obtain the initial data \( y(p_0) \) and \( u(p_0) \) in the 2DOF control system as illustrated in Fig. 1.

2. Apply the pre-filter described by (28) to \( y(p_0,z) \) and \( u(p_0,z) \).

3. Minimize the cost function (22). In order to this, perform the least squares method described in (18), (16) and (17) where \( u(p_0,z) \) and \( y(p_0,z) \) are replaced with \( F(z)u(p_0,z) \) and \( F(z)y(p_0,z) \), respectively. Then, we obtain \( p^* := \text{arg min}_p J_{FM}^F(p) \).

4. Implement \( p^* \) to the feed-forward controller in 2DOF as illustrated in Fig. 1.

As summarized in the above procedure, one of the practical advantages of the proposed method is that the required material for the optimization is only one-shot experimental data. Moreover, as stated in the above explanation, the pre-filter \( F(z) \) can be obtained without the data, which is also a practical advantage and is remarkably different from VRFT for 2DOF control systems [16].

3.3 The Case in which the Initial Experiment Is Performed in 1DOF Control Systems

The above method is used for the case in which the initial experiment is performed in the 2DOF control system. The computation is based on the least squares which does not requires the initial parameter \( p_0 \). Moreover, we have used only the relation (11) in rewriting (23) and derivation of the pre-filter described by (24). These observations imply that the initial experiment is not necessarily performed in the 2DOF control system. At the same time, there are also many cases in which the experimental data performed in the 1DOF control system can be utilized for the design of the 2DOF controller. Thus, we discuss how our proposed method can be applicable in the case in which the first experimental data is obtained in the 1DOF control system. In this case, the initial experiment is independent from \( p_0 \) we use the notations \( u_{0}(z) \) and \( y_{0}(z) \) for the initial input and output, respectively.

First, consider (22) by replacing \( u(p_0,z) \) and \( y(p_0,z) \) with \( u_{0}(z) \) and \( y_{0}(z) \), respectively. We also see that (23) holds for \( u_{0}(z) \) and \( y_{0}(z) \). This implies that the pre-filter satisfying (24) for the optimality is rewritten as

\[
F(z) y_{0}(z) = \frac{1}{C_{fb}(z)} \times \frac{P(z)C_{fb}(z)}{1 + P(z)C_{fb}(z)} r(z). \tag{30}
\]
Thus, from (30) and (31), if we apply the pre-filter described by

\[ F(z) = \frac{1}{C_{fb}(z)} \quad (32) \]

to the initial data \( u_{0}(z) \) and \( y_{0}(z) \) and minimize the cost function (22) with them, we can obtain the optimal parameter \( \rho^* \) that yields the desired response \( T_{d}(z)r(z) \) in the 2DOF control system with \( C_{fb}(z) \).

These discussions can be also summarized as follows.

**Theorem 2** Assume that the first experiment is performed in the conventional 1DOF control system illustrated in Fig. 2. By using \( C_{fb}(z) \), make the pre-filter \( F(z) \) described by (32). Then, the identity (29) holds, i.e., the minimizations of the cost function described by (8) and (22) are equivalent. \( \square \)

Thus, we can summarize the proposed method for the case in which the first experiment is performed in 1DOF control system as follows.

1. Perform a one-shot experiment and obtain the initial output \( y_{0}(z) \) and input \( u_{0}(z) \) in the 1DOF control system as illustrated in Fig. 2.
2. Apply the pre-filter described by (32) to \( y_{0}(z) \) and \( u_{0}(z) \).
3. Minimize the cost function (22) and obtain \( \rho^* \).
4. Implement \( \rho^* \) to the feed-forward controller in the 2DOF control system as illustrated in Fig. 1.

Similarly to the method proposed in Subsection 3.2, the required material for the optimization is only one-shot experimental data. This point is one of the practical utilities in the case in which the first experiment is performed in the 1DOF control system. Moreover, the pre-filter (32) is simpler than that derived as \( F(z) = T_{d}(z)/C_{fb}(z) \) for this case in the VRFT [23]. This point is also one of the advantages over VRFT.

### 3.4 The Effect of the Noise

In the practical sense, there are many cases in which it is impossible to neglect the effect of the noise. In such cases, we repeat the experiment twice under the assumption that the noises in the different experiments are uncorrelated each other. This technique and the assumption are also taken by IFT [14] and VRFT [15]. Particularly, the following explanation in our method is similar to the strategy explained in “repeated experiment” in Section 4.1 in the reference [15]. We denote the first experimental data with \( y_{n}^{(1)}(\rho_{0}, z) := y^{(1)}(\rho_{0}, z) + n_{(1)}^{(1)}(z) \), \( u_{n}^{(1)}(\rho_{0}, z) := u^{(1)}(\rho_{0}, z) + n_{u}^{(1)}(z) \) and the second experimental data with \( y_{n}^{(2)}(\rho_{0}, z) := y^{(2)}(\rho_{0}, z) + n_{(2)}^{(2)}(z) \), \( u_{n}^{(2)}(\rho_{0}, z) := u^{(2)}(\rho_{0}, z) + n_{u}^{(2)}(z) \), respectively. Here, \( n_{(i)}^{(j)}(z) \) and \( n_{u}^{(j)}(z) \) denotes the noise in the \( i \)-th experiment on the output and the input, respectively. \( y^{(i)}(\rho_{0}, z) \) and \( u^{(i)}(\rho_{0}, z) \) denotes the signal which are assumed to be not contaminated by noise. Since the experiment is performed in the closed loop, the correlation between the noise and the involved signals in the first one-shot experiment can not be neglected. However, the two experiments are performed in the different time, so it is possible to assume that \( n_{(i)}^{(j)}(z) \) and \( n_{u}^{(j)}(z) \) in the first experiment have no correlation with \( n_{(i)}^{(j)}(z) \), \( n_{u}^{(j)}(z) \) and \( u^{(j)}(\rho_{0}, z) \), where \( i, j = (1, 2) \) or \( (2, 1) \). Thus, by modifying (22) as

\[
J_{FM}^{(1)}(\rho^{'}) := \text{vec}\left[ \left( C_{ff}(\rho, z)F(z)y^{(1)}(\rho_{0}, z) - T_{d}(z)F(z)u^{(1)}(\rho_{0}, z) \right) \right]_{1}^{T} \\
\times \text{vec}\left[ \left( C_{ff}(\rho, z)F(z)y^{(2)}(\rho_{0}, z) - T_{d}(z)F(z)u^{(2)}(\rho_{0}, z) \right) \right]_{N},
\]

we can approximate the cost function so as to reduce the effect of the noise. And then the least squares method yields

\[
\rho_{FM}^{*} = (\Theta_{(1)}^{T}\Theta_{(2)})^{-1}\Theta_{(2)}^{T}\text{vec}\left[ T_{d}(z)u^{(1)}(\rho_{0}, z) \right]_{N}
\]

where \( \Theta_{(i)} \) obtained by replacing \( y_{n}^{(i)}(\rho_{0}, z) \) with \( y(\rho_{0}, z) \) in Eqs. (16) and (17). From the assumption on the uncorrelation between the included noise in the different experiment, we can regard that the product of \( \Theta_{(1)} \) and \( \Theta_{(2)} \) (or \( T_{d}(\rho_{0}, z)u_{n}^{(1)}(\rho_{0}, z) \)) enables us to regard that the correlations of the noise included in \( \Theta_{(1)} \) and \( \Theta_{(2)} \) can be reduced from the practical sense. The same discussion is applicable for the method explained in Subsection 3.3.

### 4. Experimental Results

In order to illustrate the validity of our proposed method, we give experimental results. Here, we apply our method to the cart positioning system illustrated in Fig. 3. The cart is attached to the belt and the belt is moving by the rolling of the servo motor. The input \( u \) [m] is the applied voltage for the motor. The location \( y \) (output) from the initial position of the cart is measured by the potentiometer and send to the personal computer (PC). Here, the sampling time, say \( \Delta \), is 0.001 [sec]. The feedback controller \( C_{fb}(z) \) is a conventional (discrete time) PI controller:

\[
C_{fb}(z) = 0.3 + \frac{\Delta}{z - 1}. \quad (33)
\]

The desired transfer function from \( r \) to \( y \) is also obtained by

\[
T_{d}(s) = \frac{1.246 \times 10^{-5}z - 1.242 \times 10^{-5}}{s^2 + 1.990z + 0.990}
\]

which is obtained by the discretization of

\[
T_{d}(s) = \frac{1}{(0.2s + 1)^2}.
\]

We consider the parameterized feed-forward controller \( C_{ff}(\rho, z) \) as

![Fig. 2 One-degree-of-freedom control system.](Image)

![Fig. 3 The cart positioning system.](Image)
Fig. 4 The initial input data $u(\rho_0)$.

Fig. 5 The initial output data $y(\rho_0)$ and $y_d$ (the solid line: $y(\rho_0)$, the dotted line: $y_d$).

$$C_{ff}(\rho, z) = \frac{\rho_1 z^2 + \rho_2 z + \rho_3}{z^2 - 1.990 z + 0.990}$$

where the denominator of $C_{ff}(\rho, z)$ and that of $T_d(z)$ are the same.

4.1 The Initial Experiment Is Performed in the 2DOF Control System

We use

$\rho_0 = \begin{bmatrix} 0.250 & -0.499 & 0.249 \end{bmatrix}^T$

as the initial parameter of the feed-forward controller and perform the initial experiment in the 2DOF control system illustrated in Fig. 1. In Figs. 4 and 5, we illustrate the initial input and the output data, respectively. For the later, $y_d$ is also illustrated in the same figure. We apply the pre-filter $F(z)$ described by (28). Then, we perform the minimization of $J_{FM}^p(\rho)$ described by (22). As a result, we obtain the optimal parameter

$\rho^* = \begin{bmatrix} 3.490 \times 10^{-3} & -3.110 \times 10^{-3} & -3.805 \times 10^{-4} \end{bmatrix}^T$.

By using $\rho^*$ in $C_{ff}(\rho, z)$, we again perform the experiment in the 2DOF control system and then obtain the desired output as illustrated in Fig. 6. Since $y(\rho^*)$ and $y_d$ are almost the same in Fig. 6, we see that the proposed method yields the desired output.

4.2 The Initial Experiment Is Performed in the 1DOF Control System

Here, we show the validity of our method explained in Subsection 3.3. We perform the initial experiment in the 1DOF control system illustrated in Fig. 2 with the PI controller $C_{fb}(z)$ described by (33). Similarly to the previous case, the desired transfer function from $r$ to $y$ is described by (34). In Figs. 7 and 8, we illustrate the initial input and the output data, respectively. We apply the pre-filter described by (32). We then minimize $J_{FM}^p(\rho)$ with $u(0)$ and $y(0)$ and obtain the optimal parameter

$\rho^* = \begin{bmatrix} 2.752 \times 10^{-2} & -5.114 \times 10^{-2} & 2.362 \times 10^{-2} \end{bmatrix}^T$.

By using $\rho^*$ in $C_{ff}(\rho, z)$, we then perform the experiment in the 2DOF control system illustrated in Fig. 1 and obtain the desired output as illustrated in Fig. 9. Since $y(\rho^*)$ and $y_d$ are almost the same in Fig. 9, we see that the proposed method in the case that
the pre-filter (32) is applied to the initial data obtained in the 1DOF control system also yields the desired output.

5. Conclusions

In this paper, we have proposed a new tuning method to obtain the optimal parameter of a feed-forward controller in the 2DOF control system for the sake of achieving the desired output without a mathematical model of a plant. We have introduced a new cost function consisting of the fictitious reference and the actual data for the purpose of employing the least squares method in optimization, which also leads to the possibility of analyzing how far the obtained parameter is apart from the desired one. In addition, we have derived the filter that is applied to the actual data so as to guarantee that the obtained parameter yields the desired output. Particularly, the pre-filters in our method do not require the data, which implies that we do not have to identify the pre-filter. This is a remarkable different points from VRFT. Finally, we have illustrated experimental results in order to show the utility and the validity of our proposed method.

As one of the future studies, the authors are now studying the effect of the noise in detail. In order to address the case in which the effect of the noise cannot be neglected, the proposed method should be extended to such a case. Since this issue is beyond the focus of this paper, it should be clarified in the future. The case in which the plant is with unstable zeros should be discussed in near future. The authors are also extending the proposed method here to the multi-input and multi-output case.

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