Torque Control for Automotive Engines with Variable Valves via Air and Burned Gas Flow-Based Design

Tomohiko Jimbo∗ and Yoshikazu Hayakawa**

Abstract: This paper proposes a predictive control method for automotive engines with variable valves. The control purpose is to track not only the torque reference but also the pressure reference of the surge tank in consideration for the constraint of internal exhaust gas recirculation ratio. The control inputs are the throttle angle and the intake valve lift, however, the proposed control method is based on a flow model where the mass flows through the throttle and the intake valves are regarded as the virtual control inputs. The controller designed for the SICE benchmark engine is validated by numerical simulations.

Key Words: SI engine, predictive control, constraints, quadratic programming, physical-model-based control.

1. Introduction

A lot of attention has been paid recently to torque control for automotive engines [1]–[6]. Those studies have made use of some air gas flow model based on a SISO model, neural networks, and a look-up table, but have not considered the coexisting burned gas flow directly. As it is now, engines have multiple inputs, such as the throttle valve, the intake valve lift, intake and exhaust valve timings, to improve the fuel efficiency controlling air and burned gases. In that connection, control design methods for the above MIMO system considering the burned gas is needed.

The present paper considers the torque control for a spark ignition engine with variable valves, which has been provided as a Simulink® model (called a benchmark simulator) by the SICE Research Committee on Advanced Control of Engines [7],[8].

The engine with variable valves in Fig. 1 is controlled by a continuous time input of the throttle valve and discrete event-driven inputs of the intake valve lift and the overlap of intake and exhaust valves. The engine enables the quick response of the air mass into a cylinder to improve the torque response. On the other hand, the engine may cause a misfiring of combustion at the large valves’ overlap. Therefore, a controller considering the constraint on internal exhaust gas recirculation (IEGR) ratio is required.

The present paper proposes a predictive control method for automotive engines with variable valves, which is based on flow models of air and burned gases. The control purpose is to track not only the torque reference but also the pressure reference of the surge tank in consideration for the constraint on IEGR ratio. The control inputs are the throttle angle and the intake valve lift, however, the proposed control method is based on a flow model where the mass flows through the throttle and the intake valves are regarded as the virtual control inputs. The measurable outputs are the engine speed, the throttle flow, and the intake pressure. Here, the outer air and exhaust confluence point shown in Fig. 1 are assumed to be constants.

Section 2 introduces a control oriented model. As in [9],[10], for a system with both time-dependent and crank angle-dependent dynamics, a discrete-crank angle modeling is proposed for engines with variable valves. A state-space equation is derived from the discrete-crank angle model. In Section 3, a predictive controller is proposed by using the obtained model. Constrains on bounds of inputs and IEGR ratio are transformed into ones on the flows (the virtual control inputs). The optimal mass flows are designed by a predictive controller considering the transformed constraints. An offset-free control to reference signals is achieved by using an observer. Section 4 demonstrates the effectiveness of the proposed controller by numerical simulations. The conclusion is presented in Section 5.

Notations

\( \tau \), \( \theta \), \( m \), \( v \), \( P \), \( T \), \( V \), \( m_{\text{th}} \), \( m_{\text{iv}} \), \( m_{\text{ev}} \), \( m_{\text{back}} \), \( M \), \( M_{\text{th}} \), \( M_{\text{back}} \), \( A_{\text{th}} \), \( A_{\text{iv}} \), \( A_{\text{ev}} \), \( u \), \( \tilde{u} \), \( \tilde{v} \), \( r \), \( \tilde{r} \)

∗∗ Toyota Central R&D Labs., Inc., Nagakute, Aichi 480-1192, Japan
University, Nagoya, Aichi 464-8603, Japan

E-mail: t-jmb@mosk.tytlabs.co.jp

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where \( \theta, \omega \) is crank angle and engine speed \( (\omega = d\theta/dt) \) and \( \gamma_c, \beta_c \) are internal exhaust gas recirculation (IEGR) ratios, that are the ratio of the burned gas to the air gas \( \gamma_c = \frac{M_{\text{max}} + M_{\text{back}}}{M_{\text{in}}} \), and that of the burned gas to the total gas in the cylinder \( \beta_c = \frac{M_{\text{back}}}{M_{\text{in}}} \), where \( M_{\text{back}} \) is the mass in a cylinder at intake valve open.

\[ R, \kappa \] are the gas constant and specific heat ratio.

### 2. Flow-Based Modeling

For the torque control problem with constraints of the bounds of control inputs and the IEGR ratio, a control design method based on models of air and burned gas flows is proposed. A number of previous studies have examined the modeling of internal combustion engine [11].

The gas flow of the nozzle is described by the following non-linear flow function \( \Psi \) [7],[8],[12],[13].

\[
\Psi(P_d, P_a, T_a) = \sqrt{\frac{k+1}{2k} \frac{P_a}{\sqrt{R} T_a}} \psi\left(\frac{P_d}{P_a}\right)
\]

where \( P_a \) and \( T_a \) are the upstream pressure and the temperature, respectively, \( \psi \) is the downstream pressure.

\[
\psi(x) = \begin{cases} \sqrt{\frac{k+1}{k} (x - \frac{x^{k+1}}{k+1})^2} & x \geq r_{\text{ck}} \\ \frac{x^{k+1}}{k+1} & \text{otherwise} \end{cases}
\]

\( r_{\text{ck}} \) is a constant.

For the engine with variable valves in Fig. 1, the flow of air through the throttle, \( m_t \), the intake flow through the intake valve into a cylinder, \( m_{\text{iv}} \), and the exhaust flow of the burned gas through the exhaust valve out to the exhaust confluence point, \( m_{\text{ev}} \), are obtained:

\[
m_t = A_t(u_t) \Psi(P_a, P_o, T_o)
\]

\[
m_{\text{iv}} = \begin{cases} A_{\text{iv}}(\theta, \theta_{\text{OL}}, u_{\text{VL}}) \Psi(P_c, P_a, T_a) & P_a \geq P_c \\ -A_{\text{iv}}(\theta, \theta_{\text{OL}}, u_{\text{VL}}) \Psi(P_a, P_c, T_c) & \text{otherwise} \end{cases}
\]

\[
m_{\text{ev}} = \begin{cases} A_{\text{ev}}(\theta, \theta_{\text{OL}}) \Psi(P_c, P_c, T_c) & P_c \geq P_a \\ -A_{\text{ev}}(\theta, \theta_{\text{OL}}) \Psi(P_c, P_a, T_a) & \text{otherwise} \end{cases}
\]

where \( A_t(u_t) \cong A_t^{\max}(1 - \cos(u_t)) \). And, \( A_{\text{iv}} \) and \( A_{\text{ev}} \) are determined by crank angle \( \theta, u_{\text{VL}}, \theta_{\text{OL}} \) and \( \theta_{\text{OL}} \), as shown in Fig. 2.

From the energy conservation equations of the surge tank and the cylinder, the followings are obtained:

\[
dP_a \frac{dt}{v_a} = kR(T_a m_0 - T_{\text{ai}} m_{\text{iv}})
\]

\[
dP_c \frac{dt}{v_c} = kR(T_c m_0 - T_{\text{ev}} m_{\text{ev}}) - k \frac{dV_c}{dc} \frac{dt}{v_c} + \frac{1}{v_c} (q_b + q_w)
\]

where \( q_b \) and \( q_w \) are the heat release rate and the heat transfer rate, respectively,

\[
T_{\text{in}} = \begin{cases} \frac{T_a}{T_c} & P_a \geq P_c \text{ otherwise} \\ \frac{T_c}{T_a} & P_c \geq P_a \text{ otherwise} \end{cases}
\]

Provided benchmark simulator consists of the fundamental equations with the above-mentioned ones. In the present paper, the followings are mainly assumed:

- Valves’ overlap \( \theta_{\text{OL}} \) is given as a constant.
- Fuel injection is controlled to regulate air-fuel ratio \( \alpha \) to the reference one.
- Spark timing is controlled optimally.
- Pressure \( P_c \) and the temperature \( T_c \) of the exhaust confluence point are estimated because those are usually not measured.
- \( \tilde{\tau} \) is the filtered variable of the indicated torque which is estimated by using the estimated friction and loads, or is measured.

#### 2.1 Discrete-Crank Angle Modeling

As in [9],[10], a discrete-crank angle modeling with sampling interval \( h_s = (4\pi/\{N_{\text{cyl}}\}) \text{[rad]} \) is proposed, where \( h_s = h_d/\omega \text{x}\{N_{\text{cyl}}\} \).

**2.1.1 Flow model**

For the throttle flow, the following is directly derived from (3):

\[
M_i(k) \cong \int_{h_s}^{h_s+k} m_i(t)dt \approx h_s m_i(k)
\]

For the backflow of the burned gas through both valves from the exhaust side during the valves’ overlap \( m_{\text{back}} \), strictly \( m_{\text{back}} = -m_{\text{ev}} \).

It is difficult to solve \( P_c \) analytically during the overlap because \( P_c \) changes dynamically from \( P_a \) to \( P_c \) in less time than a sampling interval. In the literature [14]–[16], a backflow has been approximated as below:

\[
m_{\text{back}} = \min(A_{\text{iv}}(\theta, A_{\text{ev}}(\theta)) \Psi(P_a, P_c, T_c)
\]

Therefore the mass of the backflow during overlap is given by

\[
\text{Effective area [cm]}
\]

Fig. 2 Effective areas of the intake and exhaust valve.
Following piece-wise linear function, $$\Psi$$ where $$\theta_{OL} = \theta_{iv} + \theta_{ev}$$, $$I_{AOL}(u_{VL}, \theta_{OL}) \pm \int_{t_{iv}}^{t_{ev}} \min(A_{iv}(\theta), A_{ev}(\theta))d\theta$$, $$t_{iv}$$ and $$t_{ev}$$ are timings at the intake valve open, the exhaust valve close, respectively, and $$\theta_{iv}$$ and $$\theta_{ev}$$ are the crank angles corresponding to $$t_{iv}$$ and $$t_{ev}$$, respectively.

In the present paper, for (10), $$\psi$$ in (2) is approximated by the following piece-wise linear function,

$$\psi_{lin}(x) = \begin{cases} \psi(1 - x) & x \geq r_{lin}^\text{in} \\ \frac{x}{r_{lin}^\text{out}} & \text{otherwise} \end{cases}, \quad (11)$$

and the downstream pressure $$P_a$$ is also replaced by $$\frac{P_a}{\sqrt{T_a}}$$ because it is assumed that the flow from exhaust confluence point to the cylinder is equal to one from the cylinder to the surge tank. As a result, the mass of the backflow is approximately proposed:

$$M_{\text{back}} = \int_{t_{iv}}^{t_{ev}} m_{\text{back}}dt = \frac{I_{AOL}(u_{VL}, \theta_{OL})}{\omega} \Psi(P_a, P_e, T_e) \quad (10)$$

where $$\theta_{OL} = \theta_{iv} + \theta_{ev}$$, $$I_{AOL}(u_{VL}, \theta_{OL}) \pm \int_{t_{iv}}^{t_{ev}} \min(A_{iv}(\theta), A_{ev}(\theta))d\theta$$, $$t_{iv}$$ and $$t_{ev}$$ are timings at the intake valve open, the exhaust valve close, respectively, and $$\theta_{iv}$$ and $$\theta_{ev}$$ are the crank angles corresponding to $$t_{iv}$$ and $$t_{ev}$$, respectively.

For the intake valve flow as is the case in the backflow, $$\psi$$ is approximated by $$\frac{P_a}{\sqrt{T_a}}$$, which mainly depends on $$\omega_{iv}$$, $$\omega_{ev}$$, and $$\theta$$.

The indicated torque model can be derived from (7) by applying analytical approximation techniques in the previous study [9],[10]. In the present paper, where it is assumed that the combustion work is $$k_7 \frac{\phi}{\sqrt{T_a}}$$ and the pumping work is $$(V_{bdc} - V_{vdc}) \frac{P_a(k) - P_e}{P_e}$$, the following simplified torque model is proposed:

$$\tau_i(k) \approx \tau_i(k) = \frac{k_5 \Omega_i(k)}{\Omega_i} - k_6 k_7 M_i(k + 1)$$

where $$k_7$$ is a parameter which means the combustion efficiency, $$k_5 = \frac{V_{bdc} - V_{vdc}}{\Omega_i}$$, $$k_7 = \frac{\phi}{\sqrt{T_a}}$$, $$\alpha$$ is the air-fuel ratio, and $$H_f$$ is the lower heating value determined by the air-fuel ratio.

In addition, because the indicated torque contains “ripple” due to intermittent combustion, the filtered torque is evaluated. Therefore, consider through the filter $$\frac{1}{1 - \zeta s}$$ with $$\frac{1}{1 - \zeta s}$$, the following torque model is derived:

$$\tau_i(k + 1) = (1 - k_9) \bar{\tau}_i(k) + k_9 \left\{ k_5 \Omega_i(k) \bar{\tau}_i(k) + k_6 k_7 M_i(k + 1) \right\} \quad (17)$$

where $$k_9 = \frac{k_9}{\zeta_{\text{filt}}}, T_i$$ is the time constant of the filter.

### 2.2 State Equation

From (8), (12), (13), (16), and (17), the discrete-time state space model is given by

$$x(k + 1) = A_r x(k) + B_r v(k) + K_r P_e \quad (18)$$

where $x(k) = [P_a(k), \bar{\tau}_i(k), \bar{\tau}_i(k), \bar{\tau}_i(k)]^T$, $v(k) = [M_i(k), \eta(k)]^T, y(k) = [P_a(k), \bar{\tau}_i(k)]^T$. $A_r = \begin{bmatrix} 1 & 0 & -k_8 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ k_8 k_6 & 0 & 0 & k_5 k_6 & 1 - k_9 \end{bmatrix}$,

$$B_r = \begin{bmatrix} 0 & k_5 \Omega_i k_6 & 0 & 0 & 0 \\ 0 & k_5 \Omega_i & 0 & 0 & 0 \end{bmatrix}, K_r = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$$,

$$C_r = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$,
\( \zeta_1, \zeta_2, \) and \( \zeta_3 \) are the time-shifted variables of \( M_i \), i.e., \( \zeta_i(k - 1) = \zeta_i(k + 1) = M_i(k) \).

3. Flow-Based Controller Design

3.1 Steady State

Consider \( \tau_i \) and \( P_o \) as free parameters. Steady states are calculated by using (8), (14), and (18). Figures 4 and 5 show \( \beta_i \) in the steady states in the case of \( \theta_{UL} = 0 \) and \( \theta_{UL} = 60 \). Respectively, under the condition, \( \omega = 2000 \) [rpm], \( P_o = P_e = 101.3 \) [kPa], \( T_o = 25 \) [degC], \( T_e = 400 \) [degC], \( k_{m} = 0.3 \), and \( k_e = 0.95 \).

In the case of constant intake valve lift, in general, when the intake pressure is small (high), the torque is low (high) and the back flow is low (low).

Firstly, from Figs. 4 and 5, in the case of variable intake valve lift, it happens that the intake pressure is high and the torque is low simultaneously. And, from the regions of the upper left of Figs. 4 and 5, it is impossible that low intake pressure causes high torque. Then, in Fig. 5, \( \beta_i \) is high at low intake pressure. That means the large mass of the backflow. Finally, it is found in Figs. 4 and 5 that \( \beta_i \) is more sensitive to \( P_o \) in the case of large overlap than the case of small overlap. Therefore, taking care of the constraint of IEGR ratio is very important in the case of large overlap.

3.2 Constraints

The followings are considered from the practical viewpoint:

(a) bounds of input levels

\[ u_{t_{\min}} \leq u_t \leq u_{t_{\max}}, \quad u_{VL_{\min}} \leq u_{VL} \leq u_{VL_{\max}} \]

(b) bounds of input speeds

\[ \delta u_{t_{\min}} \leq \delta u_t \leq \delta u_{t_{\max}}, \quad \delta u_{VL_{\min}} \leq \delta u_{VL} \leq \delta u_{VL_{\max}} \]

(c) upper bound of IEGR ratio

\[ \gamma_c \leq \gamma_{c_{\max}}, \quad \text{where } \gamma_{c_{\max}} \text{ is threshold of misfiring.} \]

In the present paper, control design based on flow \( v \) is proposed. Therefore, the constraints with respect to \( u_t \) and \( u_{VL} \) are transformed into mixed constraints of \( v(k) \) and \( x(k) \).

Firstly, constraint (a) is transformed into the mixed constraint. For the throttle valve, the following is obtained from (8):

\[ M_i^{\min} \leq M_i(k) \leq M_i^{\max} \]

where \( M_i^{\min} \) and \( M_i^{\max} \) respectively corresponds to \( u_{t_{\min}} \) and \( u_{t_{\max}} \) in (8). It is assumed that \( M_i^{\min} = 0 (u_{t_{\min}} = 0) \) in such a way that the constraint is convex for \( v(k) \) and \( x(k) \). And only for \( M_i^{\max} \), the nonlinear flow function (2) is approximated by a linear one (11) as follows:

\[ M_i^{\max}(k) = h_u A_i^{\max}\Psi_{\lin} \]

\[ = \left\{ \begin{array}{ll}
A_i(u_{t_{\max}})\delta_2(P_o - P_a(k)) & \frac{P_{\lin}(k)}{P_o} \geq \frac{P_{\lin}(k)}{P_o} \\
A_i(u_{t_{\max}})^2 c_2 P_o & \text{otherwise}
\end{array} \right. \]

\[ \delta_2 = \sqrt{\frac{\kappa}{2\rho}}, \quad c_2 = \sqrt{\frac{\kappa}{2\rho} + \frac{\kappa}{\rho} + \kappa} \]

where \( \delta_2 = \sqrt{\frac{\kappa}{2\rho}}, \quad c_2 = \sqrt{\frac{\kappa}{2\rho} + \frac{\kappa}{\rho} + \kappa} \).

For the intake valve lift, the following is obtained from (14):

\[ \eta_{\min}(k) \leq \eta(k) \leq \eta_{\max}(k) \]

where \( \eta_{\min}(k) = k_{1_{\min}} P_a(k), \quad \eta_{\max}(k) = k_{1_{\max}} P_a(k) \), \( k_{1_{\min}} \) and \( k_{1_{\max}} \) respectively corresponds to \( u_{VL_{\min}} \) and \( u_{VL_{\max}} \) via \( k_{m} \) and \( k_{e} \) in (14).

Secondly, constraint (b) is transformed into the mixed constraint as well as (a) as below:

\[ \delta M_{i_{\min}}(k) \leq \delta M_i(k) \leq \delta M_{i_{\max}}(k) \]

\[ \delta \eta_{\min}(k) \leq \delta \eta(k) \leq \delta \eta_{\max}(k) \]

where \( \delta M_{i_{\min}} \) and \( \delta M_{i_{\max}} \) respectively corresponds to \( \delta u_{t_{\min}} \) and \( \delta u_{t_{\max}} \), and \( \delta \eta_{\min} \) and \( \delta \eta_{\max} \) respectively corresponds to \( \delta u_{VL_{\min}} \) and \( \delta u_{VL_{\max}} \).

\[ \delta M_i = h_u \Psi_{\lin} \frac{\partial k}{\partial P_o} \delta u_t \]

\[ \delta \eta = P_o \frac{\partial k}{\partial P_o} \delta \eta_{VL} \]

Thirdly, constraint (c) is directly expressed from (15) by the following form of the mixed constraint:

\[ M_{i_{\min}} + M_{i_{\max}}^{\lin}(k) \leq \gamma_{c_{\max}} M_a(k + 2) \]

Finally, considering (12) and (13), the some selected constraints from (19) through (23) are combined to get the following form:

\[ C_i^T x(k) + D_i^T v(k) \leq E_i^T p_i \]

where \( p_i = [P_e, P_o, M_{i_{\max}}]^T \).

3.3 Delta Input Formation

For (18), the following \( \delta v \)-formulation is used:

\[ v(k) = v(k - 1) + \delta v(k) \]

For (18) and (25), the expanded state space model with the new states \( v(k - 1) \) is given by
\[ \xi(k + 1) = A\xi(k) + B\nu(k) + Kp_x \]
\[ y(k) = C\xi(k) \]

and for (24) and (25), the mixed constraints are given by
\[ C^T\dot{\xi}(k) + D^T\nu(k) \leq E^T p_x \]
where \( \dot{\xi}(k) = [x(k)^T, v(k - 1)^T]^T \).

\[ A = \begin{bmatrix} A_x & B_x \\ 0 & I_2 \end{bmatrix}, \quad B = \begin{bmatrix} B_x \\ I_2 \end{bmatrix}, \quad K = \begin{bmatrix} K_x \\ 0_{2 \times 2} \end{bmatrix}, \quad C = \begin{bmatrix} C_x & 0_{2 \times 2} \end{bmatrix}, \quad C^C = \begin{bmatrix} C^C_x & D^C_x \end{bmatrix} \]

subject to (26) and (27), where
\[ u_{VL}(t) \]

\[ \min \left\{ \sum_{k=0}^{N_t} e(t+k)^T Q e(t+k) + \sum_{k=1}^{N_c} \delta v(t+k-1)^T R \delta v(t+k-1) \right\} \]

\[ N_c = 1 \] if \( \nu(k) \) and \( \nu(VL) \) are directly designed, the predictive control is set-free of ˜(t), but , \( \beta^C \) and \( \beta^C \) are respectively.

\[ N_t = N_p, \quad \text{and horizon is } N_{pc} \] as nominal values, and \( \nu(t) \) is calculated as in Section 3.1.

3.4 Predictive Controller

Firstly, a predictive controller is designed to obtain the optimal \( \delta v(k) \) based on an objective function. Let \( y(k) \) track reference signal \( r(k) \), where \( r(k) = [P^*_u, \tilde{\tau}^*_i]^T \). Note that, given \( \tilde{\tau}^*_i \) and \( \beta^C \), steady state \( P^*_u \) is calculated as in Section 3.1.

Namely, the following objective function is considered:
\[ \min \left\{ \sum_{t=0}^{T} \sum_{k=0}^{N_c} e(t+k)^T Q e(t+k) + \sum_{k=1}^{N_c} \delta v(t+k-1)^T R \delta v(t+k-1) \right\} \]

subject to (26) and (27), where \( t \) is current time, \( e(t+k) \), \( y(t+k) - r(t+k) \), and prediction horizon is \( N_t \) to \( N_p \), and control horizon is \( N_c \). It is assumed that \( Q \) is a symmetric positive-semidefinite matrix and \( R \) is a symmetric positive-definite matrix.

Furthermore, the above tracking problem is easily transformed into the following form:
\[ \min_{U} \frac{1}{2} U^T H U + p(t)^T FU \]
\[ s.t. \quad GU \leq W + E p(t) \]
where \( U = [\delta v(t)^T, \cdots, \delta v(t + N - 1)^T]^T \), \( p(t) = [v(t)^T, p_x(t)^T, r(t + 1)^T, \cdots, r(t + N)^T]^T \), \( W = 0 \), and \( H \) is a symmetric positive-definite matrix.

Let \( U^*(p(t)) \) be an optimal solution of (29), the optimal variation \( \delta v(t) \) at time \( t \) consists simply of the first two components of \( U^*(p(t)) \).
\[ \delta v(t) = [I_2, 0_{2 \times 2}, \cdots, 0_{2 \times 2}] U^*(p(t)) \]

Finally, the designed optimal masses \( M_i(k) \) and \( \eta_i(k) \) are inverted from (8) and (14), respectively. Namely, the optimal \( u_i(t) \) and \( u_{VL}(t) \) are easily given by, respectively.
\[ u_i(t) = \cos^{-1} \left( 1 - \frac{M_i(t)}{h_i(t) \cdot \max \{\Psi_i(t)\}} \right) \]
\[ u_{VL}(t) = \frac{1}{k_{\nu}(t)} \left( \frac{\eta_i(t) \cdot R T_a}{P_x(t) (V_{hdc} - V_{dsc})} \cdot \omega(t) \right) \]

Figure 6 shows the block diagram of the proposed controller. Here, \( \tilde{\tau} \) indicates (30) and the actual mass corresponding to (8) and (14), respectively. And \( H \) and \( S \) are the zero-order holder and the sampler per 120 deg crank angle, respectively.

Note the following about solution of (29):

1. If \( u_t \) and \( u_{VL} \) are directly designed, the predictive control problem is treated as nonlinear programming to make it more difficult to solve the optimal sequence.
2. If matrices \( H, F, G, \) and \( E \) are constants, the problem (29) can be preliminarily solved by multiparametric programming [17]. The computational cost in the implementation may be reduced.
3. The matrices in (18) and (24) depend on driving conditions (especially, \( u_{VL} \) and \( \theta_{OL} \)). Therefore, at each time step \( t \), the problem (29) is solved by active set methods to get the optimal sequence \( U^*(p(t)) \).

3.5 Observer

The designed mass \( v(t) \) which is calculated in the controller might not be equal to the actual mass in the plant because of the error between the flow models \( g(\cdot) \) and \( \dot{g}(\cdot) \). Therefore, in order to realize offset-free control to reference signals, the predictive controller uses the estimate \( \dot{v} \) by minimum order observer instead of \( v \) of (25) as the present state. That means any plant/model mismatch is lumped into \( \dot{v} = v - \dot{v} \), the input disturbance is estimated to realize offset-free control [18].

Calculating \( v(k) \) of (25) using output \( \delta v(k) \) of the predictive controller, \( v(k) \) may exceed bounds: \( v(k) < v_{\min}(P, \dot{v}) \) or \( v(k) > v_{\max}(P, \dot{v}) \) because the input constraints are not considered in the integrator in Fig. 6. Therefore, the following countermeasure is taken:
\[ \delta v(k) = f(\delta v(k), v(k - 1), v_{\min}(k), v_{\max}(k)) \]
\[ = \begin{cases} v_{\min}(k) - v(k - 1), & v(k) < v_{\min}(k) \\ v_{\max}(k) - v(k - 1), & v(k) > v_{\max}(k) \\ \delta v(k), & \text{otherwise} \end{cases} \]

where \( v_{\min} = [M_{\min}, \eta_{\min}]^T \) and \( v_{\max} = [M_{\max}, \eta_{\max}]^T \).

4. Simulation Results

The above proposed controller is validated by using the benchmark simulator. In the following examples, parameters are set: \( \omega = 2000[\text{rpm}] \), \( \theta_{OL} = 60[\text{deg}] \), \( P_r = 101.3[\text{kPa}] \), \( T_u = 25[\text{degC}] \), and \( T_r = 400[\text{degC}] \) as engine conditions, \( k_{\eta i} = 0.3 \), \( k_e = 0.95 \), and \( T_i = 0.1 \) as nominal values, and \( \zeta_n = 4 \) and \( N_r = N_u = 6 \) as controller parameters.

4.1 Example 1: Variable \( \dot{\tau}^*_i \) and Constant \( \beta^C \) Reference Signals

Figure 7 shows \( \dot{\tau}, \alpha_{OL}, \beta_i, \) and inputs \( u_i \) and \( u_{VL} \), given variable \( \dot{\tau}^*_i \in [100, 140][\text{N} \cdot \text{m}] \) and constant \( \beta_i^C = 20[\%] \) reference signals, where \( \alpha = \text{diag}(10, 10) \), \( R = \text{diag}(1, 10) \). As shown in Fig. 7, the controller realizes offset-free of \( \dot{\tau} \) and \( P_r \), but,
model $\beta_c$ (labeled “mpc” in Fig. 7) is not reached to the reference 20[%]. The reason is there is the plant/model mismatch as calculating $P^*_a$. Figure 8 shows effects of offset-free control. Especially, $\eta$ labeled “mpc” is not equal to the estimated $\hat{\eta}$ labeled “obsv”.

4.2 Example 2: Quick Lift Control

Consider the consequence of quick lift control, where $Q = \text{diag}(10, 10)$ and $R = \text{diag}(1, 1)$. The reference torque is given as a steep square wave.

Figure 9 shows the case without considering the upper of IEGR ratio. As increasing the reference torque, at 3 second, firstly, the high intake pressure and the upward lift cause the increase of air into a cylinder to undershoot IEGR ratio. Consequently, the downward throttle angle due to the low reference pressure accelerates the increase of the backflow of burned gas to overshoot IEGR ratio. On the other hand, as decreasing the reference torque, at 6 second, firstly, the low intake pressure and the downward lift cause the decrease of air into a cylinder to overshoot IEGR ratio. Consequently, the upward throttle angle due to the high reference pressure accelerates the decrease of the backflow of burned gas to undershoot IEGR ratio. Both the overshoots of IEGR ratio should be noted. In fact, the direct approach is to consider the constraint of IEGR ratio, and the indirect approach is to smooth lift control.

Figure 10 shows the case considering the upper of IEGR ratio. This direct approach prevent the latter overshoot of IEGR ratio, but insufficiently prevent the former one, because of the short horizon.

4.3 Example 3: Effect with Constraints and Torque Priority Control

Consider $\tilde{\tau}_i^*$ and $P^*_a$ directly in (29).

Figure 11 shows the case of deceleration, that is decreasing the reference signal $\tilde{\tau}_i^*$. Here, predictive controller is executed without considering the upper bound of IEGR ratio 22[%], given reference signals $\tilde{\tau}_i^*$ and $P^*_a$. The controller realizes offset-free of $\tilde{\tau}_i$ and $P^*_a$, but between 5 and 8 second, IEGR ratio is increasing around 22[%]. In that situation, real engine may be misfiring.

Therefore, the case considering the upper bound of IEGR ratio is indicated in Fig. 12. Note that, here, considering model/plant mismatch, the upper should be set at not real one (here, 22[%]), but virtual one (here, 20[%]). Obviously, neither references 100[Nm] nor 85[kPa] can be realized between 5 and 8 second, because of the upper bound of IEGR ratio.

Consider that torque is preferentially controlled. Figure 13 shows the torque priority control. Both the torque and the intake pressure is controlled without offset between 0 and 5 second, where $Q = \text{diag}(10, 10)$ and $R = \text{diag}(1, 10)$. On the other hand, while the incompatible reference signals are given between 5 and 8 second, torque control is given the higher priority to overshoot IEGR ratio. Consequently, the upward throttle angle due to the high reference pressure accelerates the decrease of the backflow of burned gas to undershoot IEGR ratio. Both the overshoots of IEGR ratio should be noted. In fact, the direct approach is to consider the constraint of IEGR ratio, and the indirect approach is to smooth lift control.
by setting $Q = \text{diag}(0, 10)$. As a result, at all time, only torque is controlled without offset, considering the IEGR constraint.

5. Conclusions

The present paper has proposed a control design method based on models of air and burned gas flows for the automotive engine with variable valves. The features of the proposed method are as follows.

- Flow-based control design enables the engine electronic control unit to solve easily the predictive control problem with constraints (that is a quadratic programming problem, not a general nonlinear programming problem).
- Different types of inputs, the throttle angle and the in-
take valve lift, can be optimized considering the interaction, constraints such as the bounds of input levels, inputs speeds, and IEGR with the overlap.

In the future, one will challenge the followings:

- reduction of the computational cost by using multi-parametric programing and so on
- the overlap control
- external exhaust gas recirculation in addition to IEGR

References


