MIMO PID Controller Design Based on Integral-Type Optimal Servomechanism and Its Extension to Model-Following-Type

Hiroyuki Kondo* and Yoshimasa Ochi**

Abstract: This paper presents a design method of an MIMO integral preceded by proportional-derivative (I-PD) controller based on an integral-type optimal servomechanism. The proposed method consists of two steps. First, a given plant is represented in a specific state-space form, and then an integral-type optimal servo controller is designed. Although the resultant controller does not always become a typical I-PD one, when the order of a given MIMO plant is equal to or less than twice the number of the outputs, the resultant control law is equivalent to an I-PD one. Moreover, the proposed I-PD controller design can be extended to a model-following type by adding a reference model and a feedforward compensator for a desirable output response. Controller design examples and numerical simulation studies are carried out in order to demonstrate that the proposed design method has sufficient effectiveness.

Key Words: MIMO system, plant representation, PID control, linear quadratic regulator, integral-type optimal servomechanism.

1. Introduction

Proportional-integral-derivative (PID) control is most widely used in industries because the controller can be intuitively tuned due to its simple structure and the clear meaning of respective control gains. However, it is not an easy problem to devise a simple and theoretically assured design method, in spite of the simple structure of the control law. In fact, to solve the problem many studies on PID controller design have been conducted [1],[2]. Moreover, it is known that more difficulties arise in multiple-input/multiple-output (MIMO) systems than in single-input/single-output (SISO) ones from the interactions between the multiple inputs/outputs [3]. A way to guarantee control performance and robustness in PID control design is to take advantage of a state-space-based control theory. Actually many researchers have taken that way, whereas their methods often tend to be theoretically difficult and/or the design procedure is liable to become complicated.

The authors of this paper have also tackled the problem by the same way and proposed a PID controller design method for SISO systems based on Integral-type Optimal Servomechanism (IOS), which is a kind of the linear quadratic regulator (LQR). Although other PID controller design methods based on the IOS have also been proposed [4]–[6], they do not directly apply the IOS. In contrast, the authors' method [7] directly obtains the PID gains from the IOS feedback gains, since the IOS is a state-feedback control method, while enjoying the advantages of the LQR. Therefore, this design method is relatively simpler than the conventional ones and desirable control performance and stability margins that the LQR has are expected to be obtained.

The proposed controller design method for SISO systems [8] consists of two steps. First a plant transfer function is transformed into a specific state-space representation that is augmented to include the input and its derivatives in the state variables, and then an IOS controller is designed. Although the resultant controller does not generally become an integral preceded by proportional-derivative (I-PD) one, it is equivalent to a typical I-PD one in the case that the transfer function of a given plant is a strictly proper second-order rational function. Furthermore, it can be transformed into a one-degree-of-freedom PID controller and a proportional-integral preceded by derivative (PI-D) one depending on the plant or control objectives. In this paper, we propose a similar method for MIMO plants. In this method, if the order of a given MIMO plant is equal to or less than twice the number of the outputs, the resultant control law will be an I-PD one for the system. Moreover, we propose the extension to a model-following I-PD (MF-I-PD) controller.

This paper is organized as follows. In the next section, the proposed design method is described in detail. In Section 3, to illustrate the effectiveness and usefulness of the proposed method, two design examples and numerical simulation results are shown. Finally, conclusions are given with future works in Section 4.

2. Controller Design

2.1 Plant Representation

Consider a linear time-invariant plant described by the state-space representation

\[\begin{align*}
\dot{x} &= Ax + Bu, \\
y &= Cx,
\end{align*}\]

where \(x \in \mathbb{R}^n\), \(u \in \mathbb{R}^p\), and \(y \in \mathbb{R}^q\) are vectors of state variables, inputs, and outputs, respectively. The integer \(n\) and \(p\) are the numbers of state variables and inputs/outputs, respectively. \(A \in \mathbb{R}^{n \times n}\), \(B \in \mathbb{R}^{n \times p}\) and \(C \in \mathbb{R}^{q \times n}\) are constant matrices. \(C\) is assumed to be of row full rank. Additionally, the plant is assumed to be controllable and observable. For the controller design, the plant representation is transformed as follows.
First, the vector $z_i$ is defined as
\[
z_i = y^{(i-1)},
\] (2)
where the superscript $(i - 1)$ denotes $(i - 1)$-th order derivative for $i = 1, \cdots, q$ and $q \leq n$ is determined later. The vector defined by
\[
z = \begin{bmatrix} z_1^T & z_2^T & \cdots & z_q^T \end{bmatrix}^T,
\] (3)
is also described by
\[
z = M_0 x + N_0 u_0,
\] (4)
where
\[
M_0 = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{q-1} \end{bmatrix}, \quad u_0 = \begin{bmatrix} u \\ u \\ u \\ \vdots \\ u \end{bmatrix},
\] (5)
and $0_{p \times p}$ is an $n \times n$ zero matrix. The integer $q$ is determined to be the minimum one such that $M_0$ contains $n$ linearly independent rows.

Next, we choose $n$ linearly independent rows from $M_0$ and construct $n \times n$ matrix $M_s$ by stacking the $n$ rows in $M_0$. Defining an $n \times p$ matrix $E$ such that $M_s = EM_0$, we construct an $n \times (p(q-1))$ matrix $N_s = EN_0$ and a vector $z_s = Ez$. The respective linearly independent rows should be selected in incremental numerical order of row number from the first row to the last row of $M_0$. Then, the order of derivatives of the outputs comprised in $z_s$ is as low as possible. Note that when selecting the last $r$ rows ($r < p$) from $CA^{q-1}$ which includes more than $r$ rows independent of the rows selected from $CA^i (i = 0, \ldots, q - 2)$, the selection of $r$ independent rows from $CA^{q-1}$ is not unique. In such a case, designers can appropriately select the rows form $CA^{q-1}$ taking account of the availability or reliability of output measurements, especially for higher-order derivative terms, or the contribution of the derivative terms to control objectives. Then, $z_s$ is expressed by
\[
z_s = M_s x + N_s u_0,
\] (6)
Since $M_s$ is non-singular, Eq. (6) can be rewritten as
\[
x = M_s^{-1}(z_s - N_s u_0),
\] (7)
Substituting Eq. (7) to Eq. (1), we obtain
\[
\dot{z}_s = A_z z_s + B_s u_0 + N_{sq-1} u^{(q-1)},
\] (8)
where
\[
N_s = \begin{bmatrix} N_{s1} & N_{s2} & \cdots & N_{sq-2} & N_{sq-1} \end{bmatrix},
\] (9)
\[
A_z = M_s A M_s^{-1},
\] (10)
\[
B_s = -A_z N_s + [M_s B] N_{s1} N_{s2} \cdots N_{sq-1},
\] (11)
In the case of $q = 2$, $N_s = N_{s1}$ and $B_s = -A_z N_s + M_s B$. Furthermore, when $q = 1$, since $z_s = z = y$ and $C \in \mathbb{R}^{m \times n}$ is a regular matrix, the control system using $y$ as a feedback outputs vector is equivalent to a state feedback one. Therefore, designing IOS, we directly obtain an I-P control law.

From the definition of $u_0$ in Eq. (5), we have the following identity:
\[
\dot{u}_0 = B_1 u_0 + B_2 u^{(q-1)},
\] (12)
where
\[
B_1 = \begin{bmatrix} 0_{p(q-2) \times p} & I_{p(q-2)} \\ 0_{p \times p} & 0_{p \times p(q-2)} \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0_{p(q-2) \times p} \\ I_p \end{bmatrix},
\] (13)
and $I_p$ is a $p \times p$ identity matrix. With the definition of $z_0 = z_1^T$ and $C_s = CM_1$, Eqs. (8) and (12) are represented as
\[
\dot{z}_0 = A_z z_0 + B_s u^{(q-1)},
\] (14)
where
\[
A_s = \begin{bmatrix} A_z & B_1 \\ 0_{p \times p} & B_2 \end{bmatrix}, \quad B_s = \begin{bmatrix} N_{sq-1} \\ B_2 \end{bmatrix},
\] (15)
and the output equation is expressed as
\[
y = C_s z_s,
\] (16)
where
\[
C_s = C_s \begin{bmatrix} I_p & -N_0 \end{bmatrix}.
\] (17)

2.2 Controller Design Based on IOS

We design an IOS for the system composed of Eqs. (14) and (16). In this study, we employ the design method of an IOS controller by Smith and Davison [9]. With the definition of control error $e = y_{ref} - y$, where $y_{ref}$ is a constant reference output vector, another augmented plant can be defined by
\[
\frac{dx}{dt} = A_s x + \begin{bmatrix} 0_{p \times (p+q)} \\ -C_s \end{bmatrix} e + \begin{bmatrix} B_2 \\ 0_{p \times p} \end{bmatrix} u^{(q)}. \tag{18}
\]
For this augmented system, we define the quadratic cost function as
\[
J = \int_{0}^{\infty} (z_s^T Q z_s + e^T Q e + u^{(q)} R u^{(q)}) dt,
\] (19)
where $Q_s$ is a semi-positive definite matrix and $Q$ and $R$ are positive definite matrices, respectively. By using the LQR theory, we obtain the optimal control law that minimizes Eq. (19), i.e.,
\[
u^{(q)} = -K_z z_s - K_u u_0 - K_e e,
\] (20)
where $[K_z \ K_u \ K_e]$ is the LQR feedback gain matrix for the augmented state of Eq. (18). Integrating Eq. (20) yields
\[
\nu^{(q-1)} = -K_z z_s - K_u u_0 - K_e \int_{0}^{T} e dt.
\] (21)
Since the right-hand side of Eq. (21) includes $u$ and its derivatives, we obtain $u$ as follows. By the definition of $z_s$, it is expressed as
\[
z_s = \begin{bmatrix} y_{s1}^T \\ y_{s2}^T \\ \cdots \\ y_{sq-1}^T \end{bmatrix},
\] (22)
where \( y_u \) consists of the outputs corresponding to the chosen \( n \) linearly independent rows of \( M_0 \) in order to construct \( M_x \). Substituting Eq. (22) to Eq. (21) and using the definition of \( u_0 \), we obtain the control law:

\[
u = -D_u(s)^{-1} \left( K_0 y + K_1 s y + \cdots + K_{q-1} s^{q-1} y_{q-1} + K_e e \right), \tag{23}\]

where

\[
\begin{align*}
D_u(s) &= s^{q-1} I_p + (K_0 + K_1 s + \cdots + K_{q-2} s^{q-2}) , \\
K_e &= \left[ K_0 \quad K_1 \quad \cdots \quad K_{q-1} \right], \\
K_e &= \left[ K_0 \quad K_1 \quad \cdots \quad K_{q-2} \right].
\end{align*}
\]

The resultant control law of Eq. (23) does not become typical I-PD one in the case of \( q \geq 3 \), since it includes second or higher-order derivative terms. However, it becomes the I-PD control law in the case of \( q = 2 \), as shown below. Moreover, the I-P (integral preceded by proportional) control law is obtained in the case of \( q = 1 \).

In the case of \( q = 2 \) (and \( n = 2 p \)), \( M_x = [C^T (CA)^T]^T \) and \( y = y_1 \) can be chosen, and then Eq. (23) becomes

\[
u = -D_u(s)^{-1} \left( K_0 y + K_1 s y + K_e e \right) \tag{25}\]

where \( D_u(s) = s I_p + K_{q_0} \). Since the control law of Eq. (25) has the similar but not equivalent form to that of the I-PD one, we call it “pseudo I-PD” control law in this paper. Eq. (25) can be transformed into

\[
u(s) = -K_p y - (s T_D + I_p)^{-1} K_D s y + \frac{K_e}{s} e, \tag{26}\]

where \( T_D = K_{q_0} \), \( K_I = -T_D K_e \), \( K_p = T_D(K_0 - K_I) \), and \( K_D = T_D(K_1 - K_p) \). The last term on the right-hand side of Eq. (26) is of the exogenous signal \( y_{ref} \), and it can be removed, since it does not affect closed-loop stability. Therefore, the control law of Eq. (26) can be rewritten as

\[
u(s) = -K_p y - (s T_D + I_p)^{-1} K_D s y + \frac{K_e}{s} e. \tag{27}\]

Note that in the control law the derivative action is inherently given by the proper transfer function \((s T_D + I_p)^{-1} s\); hence the control law of Eq. (27) does not include any approximation for the derivative action. The block diagram of the I-PD control system is shown in Fig. 1.

![Block diagram of an I-PD control system](image)

**Fig. 1** Block diagram of an I-PD control system.

### 2.3 Extension to Model-Following I-PD Control

An extension of the IOS is the model-following servo controller (MFSC) [10], where a reference model and a feedforward compensator from the model are added to the IOS. The feedforward gain is simultaneously determined as a part of optimal control gains. Although the MFSC originally is a state feedback type, a model-following I-PD (MF-I-PD) controller of the output feedback type can be designed by using the proposed design method.

Suppose a reference model is given as

\[
\begin{align*}
\dot{x}_r &= A_r x_r + B_r r, \\
y_r &= C_r x_r,
\end{align*}
\]

where \((A_r, B_r)\) is controllable, \((A_r, C_r)\) is observable and \( r \) is constant. We define the control error as \( e = y_r - y \), and make an augmented system as

\[
\frac{d}{dt} \begin{bmatrix} \dot{z}_a \\ \dot{x}_r \\ \dot{u} \end{bmatrix} = \begin{bmatrix} A_a & 0 & 0 \\ C_r & 1 & 0 \\ 0 & 0 & A_1 \end{bmatrix} \begin{bmatrix} z_a \\ x_r \\ u \end{bmatrix} + \begin{bmatrix} B_a \\ 0 \\ 0 \end{bmatrix} u. \tag{29}\]

Similarly to Eq. (21), integrating the optimal control law given by the LQR theory yields the control law:

\[
u^{(w)} = -K_e z_r - K_e u_0 - K_e \int e \, dt + K_e x_r, \tag{30}\]

with positive definite matrices \( P_{11} \) and \( P_{12} \) that satisfy the following algebraic Riccati equations:

\[
\begin{align*}
A_{11}^T P_{11} + P_{11} A_{11} + Q_{xc} - P_{11} B_r R^{-1} B_r^T P_{11} &= 0, \\
A_{11}^T P_{12} + P_{12} A_{11} + P_{12} A_{22} - P_{12} B_r R^{-1} B_r^T P_{12} &= 0,
\end{align*}
\]

where

\[
\begin{align*}
A_{11} &= \begin{bmatrix} A_a & 0 \\ -C_a & 1 \end{bmatrix}, \\
A_{12} &= \begin{bmatrix} 0 \\ C_r \end{bmatrix}, \\
A_{22} &= A_r, \\
B_1 &= \begin{bmatrix} B_a \\ 0 \end{bmatrix}.
\end{align*}
\]

and the diagonal matrix \( Q_{xc} = \text{diag}(Q_x, Q_c) \). The last term of (30) is the feedforward control input and the first three ones are exactly the same as those of Eq. (21). In the case of \( q = 2 \) (and \( n = 2 p \)), Eq. (30) can also be transformed into

\[
u(s) = -K_p y - (s T_D + I_p)^{-1} K_D s y + \frac{K_e}{s} e \tag{36} + (s I_p + K_{q_0})^{-1} K_e x_r,
\]

Figure 2 shows a block diagram of this MF-I-PD control system, where \( P_{11}(s) = C_r(s I - A_r)^{-1} B_r \).

![Block diagram of an MF-I-PD control system](image)

**Fig. 2** Block diagram of an MF-I-PD control system.
3. Design Examples

3.1 Example 1: Stable Fourth-Order System

We consider the lateral-directional motion of the F-16 at sea level and the airspeed of 153.0 [m/sec] [11]. With the definition of the elements of $x$ are the sideslip angle $\beta$ [rad], the roll angle $\phi$ [rad], the roll rate $\rho$ [rad/sec] and the yaw rate [rad/sec], those of $u$ are the aileron angle $\delta_a$ [deg] and the rudder angle $\delta_r$ [deg], and those of $y$ are the roll angle $\phi_y$ [deg] and the sideslip angle $\beta_y$ [deg], the constant matrices in Eq. (1) are given by

$$ A = \begin{bmatrix} -0.3220 & 0.0640 & 0.0364 & -0.9917 \\ 0 & 0 & 1 & 0.0037 \\ -30.6492 & 0 & -3.6784 & 0.6646 \\ 8.5396 & 0 & -0.0254 & -0.4764 \end{bmatrix} , $$

$$ B = \begin{bmatrix} 0.0003 \\ 0 \\ -0.7333 \\ -0.0319 \end{bmatrix} , $$

$$ C = \begin{bmatrix} 0 & 57.30 & 0 & 0 \\ 57.30 & 0 & 0 & 0 \end{bmatrix} . $$

Then, the constant matrices in Eq. (6) become

$$ M_x = \begin{bmatrix} 0 & 57.30 & 0 & 0 \\ 57.30 & 0 & 0 & 0 \\ 0 & 0 & 57.30 & 0.2120 \\ -18.45 & 3.667 & 2.086 & -56.82 \end{bmatrix} , $$

$$ N_x = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.0172 & 0.0458 \end{bmatrix} . $$

Selecting the weighting matrices of the cost function as $Q_c = \text{diag}(300, 3000, 0, 0, 0, 0)$, $Q = \text{diag}(1.0 \times 10^4, 5.0 \times 10^4)$, and $R = I_2$, where “diag(·)” means a diagonal matrix which has diagonal elements (·), we obtain the gains:

$$ K_{s0} = \begin{bmatrix} -38.65 \\ 6.124 \\ 88.92 \end{bmatrix} , \quad K_{s1} = \begin{bmatrix} -4.443 \\ 4.903 \\ 0.5957 \\ 21.93 \end{bmatrix} , $$

$$ K_{s0} = \begin{bmatrix} -19.31 \\ -1.166 \\ -1.318 \\ 12.49 \end{bmatrix} , \quad K_r = \begin{bmatrix} 98.69 \\ -36.13 \\ -16.16 \\ -220.7 \end{bmatrix} . $$

Then, the parameters of Eq. (27) are

$$ K_P = \begin{bmatrix} -1.724 \\ 1.693 \\ 0.2475 \\ 5.859 \end{bmatrix} , \quad K_I = \begin{bmatrix} -5.064 \\ 2.957 \\ 0.7592 \\ 17.98 \end{bmatrix} , $$

$$ K_D = \begin{bmatrix} -0.1400 \\ 0.2455 \\ 0.01310 \\ 1.312 \end{bmatrix} , \quad T_D = \begin{bmatrix} 0.05211 \\ 0.004866 \\ 0.005501 \\ 0.08058 \end{bmatrix} . $$

One of the state-space representation for the selected reference model

$$ y_r = \frac{1}{(0.2s + 1)^2} r, $$

has constant matrices:

$$ A_r = \begin{bmatrix} -10 & -6.25 & 0 & 0 \\ 4 & 0 & 0 & 0 \\ 0 & 0 & -10 & -6.25 \\ 0 & 0 & 4 & 0 \end{bmatrix} , \quad B_r = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix} , $$

$$ C_r = \begin{bmatrix} 0 & 3.125 & 0 & 0 \\ 0 & 0 & 0 & 3.125 \end{bmatrix} . $$

With this reference model, we obtain the feedforward gain matrix:

$$ K_f = \begin{bmatrix} -26.75 & -83.54 & 12.92 & 36.50 \\ 4.189 & 13.31 & 76.77 & 219.0 \end{bmatrix} . $$

Figures 3 and 4 show time responses to the step reference output $y_{ref} = [45 5]^T$, where input signals $\delta_a$ and $\delta_r$ are re-
spectively limited by ±21.5 [deg] and ±30.0 [deg], to compare the inputs and outputs of the two-input-two-output (2I2O) controllers, i.e., the pseudo I-PD of Eq. (25) and the proposed I-PD of Eq. (27), with 2I2O control system composed of the two SISO controllers by [7]. In order to obtain the two SISO controllers by the design method of [7], it is required to separate the two SISO systems from the given original 2I2O plant and to reduce the order of the respective fourth-order SISO systems to the second. However, since the order of the given 2I2O plant is twice the number of the outputs, the proposed MIMO I-PD controller design method can directly be applied. Although the initial responses by all the 2I2O controllers are slightly slower than those by the two SISO ones, the settling time by all the 2I2O controllers is shorter than the SISO ones. Moreover, the proposed I-PD controller achieves the shortest rise time and settling time among the 2I2O controllers.

Figures 5 and 6 show time responses by the proposed MF-I-PD controller and the above I-PD one with the reference model \( P_R(s) \). The solid lines follow the reference outputs closer than the dashed-dotted lines, which means that the extension to the model-following type realizes higher control performance.

Table 1 summarizes the robust stability margins obtained by the respective controllers in four metrics [12]–[14], where \( 1/\| KS \|_{\infty} \), \( 1/\| T \|_{\infty} \), \( 1/\| S \|_{\infty} \), and \( b_{P,K} \) are the metric for additive uncertainty, the inverse of maximum complementary sensitivity, that of maximum sensitivity, and generalized stability margin, respectively. Note that the robust stability margins by the proposed I-PD controller are equal to those of the pseudo I-PD one, i.e. the transformation into Eq. (27) does not affect closed-loop stability of Eq. (25). All the 2I2O controllers give larger robust stability margins than those of the SISO system. Particularly, the proposed I-PD controller gives the largest \( b_{P,K} \approx 1/\| KS \|_{\infty} \) and sufficient \( 1/\| T \|_{\infty} \) and \( 1/\| S \|_{\infty} \).

3.2 Example 2: Unstable Fourth-Order System

We consider an unstable 2I2O system with the constant matrices given by

\[
A = \begin{bmatrix}
-2 & 0 & 1 & 0 \\
1 & -2 & 1 & 0 \\
1 & -2 & -1 & 1 \\
0 & 4 & 1 & 0
\end{bmatrix},
B = \begin{bmatrix}
1 & 0 \\
0 & 1 \\
2 & 1
\end{bmatrix},
C = \begin{bmatrix}
1 & 1 & 0 & 0 \\
1 & 0 & -1 & 0
\end{bmatrix}.
\]

Selecting the weighting matrices of the cost function as

\[
Q_z = \text{diag}(1.0 \times 10^4, 2.0 \times 10^3, 0, 200, 800, 100),
Q = \text{diag}(1.0 \times 10^2, 5.0 \times 10^3),
\]

and \( R = I_2 \), we obtain the parameters of Eq. (27):

\[
K_P = \begin{bmatrix}
9.743 & -0.2541 \\
7.981 & -24.89
\end{bmatrix},
K_I = \begin{bmatrix}
18.62 & 0.2334 \\
-35.85 & -37.69
\end{bmatrix},
K_D = \begin{bmatrix}
0.6033 & 0.1711 \\
-3.810 & 1.959
\end{bmatrix},
T_D = \begin{bmatrix}
0.05482 & 0.02177 \\
-0.3270 & 0.4359
\end{bmatrix}.
\]

With the reference model of Eq. (41), we obtain the feedforward gain matrix

\[
K_f = \begin{bmatrix}
92.18 & 267.2 & 12.17 & 32.46 \\
49.48 & 137.3 & -26.70 & -70.90
\end{bmatrix}.
\]

Table 2 summarizes the robust stability margins obtained by the respective controllers in four metrics [12]–[14]. \( b_{P,K} \) and
$1/\|KS\|_{\infty}$ are not large, whereas $1/\|T\|_{\infty}$ and $1/\|S\|_{\infty}$ are remarkably large.

Figure 7 shows time responses to the step reference output $y_{ref} = [1 \ 2]^T$ by the proposed MF-I-PD controller and the above I-PD one with the reference model $P_R(s)$ of Eq. (41). Also in this example, since the solid lines follow the reference outputs closer than the dashed lines, the extension to the model-following type gives higher control performance.

![Figure 7](image)

Table 2: Robust stability margins (Example 2).

<table>
<thead>
<tr>
<th>Controller</th>
<th>$1/|KS|_{\infty}$</th>
<th>$1/|T|_{\infty}$</th>
<th>$1/|S|_{\infty}$</th>
<th>$b_{p,K}$</th>
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4. Conclusion

This paper has proposed an MIMO I-PD controller design method and its extension to the model-following type. The design examples and simulation results demonstrate that the design method is a practical and useful design tool.

The proposed method provides an I-PD/MF-I-PD controller only in the case of $n \leq 2p$. The future work is to apply the method to the higher-order systems with $n > 2p$ by combining with plant model reduction.

References


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