Fictitious Reference Iterative Tuning for Non-Minimum Phase Systems in the IMC Architecture: Simultaneous Attainment of Controllers and Models

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Abstract: This paper provides a practical and meaningful application of controller parameter tuning. Here, we propose a simultaneous attainment of a desired controller and a mathematical model of a plant by utilizing the fictitious reference iterative tuning (FRIT), which is a useful method of controller parameter tuning with only one-shot experimental data, in the internal model control (IMC) architecture. Particularly, this paper focuses on systems with unstable zeros which cannot be eliminated in many applications. We explain how the utilization of the FRIT is effective for obtaining not only the desired control parameter values but also an appropriate mathematical model of the plant. In order to show the effectiveness and the validity of the proposed method, we give illustrative examples.

Key Words: controller parameter tuning, fictitious reference iterative tuning (FRIT), internal model control (IMC), non-minimum phase systems.

1. Introduction

Internal model control (IMC) [1] is one of the effective approaches to the achievement of a desired tracking property, which is realized as shown in Fig. 1 where $P$ is a plant, $\hat{P}$ is a model, $C_{IMC}$ is a controller, $r$ is the reference signal, $u$ is the plant input, and $y$ is the plant output. In fact, since the architecture of the IMC is intuitively understandable, it is widely utilized in many applications, e.g., [2]–[4]. Similarly to most other control architectures, we need an appropriate mathematical model in order to achieve the desired tracking property in the IMC architecture. The most rational approach for this purpose is to execute an identification.

![Fig. 1 Internal model control architecture.](image)

On another front, there are also many industrial processes in which it is impossible to apply a persistent excited signal required in the identification to an actual plant in terms of a safe operation of the plant. In some cases, it is difficult to take a time for an experiment of the identification in terms of the delivery period of industrial products. In these cases, synthesis of a controller with the direct utilization of the data is an effective way. Moreover, since the trajectories in which the dynamics directly appear have fruitful information of a system, it is also expected to obtain a controller which reflects the dynamics of a plant more directly than the model-based control design. From these backgrounds, there are some studies on the controller synthesis with the direct use of the data [5]–[7]. Particularly, for the cases where a controller with a tunable parameter has already been implemented, the iterative feedback tuning (IFT, [8]), the virtual reference feedback tuning (VRFT, [9]), and the fictitious reference iterative tuning (FRIT, [10]) are effective and useful methods.

The IFT is a tuning method that iteratively updates the variable parameters of a controller so as to minimize a performance index. This minimization can be computed by a nonlinear optimization technique like the Gauss-Newton method where required quantities consist of the experimental data. This also means that the IFT requires many experiments in order to update the parameters of a controller so as to minimize the performance index. On this point, the VRFT and the FRIT need only one-shot experimental data in the optimization for the achievement of a desired specification. This fact means that these two methods have a great advantage compared with the IFT in the sense that the time and cost for obtaining the optimal parameter are drastically reduced. Particularly, the FRIT considers the minimization of the error between the fictitious output and the actual one while the VRFT focus on the error between virtual input and the actual one. Then, the FRIT is intuitively understandable for the case in which the specification is given for the achievement of a desired output. In addition, the FRIT does not involve the computation of a non-proper transfer function which is required in the VRFT for the computation of the virtual reference signal.

By the way, since the IMC explicitly contains a model of a plant, the achievement of a desired output by some sort of method based on the direct use of the data might yield a model of the plant. From practical points of view, the obtained model can be utilized for finding out information on model uncertainties, monitoring of the actual status of a plant, and detection of an aging variation of a plant. From theoretical points of view, there is a crucial interplay which cannot be separated between a mathematical model and a designed controller as stated in [11]. As for this point, it is natural to treat the data as an interface.
that connects a model and a controller from a broader perspective. Thus, it is meaningful to simultaneously obtain not only a desired controller but also a mathematical model of the plant. In [12], although such a simultaneous attainment of a controller and a model in the IMC architecture was discussed, many iterative experiments for the identification and the controller design was required. The application of the IFT for the IMC studied in [13] also requires many experiments and it does not yield a mathematical model of the plant.

From these backgrounds, we proposed a simultaneous attainment of the desired controller and a mathematical model by utilizing the FRIT for the tuning of a controller in the IMC architecture in [14]. However, the result in [14] is applicable to a system with only stable zeros. On the other hand, there are many applications in which plants include unstable zeros, e.g., the dynamics of the horizontal acceleration with respect to the rudder angle of the rear wheel in a 4WD car [15], the horizontal velocity dynamics of a jet aircraft with respect to an elevator step deflection [16], the dynamics of mechanical systems in which the sensor and the actuator are located in the different points [17], and other applications\(^1\). In these applications, the unstable zeros cause an undershoot in the initial step response [19],[20], and are also related to an overshoot [21]. Thus, since unstable zeros cause a deterioration of the performance of a control system, it is important to consider them. In [22], the application of the VRFT in the IMC architecture with a time-delay case was also discussed. However, the result in [22] assumes that the length of the time-delay is known and cannot be applied for a simultaneous attainment of a controller and a model. Quite recently, the reference [23] addresses the application of the VRFT for non-minimum phase systems under the situation where the unstable zeros are unknown. However, the result given in [23] assumes that the structure of a controller is restricted as linearly parameterized one. Moreover, it cannot be applied for the simultaneous attainment\(^2\).

From these reasons, this paper addresses an application of the FRIT in the IMC architecture for the simultaneous attainment of a controller and a model of a non-minimum phase system. We explain how the utilization of the FRIT is effective for obtaining not only the desired control parameter values but also an appropriate mathematical model of the plant. Moreover, since we employ the FRIT, we show that such a simultaneous attainment can be achieved by using only one-shot experiment data. Finally, in order to show the effectiveness and the validity of the proposed method, we give illustrative examples.

2. Preliminaries

2.1 Notations

Let \( \mathbb{R} \) and \( \mathbb{R}^n \) denote the set of real numbers and that of real vectors of size \( n \), respectively. Let \( z \) denote the \( z \)-operator. When \( w \) is the discrete time signal, \( w_t \) denotes the value at the time \( t \). For a discrete time signal \( w = \{w_0, w_1, \ldots, w_k, \ldots\} \), its \( z \)-transformed representation is \( w(z) = \sum_{k=0}^{\infty} w_k z^{-k} \). For a \( w(z) \), we use the measure defined by

\[
\|w(z)\|_N = \sqrt{\frac{1}{N} \sum_{k=0}^{N-1} w_k^2} \tag{1}
\]

which was introduced in [7]. For \( w(z) = \sum_{k=0}^{\infty} w_k z^{-k} \), let \( [w(z)]_N \) denote

\[
[w(z)]_N := \begin{bmatrix} w_0 & w_1 & \cdots & w_{N-1} \end{bmatrix}^T \in \mathbb{R}^N. \tag{2}
\]

Similarly, for a transfer function \( G(z) \), let \( [G(z)]_N \) denote

\[
[G(z)]_N := \begin{bmatrix} G_0 & G_1 & \cdots & G_{N-1} \end{bmatrix}^T \in \mathbb{R}^N \tag{3}
\]

where \( G_i \) is the \( i \)-th Markov parameter of \( G(z) \). Finally, throughout this paper, we often omit ‘\( z \)’ from a rational function (e.g., ‘\( G(z)’ \) instead of ‘\( G(z)’ \) and \( z \)-transformed time series (e.g., ‘\( y(z)’ \) instead of ‘\( y(z)’ \) if it clearly follows from the context that they are \( z \)-transformed representations.

2.2 Assumptions

In this paper, we address a linear, time-invariant, single-input single-output, strictly proper, stable, and non-minimum phase plant. Let \( P(z) \) denote a transfer function of a plant. Let \( D(z) \) denote the denominator of \( P(z) \). Let \( N_m(z) \) and \( N_n(z) \) denote the polynomials whose roots are stable- and unstable zeros of \( P(z) \), respectively. Assume that \( P(z) \) is unknown except the degrees of \( N_m(z) \), \( N_n(z) \) and \( D(z) \). In order to embed the internal model of the plant to the IMC, we should distinguish the minimum phase part and the non-minimum phase one of the mathematical model as we will discuss later. Let \( N_m^d(z) \) denote the reciprocal polynomial\(^3\) of \( N_m(z) \). By using this, \( P(z) \) can be described as

\[
P(z) = P_m(z)P_n(z) = \frac{N_m(z)N_m^d(z)}{D(z)} = \frac{N_m(z)N_m^d(z)}{P_d(z)P_r(z)} \tag{4}
\]

where \( P_m(z) \) and \( P_n(z) \) are the minimum- and the non-minimum phase part of \( P(z) \), respectively. Note that \( P_d(z) \) is an inner function. Such a factorization technique to handle the unstable zeros has already been introduced in [1],[3], and [12]. Along this factorization of \( P(z) \), we parametrize the internal mathematical model \( \hat{P}(z) \) embedded in the IMC as

\[
\hat{P}(P_m, P_n, \rho_m) := \hat{P}_m(P_m, P_n, \rho_m) \hat{P}_n(P_n, z) \quad \hat{P}_m(P_m, P_n, \rho_m) := \frac{\sum_{i=0}^{\mu} b_i z^{-i}}{\sum_{i=0}^{\nu} a_i z^{-i}} + 1 \quad \hat{P}_n(P_n, z) := \frac{\sum_{i=0}^{d} c_i z^{-i}}{\sum_{i=0}^{d} c_i z^{-i}} \tag{5}
\]

with unknown parameter vectors

\[
\rho_m := [a_1 \cdots a_\mu b_0 \cdots b_\nu]^T \in \mathbb{R}^{\nu+\mu+1} \quad \rho_n := [c_0 \cdots c_d]^T \in \mathbb{R}^{d+1}. \tag{6}
\]

\( \hat{P}_m(P_m, \rho_m) \) and \( \hat{P}_n(P_n, \rho_n) \) are the parameterized minimum- and non-minimum phase part of the plant, respectively. We also parametrize the feedback controller \( C_{IMC} \) as

\[
C_{IMC}(\rho_c) := \frac{\sum_{i=0}^{\kappa} g_i z^{-i}}{\sum_{i=0}^{\kappa} f_i z^{-i}} + 1 \tag{7}
\]

with an unknown parameter vector

\[
\rho_c := [f_1 \cdots f_\kappa g_0 \cdots g_\kappa]^T \in \mathbb{R}^{\kappa+\nu+1}. \tag{8}
\]

\(^1\) The reciprocal polynomial of \( a(z) = \sum_{i=0}^{\mu} a_i z^{-i} \) with \( a_0 \neq 0 \) is defined by \( \sum_{i=0}^{\mu} a_i z^{-i} \).
In the following, we use the notation \( \rho := [g_{\rho}^T\, P_m^T\, \rho_{\rho}^T]^T \).

The closed loop in the IMC structure with a tunable parameter \( \rho \) is illustrated in Fig. 2.

![Fig. 2 IMC with a tunable parameter.](image)

Since the output and input in this closed loop depend on \( \rho \), they are denoted with \( y(\rho, z) \) and \( u(\rho, z) \), respectively. The closed loop transfer function with \( \rho \) from the reference \( r(z) \) to \( y(\rho, z) \) is denoted as \( G_{\rho r}(\rho, z) \).

### 3. Problem setting

The objective of this paper is to attain the desired output and the model of a plant with the direct utilization of the data. In order to consider this, we firstly give a reference model as the desired closed loop transfer function. It should be noted that the transient response is directly related to the limitation of the performance with which a plant is. That is, if a plant is a non-minimum phase system, a reference model should be given so as to include the unstable zeros of the plant as its own zeros. However, we have no information of the plant model in our setting. Thus, one of the rational ways is to include the unknown unstable zeros of the plant as a parameterized rational function in the reference model. This strategy has already been introduced in [24] where the IFT is applied to non-minimum phase systems\(^4\). In [24], the reference model is described by using the sum of the linear combination of Laguerre functions with unknown coefficients to approximate the non-minimum phase part of a plant. The reference [23] in which the VRFT is applied to non-minimum phase systems also proposes that the unknown zeros are added to a given reference model as a linearly-parameterized numerator.

Differently from [24] and [23], we use the inner function whose numerator consists of the unknown unstable zeros of a plant. This is the reason why we describe the model of the plant as (5). Let \( T_{dn}(z) \) be the minimum phase reference model which is initially given by the designer and is strictly proper. Then, we modify the reference model as

\[
T_{d}(\rho_n, z) = T_{dn}(z)P_{\rho_n}(\rho_n, z) \tag{9}
\]

and let

\[
y_d(\rho_n, z) := T_{d}(\rho_n, z)r(z) \tag{10}
\]

denote the desired output. One of the reasons why we use the inner function \( P_{\rho_n}(\rho_n) \) is to keep at least the gain characteristics of the initial reference model \( T_{dn}(z) \) for any \( \rho_n \). In addition, since \( T_{d}(\rho_n, 1) = T_{dn}(1) \) also holds for any \( \rho_n \), we do not have to modify the steady state gain after the estimation of the unstable zeros of a plant. This is also another different point from the method in [23] where the steady state gain must be modified after the estimation of the zeros.

Under the above setting, the problem is to find a parameter \( \rho \) such that \( y(\rho, z) = G_{\rho r}(\rho, z)r(z) \) achieves the desired output \( y_d(\rho_n, z) \) and simultaneously the internal model \( \hat{P}(\rho_n, \rho_n, z) \) approximates the actual plant \( P(z) \) as closely as possible with only one-shot experiment.

### 4. Fictitious Reference Iterative Tuning

In this section, we review the fictitious reference iterative tuning (FRIT) [10], which is a controller parameter tuning that enables one to obtain the optimal parameter of a controller with only one-shot experiment. Figure 3 illustrates a conventional feedback control system that consists of a plant and a controller \( C(\rho, z) \) with a tunable parameter \( \rho \).

![Fig. 3 A feedback system with a parameterized controller.](image)

First, by using the initial parameter \( \rho^0 \), perform the first experiment with \( C(\rho^0, z) \) and obtain the initial data \( u^0(z) := u(\rho^0, z) \) and \( y^0(z) := y(\rho^0, z) \). Here we also assume that \( C(\rho^0, z) \) tentatively stabilizes the closed loop so as to yield the bounded input and output. By using \( u^0(z) \) and \( y^0(z) \), we compute the fictitious reference signal \( \hat{r}(\rho, z) \) (which was introduced by [27]) in the unfalsified control framework described by

\[
\hat{r}(\rho, z) = C(\rho, z)^{-1}u^0(z) + y^0(z). \tag{11}
\]

Next, we introduce the following cost function described by

\[
J_{\rho}(\rho) = \|y^0(z) - T_{d}(z)\hat{r}(\rho, z)\|^2_{\tilde{N}}. \tag{12}
\]

Then we minimize \( J_{\rho}(\rho) \) and implement \( \hat{\rho}^* := \arg\min_{\rho} J_{\rho}(\rho) \) to the controller. Note that the cost function (12) with \( \hat{r}(\rho, z) \) described by (11) requires only \( u^0(z) \) and \( y^0(z) \). This means that the minimization of (12) can be performed off-line by using only one-shot experimental data. As for the relationship between the minimization of (12) and that of \( \|y(\rho, z) - T_{d}(z)\hat{r}(\rho, z)\|^2_{\tilde{N}} \) which is initially given as the desired specification, we obtain the following result.

**Proposition 1** For a parameter \( \hat{\rho}^* \), \( \|y(\rho^*, z) - T_{d}(z)\hat{r}(\rho^*, z)\|^2_{\tilde{N}} = 0 \) holds if and only if \( J_{\rho}(\hat{\rho}^*) = 0 \) holds.

See Theorem 3.1 in [10] for the detailed proof and discussions. This proposition implicitly means that the minimization of \( J_{\rho}(\rho) \) is deeply related to that of \( \|y(\rho, z) - T_{d}(z)\hat{r}(\rho, z)\|^2_{\tilde{N}} \).

### 5. FRIT for Non-Minimum Phase Systems in IMC

#### 5.1 Utilization of FRIT for the Simultaneous Attainment

Consider the feedback system with the IMC illustrated in Fig. 1. The transfer function from \( r \) to \( y \) is described by

\[
G_{\rho y} = \frac{C_{\rho IMC}P}{1 + C_{\rho IMC}(P - \tilde{P})}. \tag{13}
\]

If it is possible to put \( \tilde{P} = P \), then the feedback controller described by

\[
C_{\rho IMC} = T_{dn}\tilde{P}_m^{-1} \tag{14}
\]

enables us to see that

\[
G_{\rho y} = \frac{C_{\rho IMC}P}{1 + C_{\rho IMC}(P - \tilde{P})} = T_{dn}\tilde{P}_m^{-1}P_mP_n = T_{dn}P_n \tag{15}
\]
holds. Conversely, if $T_{dm}\hat{P}_n = G_{ry}$ holds by using $C_{IMC}$ described by (14), then we see
\[
T_{dm}\hat{P}_n = G_{ry} = \frac{C_{IMC}P}{1 + C_{IMC}(P - \hat{P})} = \frac{T_{dm}\hat{P}_n^{-1}P}{1 + T_{dm}\hat{P}_n^{-1}(P - \hat{P})}.
\]
Multiplying the above equality by $1 + T_{dm}\hat{P}_n^{-1}(P - \hat{P})$ yields
\[
T_{dm}\hat{P}_n(1 + T_{dm}\hat{P}_n^{-1}(P - \hat{P})) = T_{dm}\hat{P}_n^{-1}P.
\]
By rewriting the left-hand side of the above equality, we obtain
\[
T_{dm}\hat{P}_n^{-1}(P + T_{dm}\hat{P}_n(P - \hat{P})) = T_{dm}\hat{P}_n^{-1}P.
\]
Moreover, by arranging the above equality, we obtain
\[
T_{dm}\hat{P}_n^{-1}(1 - T_{dm}\hat{P}_n)(P - \hat{P}) = 0. \tag{16}
\]
From (16), we see that $P(z) = P(z)$ holds for almost every complex numbers $z$ except at most a finite number of the zeros of $T_{dm}\hat{P}_n^{-1}(1 - T_{dm}\hat{P}_n)$. In a practical sense, since $(1 - T_{dm}\hat{P}_n)$ is small at low-frequencies, $P - \hat{P}$ need not be zero at low-frequencies. More detailed discussions in the frequency domain are given in Section 5.2.

From the above observation, since it is expected that the structure of (14) is useful for the simultaneous attainment of a model and a controller, we parameterize the feedback controller $C_{IMC}$ in Fig. 2 as
\[
C_{IMC}(\rho_n, \rho_m) = T_{dm}\hat{P}_n(\rho_m, \rho_n)^{-1} \tag{17}
\]
which is illustrated in Fig. 4. Since $C_{IMC}$ (17) is parameterized by $\rho_m$ and $\rho_n$ instead of $P_c$, we denote $\rho := [\rho_m^T \rho_n^T]^T$ henceforth. The controller (17) plays a crucial role in the simultaneous attainment of the desired response and a mathematical model of the plant as the following discussion.

![Fig. 4 IMC with the controller described by Eq. (17).](image)

We apply the FRIT to the IMC with $C_{IMC}(\rho)$ described by (17) in Fig. 4. Firstly, we set the initial parameter $\rho_m^0$ and $\rho_n^0$. Then we perform a one-shot experiment in the IMC illustrated in Fig. 4 and obtain the initial data $u^0$ and $y^0$. In order to apply the FRIT, we compute the fictitious reference
\[
\tilde{y}(\rho) = \frac{1 - T_{dm}\hat{P}_n(\rho_m)}{T_{dm}}\hat{P}_n(\rho_m)u^0 + y^0 \tag{18}
\]
which can be obtained by substituting the dotted line region in Fig. 4 to $C(\rho)$ in (11) as
\[
C(\rho) = \frac{T_{dm}}{1 - T_{dm}\hat{P}_n(\rho_m)}\tilde{P}_n(\rho_m)^{-1}. \tag{19}
\]

We then minimize the cost function which corresponds to (12) and is described by
\[
J_F(\rho) = \|y^0 - T_d(\rho_m)\tilde{r}(\rho)\|_N^2 \tag{20}
\]
with respect to $\rho$. In the following, we consider the relationship among the minimization of $J_F(\rho)$, the achievability of the desired output, and the attainment of the mathematical model.

By using the actual input-output relation $y^0 = Pu^0$, the output of $G_{ry}(\rho) = PC(\rho)/(1 + PC(\rho))$ with respect to $\tilde{r}(\rho)$ described by (11) is obtained as
\[
G_{ry}(\rho)\tilde{r}(\rho) = \frac{PC(\rho)}{1 + PC(\rho)}(C(\rho)^{-1}u^0 + y^0) = y^0. \tag{21}
\]

By using (21), the cost function $J_F(\rho)$ can be rewritten by
\[
J_F(\rho) = \left\| \left(1 - \frac{T_d(\rho_m)}{G_{ry}(\rho)} \right)y^0 \right\|^2_N. \tag{22}
\]

On the other hand, together with (9), (13), and (17), note that $G_{ry}(\rho, z)$ can also be written by
\[
G_{ry}(\rho) = \frac{T_{dm}\hat{P}_n(\rho)^{-1}P}{1 + T_{dm}\hat{P}_n(\rho)^{-1}(P - \hat{P}(\rho))}. \tag{23}
\]

Substituting (23) and (9) to the right hand side of (22) yields
\[
J_F(\rho) = \left\| \left(1 - T_{dm}\hat{P}_n(\rho_m) + T_{dm}\hat{P}_n(\rho)^{-1}(P - \hat{P}(\rho)) \right) \right\|^2_N.
\]

Summing up (22) and (24), we can obtain the following theorem which enables us to see the intuitive effectiveness of the utilization of the FRIT for the simultaneous attainment.

**Theorem 1** Consider the IMC illustrated in Fig. 4. Assume that $T_{dm}(z)$ is given as to be strictly proper. Let $p$ denote the first index such that $y^0_i = 0$ for all $i < p$ and $y^0_p \neq 0$. Then, for some parameter $\rho'$, the following statements are equivalent:

(a) $J_F(\rho') = 0$,

(b) $\left[1 - \frac{T_d(\rho')}{G_{ry}(\rho')} \right]_{N-p} = 0$,

(c) $\left[1 - \frac{P(\rho')}{P} \right]_{N-p} = 0$.

**Proof** The equivalence of the statements (a) and (b) is clear from (22) and Lemma 1 in Appendix. Next, from (24), the statement (a) is equivalent to
\[
\left[1 - \frac{P(\rho)}{P} \right]_{N-p} = 0. \tag{25}
\]
Let $p'$ denote the first index such that $((1 - T_d(\rho_m))y^0_i)_i = 0$ for all $i < p'$ and $((1 - T_d(\rho_m))y^0_p)_{p'} \neq 0$. Since $T_{dm}$ is strictly
proper, $1 - T_{dun}\tilde{P}(\rho_n) = 1 - T_d(\rho_n)$ is bi-proper independently of $\rho_n$. This implies $\rho' = p$. Hence, by Lemma 1, (25) is also equivalent to the statement (c).

From this theorem, if $J_F(p') = 0$ can be achieved at $\rho^*$, then the first $N = p$ Markov parameters of the relative error on the achievement of the desired output (i.e., $1 - T_d(p'/\tilde{G}_r(p'))$) and that of the accuracy of the obtained model (i.e., $1 - \tilde{P}(p)/P$) are equal to zero. This is the effectiveness of the utilization of the FRIT for the simultaneous attainment if the minimization can be ideally achieved.

5.2 On the Reducing of $J_F(\rho)$

For the cases where it is difficult to achieve $J_F(\rho) = 0$, we consider the meaning of the reducing of $J_F(\rho)$. In order to do this, we remind (22) and (24). Then we can obtain the following theorem, which is a trivial consequence of (22) and (24).

Theorem 2 Consider the IMC illustrated in Fig. 4. Assume that $T_d(\rho_n)$ is given as to be strictly proper. Then, for some parameters $\rho'$ and $\rho''$, the following statements are equivalent:

(a) $J_F(p') > J_F(p'')$,

$$n\left\|1 - \frac{T_d(p_n)}{G_r(p')}\right\|^2_n > \left\|1 - \frac{T_d(p_n)}{G_r(p'')}\right\|^2_n,$$

(c) $n\left\|1 - \frac{\tilde{P}(p')}{P}\right\|^2_n (1 - T_d(p')) y^0_n > \left\|1 - \frac{\tilde{P}(p'')}{P}\right\|^2_n (1 - T_d(p'')) y^0_n$.

The statement (b) is related to the achievement of the desired output since the relative error of the closed loop system consists of these two experiment data so as to can-

remark 3 As shown in (20), the minimization of $J_F(\rho)$ is done with a nonlinear optimization. This implies that the result depends on the initial input and output data. A theoretical analysis on the effect of the initial experiment is also one of the future important studies.

5.4 Algorithm

We can summarize the proposed method as follows.

0. Prepare the initial parameter $\rho^0 := [\rho^0_0 \; \rho^0_1 \; \rho^0_2]^T$, and give the minimum phase part of the desired reference model $T_{dun}$.

1. In Fig. 4, we implement $\rho^0$.

2. Perform one-shot experiment and obtain $\tilde{u}^0$ and $\tilde{y}^0$.

3. Perform FRIT: construct the cost function $J_F(\rho)$ by using the fictitious reference $\tilde{r}_n(\rho)$ described by (18). And then minimize $J_F(\rho)$ by an off-line non-linear optimization (e.g., the Gauss-Newton method).

4. We obtain the optimal parameter $\rho' := \arg \min_{\rho'} J_F(\rho)$ which yields the desired controller and the mathematical model of the plant.
We take the internal model described by sampling period 0.1 [sec]. In Fig. 5, the initial output the initial experiment in the IMC structure in Fig. 4 with the \( \rho \) reference signal \( [1 - \rho_1 - \rho_2 - \rho_3]^T \) and \( \rho_4 \) are unknown parameters. We give the desired reference model which includes the unknown non-minimum phase part \( \hat{P}_d(\rho_n) \) is given by

\[
T_d(\rho_n) = T_{dn}\hat{P}_d(\rho_n) = \frac{0.1(z + 0.8)}{(z - 0.7)(z - 0.4)} \frac{z + \rho_4}{1 + \rho_4 z} \tag{28}
\]

Under the above setting, we set the initial parameter \( \rho_n^0 = [1 - 0.8 - 0.1]^T \) and \( \rho_0 = -1.1 \). Then, we perform the initial experiment in the IMC structure in Fig. 4 with the sampling period 0.1 [sec]. In Fig. 5, the initial output \( y^0 \), the reference signal \( r \), and \( y_d(\rho_n^0) = T_d(\rho_n^0) r \) are drawn by the solid line, the dot-and-dash line, and the dotted line, respectively.

We then apply the proposed method with FRIT. As a result, we obtain the optimal parameter as \( \rho_n^* = [1.000 - 1.000 0.160]^T \) and \( \rho_0^* = -1.5000 \). We perform the experiment by using \( \rho^* := [\rho_n^*^T \rho_0^*]^T \) and illustrate the result in Fig. 6. In this figure, the actual output with the optimal parameter \( y(\rho^*) \), the reference signal \( r \), and tuned desired output \( y_d(\rho_n^*) \) are drawn by the solid line, the dot-and-dash line, and the dotted line, respectively. From Fig. 6, we see that \( y(\rho^*) \) and the desired output \( y_d(\rho_n^*) \) are almost the same, which implies that the desired output is achieved by using \( \rho^* \).

The internal model by using \( \rho^* \) is obtained as

\[
\hat{P}(\rho^*) = \hat{P}_w(\rho_n^*)\hat{P}_d(\rho_n^*) = \frac{z - 1.500}{1.000z^2 - 1.000z + 0.160} = \frac{z - 1.500}{(z - 0.200)(z - 0.800)}. \tag{29}
\]

Compared with the pole and the zeros of the actual plant (26), we see that they are also well-identified.

6.2 Example 2

In order to show that the proposed method is applicable for the case in which the structure of \( P \) is unknown, we give the following example where the structures of \( P \) and \( \hat{P} \) are different.

For the unknown plant which is assumed to be described by

\[
P = \frac{z - 1.5}{z^3 - 0.85z^2 - 0.06z + 0.0135} = \frac{z - 1.5}{(z - 0.1)(z + 0.15)(z - 0.9)}. \tag{30}
\]

we use the internal model (27) and the reference model (28). We set the initial parameter \( \rho_n^0 = [0.9 - 0.7 - 0.1]^T \) and
performing the experiment with over the frequency range of the reference model and \( \tilde{\rho} \) respectively. In these two figures, characteristics of \( \tilde{\rho} \) with respect to the frequency response. Their magnitude and transfer function are di

pare the poles \( \rho \) of the actual plant \( P \) and those of \( \tilde{P} \) since the structure of transfer function are different. Thus, we compare \( P \) and \( \tilde{P}(\rho^*) \) with respect to the frequency response. Their magnitude and phase characteristics are illustrated in Figs. 9 and 10, respectively. In these two figures, characteristics of \( P(e^{j\omega}) \), \( \tilde{P}(\rho^*, e^{j\omega}) \) and \( \tilde{P}(\rho^*, e^{j\omega}) \) are drawn by the dot-and-dash line, the dotted line, and the solid line, respectively. In addition, characteristics of \( \tilde{T}(\rho^*, e^{j\omega}) \) are drawn by the dashed line. The frequency characteristics of \( \tilde{P}(\rho^*, e^{j\omega}) \) is remarkably closer to that of the actual plant \( P(e^{j\omega}) \) rather than that of the initial model \( \tilde{P}(\rho^*, e^{j\omega}) \) over the frequency range of the reference model \( \tilde{T}(\rho^*, e^{j\omega}) \). Thus, we see that the dynamics of the actual plant is also well-identified as the obtained internal model in the frequency range for the achievement of the desired tracking.

7. Concluding Remarks

In this paper, we have addressed a practical and meaningful application of controller parameter tuning. Here, we have proposed a simultaneous attainment of the desired controller and a mathematical model by utilizing the FRIT, which is a useful method of controller parameter tuning with only one-shot experimental data, in the IMC architecture for non-minimum phase systems.

In addition to Remarks 2 and 3, the case where the number of the unstable zeros is unknown should be also more theoretically analyzed. These issues will be clarified by further research.

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References


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Appendix Lemma for the proof of Theorem 1

Lemma 1 Let \( G(z) \) be a discrete time transfer function. Let \( u(z) \) be a \( z \)-transformed discrete time signal and \( p \) denote the first index such that \( u_p \neq 0 \) and \( u_i = 0 \) for all \( i < p \). Then

\[
\begin{align*}
[G(z)u(z)]_N &= 0 \iff [G(z)]_{N,p} = 0. \tag{A.1}
\end{align*}
\]

Proof Since “\( \iff \)” is trivial, we focus on the proof of “\( \Rightarrow \)”. Since the first \( p \) samples of \( u_i \) \( (i = 0, 1, \ldots, p - 1) \) are equal to zero, \( [G(z)u(z)]_N = 0 \) implies

\[
\begin{bmatrix}
G_0 & 0 & \cdots & 0 \\
G_1 & G_0 & \cdots & 0 \\
\vdots & \ddots & \ddots & \vdots \\
G_{N-p-1} & \cdots & G_1 & G_0
\end{bmatrix}
\begin{bmatrix}
\begin{bmatrix}
u_p \\
u_{p+1} \\
\vdots \\
u_{N-1}
\end{bmatrix}
\end{bmatrix} = 0 \tag{A.2}
\]

where \( G_i \) be the \( i \)-th Markov parameter of \( G(z) \). Eq. (A.2) is also equivalent to

\[
\begin{bmatrix}
u_p & 0 & \cdots & 0 \\
u_{p+1} & \begin{bmatrix}G_0 & \cdots & 0 & 0 \end{bmatrix} \\
\vdots & \ddots & \ddots & \ddots \\
u_{N-1} & \cdots & \begin{bmatrix}G_1 & \cdots & 0 & 0 \end{bmatrix}
\end{bmatrix} = 0. \tag{A.3}
\]

It follows from \( u_p \neq 0 \) that the Toeplitz matrix in the left hand side is non-singular, which implies \( [G(z)]_{N,p} = 0 \). This completes the proof of the lemma. \( \Box \)

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