Controller Parameter Tuning for Systems with Hysteresis and Its Application to Shape Memory Alloy Actuators

Yuji WAKASA*, Shinji KANAGAWA**, Kanya TANAKA*, and Yuki NISHIMURA*

Abstract: This paper proposes a simple controller parameter tuning method that can compensate for hysteresis. The proposed method is based on the so-called fictitious reference iterative tuning (FRIT) technique which can easily tune controller parameters such as proportional-integral-derivative gains using a one-shot closed-loop experimental data. In the proposed framework, a simple hysteresis model is introduced to a control system, and its inverse is used as a hysteresis compensator. Since the hysteresis model is characterized with only three parameters, the related computational burden is moderate in the parameter tuning process. Also, the proposed FRIT method needs an only one-shot experiment as in the standard FRIT one, which implies that the feature of FRIT is well-maintained. In the optimization process, the so-called covariance matrix adaptation evolution strategy is used for simultaneously searching hysteresis parameters as well as controller parameters. The proposed FRIT method is applied to an experimental control system that comprises a shape memory alloy actuator, and its effectiveness is verified.

Key Words: controller parameter tuning, PID control, hysteresis, shape memory alloy actuator, covariance matrix adaptation evolution strategy.

1. Introduction

Shape memory alloy (SMA) actuators exhibit high power-to-weight ratios and can produce a relatively large displacement in comparison with other devices such as piezoelectric actuators. Because of these excellent characteristics, SMA actuators have received considerable attention as effective actuators for various applications in which space and weight constraints are critical design requirements. These applications include microprocessors, minimally invasive surgery, and so on; see, for instance, [1] and [2]. However, SMA actuators intrinsically have hysteretic nonlinearity, which often leads to inferior control performance when using simple control methods such as proportional-integral-derivative (PID) control.

In order to cope with this nonlinearity, several modeling and control methods for SMA actuators have been proposed using the Preisach model [3] and iterative learning control [4]. These methods require the implementation of complicated algorithms and iterative experiments to obtain input/output data. In practical terms, however, it is often desirable that a control method be simple but be able to achieve a reasonable level of control performance.

On the other hand, fictitious reference iterative tuning (FRIT) in [5] is a simple tuning scheme for controller parameters that can be carried out using just a one-shot closed-loop experiment. An extended version of the FRIT method has been proposed in [6], and its effectiveness is demonstrated by means of simulations of chemical process applications. However, these FRIT methods were originally developed for linear systems and often fail to provide good control performance for SMA actuators because the dynamics of SMA actuators are nonlinear as a result of hysteresis. In [7], to overcome this difficulty, an FRIT method combined with hysteresis compensation has been proposed. However it requires two experiments—an open-loop experiment and a closed-loop experiment, and the hysteresis compensator is characterized with many parameters, which results in a large amount of computation.

In this paper, we propose an FRIT method that can compensate for hysteresis and can resolve the abovementioned problems in [7]. Namely, in the proposed FRIT method, just a one-shot closed-loop experiment is required, and the hysteresis compensator is characterized with only a few parameters. We verify the effectiveness of the proposed method by applying it to an experimental SMA actuator system.

The rest of this paper is organized as follows. In Section 2, we present a framework for feedback control and hysteresis compensation. In Section 3, we propose an FRIT method for tuning the hysteresis parameters as well as the controller parameters. Then, in Section 4, we describe an algorithm for solving the optimization problem in the FRIT procedure. The algorithm is the so-called covariance matrix adaptation evolution strategy (CMA-ES) which is a kind of stochastic multi-point search techniques. Section 5 presents the experimental results. Finally, Section 6 presents our conclusions and outlines the scope for future research.

2. System Configuration

We consider a closed-loop system configuration, as shown in Fig. 1, where a system with hysteresis is assumed to consist of a hysteresis operator and a linear system $G(z)$ connected in series, and an overall controller consists of a PID controller $C(z, \theta)$ and a hysteresis compensator. Although in this we deal with a PID controller as the typical controller, we can deal with parameterized linear controllers with other structures as in [5]. In the

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figure, \(u(k), y(k), r(k), \) and \(e(k)\) denote the control input, control output, reference signal, and tracking error, respectively. The transfer function of the PID controller can be expressed as

\[
C(z, \theta) = \frac{K_P(1 - z^{-1}) + K_I + K_D(1 - z^{-1})^2}{1 - z^{-1}},
\]

where \(K_P, K_I,\) and \(K_D\) are the proportional, integral, and derivative gains, respectively, and \(\theta = (K_P, K_I, K_D)^T\) includes the PID gains to be tuned.

The basic idea of this system configuration is as follows. If the inverse of the hysteresis operator is used as a hysteresis compensator as in [3] and [7], the hysteretic property in a system with hysteresis is neutralized by the hysteresis compensator. In such an ideal case, only the linear system \(G(z)\) is used as a hysteresis compensator as in [3] and [7], the hysteretic property in a system with hysteresis is neutralized by the hysteresis compensator; see the references for details. Furthermore, the inverse of the proposed hysteresis model can be simply obtained as stated in the following theorem.

**Theorem 1** Consider the following system \(\hat{H}_\rho\) defined by the function \(\hat{H}_\rho:\)

\[
\hat{H}_\rho : u(k) = \hat{H}_\rho(h(k), h(k - 1)) = -(\log \xi)/\gamma,
\]

where \(\xi\) is the positive root of the quadratic equation

\[
a_2\xi^2 + a_1\xi + a_0 = 0,
\]

\[
a_2 = e^{(\alpha + \beta)\gamma} h(k),
\]

\[
a_1 = (e^{\alpha\gamma} + e^{\beta\gamma})h(k) - (e^{\alpha\gamma} - e^{\beta\gamma})h(k - 1) - e^{\beta\gamma},
\]

\[
a_0 = h(k) - 1.
\]

Then System \(\hat{H}_\rho\) is the inverse of System \(H_\rho\).

**Proof:** Letting \(\xi = e^{-\gamma h(k)}\) and arranging (1) with respect to \(\xi\), we obtain (3). Since the range of the function \(H_\rho\) is \((0, 1)\) from its definition, it follows that \(a_2 > 0\) and \(a_0 < 0\). Therefore, (3) must have one positive and one negative root. We obtain (2) from \(\xi = e^{-\gamma h(k)}\) for the positive root \(\xi\). Therefore, we can conclude that if the initial values \(h(0)\) of \(\hat{H}_\rho\) and \(\hat{H}_\rho\) are the same, then both \(H_\rho \circ \hat{H}_\rho\) and \(\hat{H}_\rho \circ H_\rho\) are the identity mappings.

Note that, in [8] and [7], the pseudoinverse model of the HRNN is used as a hysteresis compensator; see the references for details. However, the output of the hysteresis compensator

![Fig. 1 Configuration of the control system.](image1)

![Fig. 2 Example of dynamical behavior of the hysteresis model with \(\alpha = 1, \beta = 0, \gamma = 20,\) and \(h(0) = 0.0001\).](image2)
in [8] and [7] is computed iteratively, and the precise inverse computation requires a large computational burden. In contrast, the output of the inverse model $\hat{M}_P$ is computed non-iteratively and provides satisfactory results as shown later in the experiments.

Remark 1 We implement the inverse model $\hat{M}_P$ as the hysteresis compensator in the system configuration shown in Fig. 2. As defined, the domain of the function $\hat{M}_P$ is restricted to $(0, 1)$ while the output of the PID controller, i.e., the input of the hysteresis compensator, may exceed the interval $(0, 1)$. Therefore, we have to restrict the output $\hat{h}(k)$ of the PID controller in the implementation by setting it as $\epsilon(>0)$ and $1-\epsilon$ if $\hat{h}(k) \leq 0$ and $\hat{h}(k) \geq 1$, respectively. Here, we usually set $\epsilon$ as a small positive number. At least, there are two possible choices to carry out this modification of $\hat{h}(k)$. One is to change the state of the integrator in the PID controller, and the other is simply to clip $\hat{h}(k)$. We adopt the latter method in the experimental results in Section 5.

Remark 2 When the input to the hysteresis compensator is close to 0 or 1, the output of the hysteresis compensator is generally a large negative or positive value, respectively. As a result, the output of the hysteresis compensator is often saturated and has to be limited within an allowable range of the input to the plant. It is desirable to avoid this restriction of the input because it may lead to deterioration of control performance. In many cases, however, the PID gains and hysteresis parameters can be appropriately tuned in the proposed FRIT so that the output of the hysteresis compensator, i.e., the input to the plant, tends to be within the allowable range. We can verify this in the experimental results in Section 5.

### 3. FRIT under Hysteresis Compensation

In FRIT, we calculate a fictitious reference signal $\tilde{r}$ based on input/output data obtained from a closed-loop experiment. The method for calculating $\tilde{r}$ under the configuration shown in Fig. 1 is different from that used in the standard FRIT described in [5]. Assuming that there is no hysteresis compensation and that we can obtain input/output data $u_0(k), y_0(k), k = 1, \ldots, N$, for an initial controller parameter $\theta_0$ and a reference signal $r(k)$, then, in the standard FRIT, the fictitious reference signal is calculated as follows:

$$\tilde{r}(\theta, k) = C(z, \theta)^{-1}u_0(k) + y_0(k).$$

However, the fictitious reference signal in our scheme is calculated as follows:

$$\tilde{r}(x, k) = C(z, \theta)^{-1}\hat{h}(k) + y_0(k),$$

$$\hat{h}(k) = H_P[(\hat{h}(k-1), u_0(k))],$$

where $x = (\theta^T, \rho^T)^T$. Note that the hysteresis function $H_P$ is included in the calculation of the fictitious reference signal.

We summarize the FRIT procedure under hysteresis compensation as follows.

**FRIT procedure under hysteresis compensation**

**Step 1.** Set an initial PID parameter $\theta_0$, a reference signal $r(k)$, $k = 1, \ldots, N$, and a reference model $M(z)$.

**Step 2.** From a closed-loop experiment that is performed without hysteresis compensation, we obtain the input/output data $u_0(k), y_0(k), k = 1, \ldots, N$, for the reference signal $r(k)$.

**Step 3.** Calculate the fictitious reference signal $\tilde{r}(x, k)$ according to (4). Then, find the optimal (or suboptimal) tuning parameter $x^*$ that minimizes the performance index

$$J(x) = \sum_{k=1}^{N} (y_0(k) - M(z)\tilde{r}(x, k))^2,$$

to obtain the optimal (or suboptimal) PID gains $\theta^*$ and hysteresis parameters $\rho^*$.

Since the minimization problem in Step 3 is usually nonconvex, an optimization technique of stochastic multi-point search is practical and effective for solving this type of problem. In this paper, we use the CMA-ES algorithm [9] to solve the problem as in [7] and [10].

### 4. CMA-ES

The CMA-ES algorithm is based on an evolution strategy where search points are generated according to a multivariate normal distribution, and the corresponding covariance matrix is updated through the evaluation of the search points. Like genetic algorithms (GAs) and particle swarm optimization (PSO), the CMA-ES algorithm does not require that the objective function be differentiable. The GAs and PSO algorithms are relatively well known in the control engineering community; see, for instance, [6], [11], and [12]. On the other hand, there are only a few applications of the CMA-ES to control problems. In [9] and [13], it is demonstrated that the CMA-ES algorithm outperforms the GAs and PSO algorithms through some benchmark and control problems.

In this kind of heuristic optimization techniques, it is an important aspect to take into account a tradeoff between exploration and exploitation in the process of finding a solution. While high exploration gives high possibility of finding the global solution, high exploitation leads to high convergence speed. To construct a good algorithm, we usually need to adjust some tuning parameters in such an algorithm, and therefore, a small number of such tuning parameters is desirable. In this sense, the CMA-ES is practical and tractable because the CMA-ES is well-tuned and there are few tuning parameters left to the user.

The algorithm mainly consists of the generation of search points, the update of the mean of the search points, the update of the covariance matrix of the search distribution, and the update of the step size parameter.

The dimension of the search space is denoted by $n$. The outline of the CMA-ES algorithm is concisely but completely shown as follows.

**CMA-ES algorithm**

**Step 1.** (Setting of parameters) Set the number of search points $d$, the number of selected search points $\mu$, the weights $w_i, i = 1, \ldots, \mu$, and the other parameters $\mu_{eff}, \mu_{cov}$, $d_r$, $c_r$, $c_c$, $c_{cov}$. (We will explain how to determine these parameters later.)
Step 2. (Initialization) Initialize the generation number \( g = 0 \), the covariance matrix \( C^{(0)} = I \), the step size parameter \( \sigma^{(0)} = 0.5 \), and the evolution path parameters \( p^{(0)}_e \), \( p^{(0)}_c = 0 \). Set the initial mean \( m^{(0)} \in \mathcal{D} \) where \( \mathcal{D} \subseteq \mathbb{R}^n \) is an initial region.

Step 3. (Generation of search points) Search points are generated according to the normal distribution as follows:
\[
x^{(g+1)}_j \sim \mathcal{N}(m^{(g)},\sigma^{(g)})^2 C^{(g)} \quad \text{for} \quad j = 1, \ldots, A.
\]

Step 4. (Update of the mean) Evaluate the values of the performance index \( J(x^{(g+1)}_j) \) for the search points, and select \( \mu \) best points \( x^{(g+1)}_{i\lambda}, i = 1, \ldots, \mu \). Update the mean of the search distribution as follows:
\[
m^{(g+1)} = \frac{1}{\mu} \sum_{i=1}^{\mu} x^{(g+1)}_{i\lambda}.
\]

Step 5. (Update of the covariance matrix) Update the covariance matrix as follows:
\[
C^{(g+1)} = (1 - c_{cov}) C^{(g)} + c_{cov} \left(1 - \frac{1}{\mu_{cov}}\right)
\sum_{i=1}^{\mu} \left(x^{(g+1)}_{i\lambda} - m^{(g+1)}\right)\left(x^{(g+1)}_{i\lambda} - m^{(g+1)}\right)^T
+ \frac{c_{cov}}{\mu_{cov}} \left(p^{(g+1)}_c - p^{(g)}_c\right)^T + \delta(H_{c\sigma}) C^{(g)}
\]
where
\[
p^{(g+1)}_c = (1 - c_e) p^{(g)}_c + H_{c\sigma} \sqrt{c_e(2 - c_e) \sigma^{(g)}}\frac{m^{(g+1)} - m^{(g)}}{\sigma^{(g)}}
\]
\[
\delta(H_{c\sigma}) = \begin{cases} 1 & \text{if } \frac{\|p^{(g+1)}_c\|}{\sqrt{1 - c_{cov}(2 - c_e) \sigma^{(g)}}} < (1.4 + \frac{2}{n+1}) E\|N(0, I)\| \\ 0 & \text{otherwise,} \end{cases}
\]
\[
E\|N(0, I)\| = \sqrt{n} \left(1 - \frac{1}{4n} + \frac{1}{21n^2}\right).
\]

Step 6. (Update of the step size parameter) Update the step size parameter as follows:
\[
\sigma^{(g+1)} = \sigma^{(g)} \exp \left(\frac{c_e}{d_{c\sigma}} \left(\frac{\|p^{(g+1)}_c\|}{E\|N(0, I)\|} - 1\right)\right)
\]
where
\[
p^{(g+1)}_c = (1 - c_e) p^{(g)}_c + \sqrt{c_e(2 - c_e) \mu_{eff}} \left(C^{(g)}\right)^{1/2} \frac{m^{(g+1)} - m^{(g)}}{\sigma^{(g)}}
\]

Step 7. (Stopping criterion) If a stopping criterion such as the maximum number of generations is satisfied, stop. Otherwise, set \( g = g + 1 \) and go to Step 3.

Although the above algorithm includes some parameters to be set in Step 1, these parameters are automatically determined in the CMA-ES algorithm developed from theoretical and empirical viewpoints; see [9] for details. Therefore, note that the CMA-ES algorithm does not need so much effort for parameter tuning. Basically, we adopt such parameter settings in the CMA-ES algorithm. The parameters are determined as follows:
\[
\lambda = 4 + [3 \ln n],
\mu = (\lambda - 1)/2,
w_i = \frac{w'_i}{\sum_{i=1}^n w_i},
w'_i = \ln((\lambda - 1)/2 + 1) - \ln i,
\mu_{eff} = \frac{1}{\sum_{i=1}^n w_i^2},
\mu_{cov} = \mu_{eff},
d_{c\sigma} = 1 + 2 \max \left(0, \sqrt{n\mu_{eff} - 1}/n - 1\right) + c_{c\sigma},
c_{c\sigma} = \frac{\mu_{eff} + 2}{n + \mu_{eff} + 3},
c_c = \frac{4}{n + 4},
c_{cov} = \frac{1}{\mu_{cov}} \left(\frac{2}{n + \sqrt{2\mu_{eff}}} + \left(1 - \frac{1}{\mu_{cov}}\right)\right) \cdot \min \left(1, \frac{2\mu_{cov} - 1}{(n + 2)^2 + \mu_{cov}}\right).
\]

Here, we give some comments on the parameter settings. In Step 5 of the algorithm, the covariance matrix is updated mainly based on the previous covariance matrix, the current covariance matrix of the \( \mu \) best points, and transition information \( p^{(g+1)}_c \) on the distribution mean \( m^{(g+1)} \). These three kinds of information are weighted by the parameters \( c_{cov} \) and \( \mu_{cov} \). The parameter \( d_{c\sigma} \) is a damping factor that scales the change magnitude of \( \ln \sigma^{(g)} \). The parameters \( c_{c\sigma} \) and \( c_c \) are weighting factors to be used for calculating \( p^{(g+1)}_c \) and \( \sigma^{(g+1)} \), respectively. Although \( \mu_{cov} \) can be selected independently from \( \mu_{eff} \), \( \mu_{cov} = \mu_{eff} \) is often appropriate. For more detailed explanation on the parameter settings, see [9].

5. Experimental Results

Figure 3 shows the configuration of the experimental system used in this study. A pulley of diameter 65 mm is rotated using an SMA coil (Toki Corporation, BMX150) of length 50 mm which is lengthened from its original length 25 mm, and a linear spring of stiffness 8.8 N/m is placed antagonistically with respect to the SMA coil. The rotation angle of the pulley can be measured using a potentiometer connected to the pulley. The output voltage 1 V of the potentiometer corresponds to the rotation angle of 25° which also corresponds to the placement 14.17 mm of the SMA coil. The SMA coil actuator is contracted by electricity, and the maximum voltage applied is limited depending on the original length of the SMA coil. In this experiment, the maximum voltage is set to 2.5 V. The control input and output in this system are the applied voltage to the SMA coil and the output voltage of the potentiometer, respectively. The sampling time of the A/D and D/A converters is set to 10 ms.
\[ x = (\theta, \phi)^T = (K_P, K_I, K_D, \alpha, \beta, \gamma)^T \] is replaced with \( x = \theta = (K_P, K_I, K_D)^T \). The initial PID gains are set to \( \theta_0 = (3, 0.002, 0.1)^T \), and the reference signal \( r(k), k = 1, \ldots, 25000 \) is set to a waveform consisting of sinusoidal signals with variable amplitude. We obtained the input/output data from a closed-loop experiment with the initial PID gains as shown in Fig. 4.

**Case 1** At first, we show the experimental results by the conventional FRIT method. We set a sufficiently large search region as \( \{x | x_{lb} \leq x \leq x_{ub}\} \) with \( x_{lb} = (0, 0, 0, 0, 0, 0, 0, 0)^T \) and \( x_{ub} = (20, 1, 10000)^T \), so that the obtained PID gains \( x^* = (4.2795, 0.0096, 2723.7229)^T \) are not on the boundaries of the search region. The reference model is given by \( M(z) = (0.0001974z + 0.0001947)/(z^2 - 1.96z + 0.9608) \) which is obtained by discretizing the continuous-time system \( 1/(0.5s + 1)^2 \). In the CMA-ES algorithm, the maximum number of generations was set to 700. The algorithm programmed in MATLAB was run on a computer with 2.4GHz Core 2 Duo CPU and 1024MB RAM. The computation time was 297.84 s. The control output and input by the experiment with the obtained PID gains are shown in Fig. 5. In Fig. 5, the desired output is the output of the reference model driven by the reference signal. We see from the figure that the control input is mostly saturated, and the control output is far from the desired output.

**Case 2** In order to avoid the input saturation in Case 1, we set a relatively small search region by replacing the upper bound with \( x_{ub} = (5, 1, 20)^T \). Under this setting, we obtained the PID gains \( x^* = \theta^* = (5, 1, 0)^T \) by the conventional FRIT method. We show the control output and input obtained by implementing the PID gains in Fig. 6. In this case, we see that the control output does not track the desired output precisely, while the frequency of input saturation is decreased and improved in comparison with Case 1.

As shown above, it is often difficult to give good control performance by the conventional FRIT method due to the hysteresis and input saturation. Now we show the experimental results by the proposed FRIT method. We set the search region with the lower/upper bounds \( x_{lb} = (0, 0, 0, 0, -2, 0)^T \), \( x_{ub} = (5, 1, 20, 4, 2, 50)^T \) and obtained the PID gains \( \theta^* = (2.8427, 0.0055, 0)^T \) and the hysteresis parameters \( \rho^* = (2.8753, 0.1478, 5.8365)^T \). Note that the obtained solutions are not on the boundaries of the search region. The computation time was 413.17 s. We show the control output and input by the proposed FRIT method in Fig. 7. The parameter \( z \) is set to \( 10^{-5} \). It is evident from the figure that the control output tracks the desired output precisely while the control input is not saturated with 2.5 V.
5.2 Comparison of Control Performance under Various Conditions

To show the effectiveness of the proposed FRIT method for various cases, we carried out experiments by changing some conditions of Case 2 in Example 1. The control performance is evaluated by the performance index (5) where the initial output \( y_0(k) \) is replaced with the resulting output \( y(k) \). For each case, we describe the condition changed from Case 2 as follows.

Case 3 The time interval of the reference signal is changed from 250 s to 200 s, while the waveform is the same as in Case 2. Namely, the reference signal in this case varies faster than that in Case 2.

Case 4 The maximum amplitude of the reference signal is changed from 1.5 V to 1.0 V, while the waveform is the same as in Case 2.

Case 5 The initial PID gains are changed to \( \theta_0 = (1, 0.001, 1)^T \).

Case 6 The time constant of the reference model is changed from 0.5 to 0.2.

In Table 1, we show the performance index values for Cases 1–6 where \( J_0 = \sum_{k=1}^{N} (y_0(k) - r(k))^2 \) is used for the initial experiments. We see from the table that the proposed FRIT method gives better performance than the conventional one for all the cases.

### Table 1 Control performance.

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
<th>Case 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial experiment</td>
<td>323.3357</td>
<td>323.3357</td>
<td>336.9159</td>
<td>267.8111</td>
<td>1305.9920</td>
<td>323.3357</td>
</tr>
<tr>
<td>Conventional FRIT method</td>
<td>3368.7894</td>
<td>934.8809</td>
<td>549.2595</td>
<td>210.5005</td>
<td>445.8696</td>
<td>1552.9678</td>
</tr>
<tr>
<td>Proposed FRIT method</td>
<td>5.0739</td>
<td>20.3631</td>
<td>6.0969</td>
<td>33.5552</td>
<td>6.3279</td>
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</tr>
</tbody>
</table>

6. Conclusion

In this paper, we have proposed an FRIT method combined with hysteresis compensation and verified its effectiveness by experiments. It should be noted that the introduced hysteresis model is well-suited to both the offline optimization in the FRIT procedure and the online computation in the hysteresis compensator while the approximation capability of the model is satisfactory.

As is the case of dead-zone in [10], the idea of the proposed FRIT approach is to utilize partial information of nonlinearity in control systems. The same idea may be applicable to the so-called virtual reference feedback tuning (VRFT) in [14] which is similar to FRIT. Also, this kind of approach may be effective for other nonlinear properties such as saturation, which is one of the future research directions.

References


Appendix Modeling by Using the Proposed Hysteresis Model

We have good control results in the experiments by the proposed FRIT although we do not require any model of the plant. It is because the proposed hysteresis model has a good approximation ability for the SMA actuator system. In this appendix, we briefly demonstrate this fact by modeling based on the proposed model and compare the results with modeling based on a linear model.

We first consider to approximate the SMA actuator system as a series connection of the proposed hysteresis model and a
discretized model of the first-order system $K_a/(T_a s + 1)$. In this case, the parameter for characterizing the SMA actuator system is $x = (\rho, K_a, T_a)^T$. For modeling, we obtained a pair of the input and output of the SMA actuator system as shown in Fig. A.1. Then, we minimize the following function by using the CMA-ES algorithm:

$$J_a(x) = \sum_{k=1}^{N} (y(k) - y_a(k))^2,$$

where $y(k)$ is the measured output and $y_a(k)$ is the model output. Consequently, when the same input as that for modeling was given to the optimized model, we obtained the model output and the relationship between the input and output as shown in Fig. A.2.

For comparison, we carried out the system identification method for linear systems by using the MATLAB command `n4sid`. As a result, the optimal (recommended) order of the estimated model was returned as 2 by the command, and we obtained the results as shown in Fig. A.3.

It is evident from Figs. A.2 and A.3 that the proposed model can approximate the SMA actuator system adequately in comparison with the typical system identification method.

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