Fault-Tolerant Control for LPV Systems Based on Fault Compensator

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Abstract: This paper proposes a fault-tolerant control (FTC) scheme for polytopic linear parameter varying (LPV) systems. First, a fault compensator is proposed. Its structure is simple, but it is effective against actuator faults. Then, in order to show its basic idea, the authors consider a state feedback FTC scheme based on the fault compensator. Next, the FTC algorithm is extended to an output feedback FTC scheme. The FTC scheme for LPV systems involves a fault compensator based on the estimated fault and an observer based on linear matrix inequality (LMI). The proposed FTC method is applicable to a variety of systems and guarantees bounded states of the system in the event of actuator faults. Numerical examples are given to demonstrate its effectiveness.

Key Words: fault tolerant control, linear parameter varying system, linear matrix inequality.

1. Introduction

Fault-tolerant control (FTC) is a control algorithm to satisfy system stability and nominal performance in the presence of faults, and it reduces the damage to systems. Over the past few decades, researchers and engineers have made many efforts for FTC to prevent accidents, since there has been a sharp rise in the number of accidents. Such efforts can be found in books [1]–[4] and review papers [5]–[8].

Especially, Zhang and Jiang [8] summarized main results of FTC methods against various types of faults recently. Among such faults in practical engineering systems, actuator faults seem to be important. Therefore, many control algorithms against such actuator faults have been proposed in [9]–[12]. In these methods, it is assumed that the system state is measurable, which may be restrictive for practical systems. Therefore, concerning the output feedback FTC scheme, various control algorithms have been studied in [13]–[17]. However, since these control methods handle only linear time-invariant (LTI) systems, it may be desirable to develop FTC algorithms which can deal with more general class of systems such as linear time-varying (LTV) systems and linear parameter-varying (LPV) systems [18]–[24]. More importantly, in most of control methods for FTC, the controller is designed under abnormal conditions that faults exist, and the design method for abnormal conditions is more complex and difficult than that for normal conditions. For this reason, most of FTC methods are difficult to understand for practical engineers, because there are not so many people who have the expertise on control theory in industry [9]–[24]. For example, as for the FTC of LPV systems which is closely related to this paper, the existing methods in [18]–[24] require the redesign of the controller gain after faults or adopt the complex gain scheduling strategy. Therefore, engineers always ask for a simple algorithm which is easy to understand, but effective. In spite of such practical needs, there are not so many FTC algorithms for practical engineers.

From the above viewpoint, the goal of this paper is to propose a new FTC scheme which has the following three merits: First, it is based on a simple structure, but effective to compensate the effect of faults. Second, it is a control algorithm based on output feedback methods. Third, it is a control scheme for polytopic linear parameter varying (LPV) systems.

In this paper, we address a new FTC algorithm for polytopic LPV systems. It is assumed that the time-varying parameter is measurable on-line, but its future behavior contained in a given polytope is uncertain. First, a fault compensator is proposed. The fault compensator produces the signal to compensate the effect of actuator faults from the difference between desirable control inputs and manipulated control inputs with actuator faults. Then, in order to show its basic idea, we consider a state feedback FTC scheme based on the fault compensator. In the proposed FTC scheme, the fault compensator is just added to the original controller which is already designed for normal conditions. Nevertheless, the proposed FTC scheme compensates actuator faults effectively and guarantees bounded states of the system well. Next, the algorithm is extended to an output feedback FTC scheme. The FTC scheme for LPV systems involves the fault compensator based on the estimated fault and the observer based on linear matrix inequality (LMI). Numerical examples are given to demonstrate its effectiveness.

Finally, note that this paper proposes here a very simple FTC method which (i) can be used by practical engineers, (ii) does not make the resulting control system unstable (in the sense that the state is bounded), and (iii) keeps the normal performance as much as possible even if a fault occurs. The second feature is proven theoretically and the third one is verified by the numerical examples.

Notation: The symbol $*$ is used for convenience to denote the symmetric elements in a symmetric matrix. We denote the transpose and inverse of a matrix $M$ by $M^T$ and $M^{-1}$, respectively. The notation $M > 0$ $(M \geq 0)$ means that $M$ is a symmetric positive (semi-) definite matrix. The notation $\| \cdot \|$ denotes Euclidean norm. $\lambda_{\text{max}}(\cdot)$ and $\lambda_{\text{min}}(\cdot)$ denote the largest and the

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smallest eigenvalues of $(\cdot)$, respectively.

2. Problem Statement

2.1 System Description

Consider the discrete-time LPV system represented by

$$
x(k + 1) = A(p(k))x(k) + B(p(k))u(k),
$$

$$
y(k) = Cx(k),
$$

where $x(k) \in \mathbb{R}^n$ is the state, $y(k) \in \mathbb{R}^m$ is the output which is measurable, $u(k) \in \mathbb{R}^m$ is the manipulated control input, and $p(k)$ is the time-varying parameter. We assume that $[A(p(k))B(p(k))]$ varies inside a corresponding polytope $\Omega$ whose vertices consist of $\ell$ local system matrices, i.e.,

$$
[A(p(k))B(p(k))] \in \Omega, \quad k \geq 0,
$$

$$
\Omega := C_o[[A_1|B_1], [A_2|B_2], \ldots, [A_\ell|B_\ell]],
$$

where $C_o$ denotes the convex hull. In this paper, we assume that $p(k)$ is measurable at each step $k$. Then, the current system matrices $[A(p(k))B(p(k))]$ are known at each step $k$. However, the future ones $[A(p(k+i))B(p(k+i))]$, $i \geq 1$, which vary inside the prescribed polytope $\Omega$, are uncertain.

2.2 Actuator Fault

The illustration for modeling of the actuator fault is shown in Fig. 1. Let us first consider normal conditions that there is no actuator fault. Then, the manipulated control input $u(k)$ modeled by control signal is represented as follows:

$$
u(k) = u_c(k),
$$

where $u_c(k) \in \mathbb{R}^m$ is the control signal given by a controller, and it enters the actuator as the reference input for generating the actuator output. In addition, the control signal $u_c(k)$ is represented as follows:

$$
u_c(k) = -Kx(k),
$$

where $K$ is the control gain such that $(A(p(k)) - B(p(k))K)$ is quadratically stable for any $[A(p(k))B(p(k))] \in \Omega$, i.e., there exists $P > 0$ satisfying

$$
(A_j - B_jK)^T P(A_j - B_jK) - P < 0, \quad j = 1, 2, \ldots, \ell.
$$

Next, in order to model actuator faults, we consider the following manipulated control input $u(k)$ with actuator faults:

$$
u(k) = u_c(k) + f_a(k),
$$

where $f_a(k) \in \mathbb{R}^m$ is the actuator fault which is unmeasurable. Here, it is represented as follows:

$$
\begin{cases}
    f_a(k) = 0, & k < \tau, \\
    f_a(k) \neq 0, & k \geq \tau
\end{cases}
$$

where $\tau$ is the fault occurrence time which is unknown. In addition, it is assumed that

$$
\|\Delta f_a(k)\| \leq \alpha, \quad k \geq \tau,
$$

where $\Delta f_a(k) := f_a(k) - f_a(k - 1)$. Note that $\alpha$ specifies the changing speed of the fault and is given in advance.

3. State Feedback FTC Based on Fault Compensator

In this section, we propose a simple FTC scheme which does not make the system unstable and gives practically nice performance. Here, in order to show its basic idea, it is assumed that the state $x$ of the system (1) and the manipulated control input $u$ of (7) are measurable. Then, we propose a simple fault compensator for a new state feedback FTC scheme.

Consider the FTC scheme represented in Fig. 2. It is the state feedback FTC scheme based on the fault compensator represented as follows:

$$
z(k + 1) = \begin{bmatrix} u(k) \\ u_c(k) \end{bmatrix},
$$

$$
v(k) = [1 -1]z(k),
$$

where $v(k) \in \mathbb{R}^m$ is the fault compensation input which is produced by the fault compensator. Here, in order to compensate the effect of faults and guarantee bounded states of the system, the control signal $u_c(k)$ is considered as follows:

$$
u_c(k) = -Kx(k) - v(k).
$$

The control gain $K$ of (11) is given for normal conditions. There are many FTC methods which redesign or switch the control gain after the fault is detected [18],[21],[22]. However, in the proposed FTC algorithm, without redesigning or switching the control gain for abnormal conditions, the control signal (11) guarantees bounded states of the system, since the fault compensation input $v(k)$ continues the effort to compensate the actuator faults.

**Theorem 1** Consider the system in Fig. 2. If there exist $P > 0$, $G > 0$, and $\mu > 0$ satisfying

$$
(A_j - B_jK)^T P(A_j - B_jK) - P + \frac{1}{\mu}G < 0, \quad j = 1, 2, \ldots, \ell
$$

for a given $K$, then the state $x(k)$ of the system (1) is bounded, where $P \in \mathbb{R}^{n \times n}$, $G \in \mathbb{R}^{n \times n}$, and $\mu \in \mathbb{R}$.

For the proof of Theorem 1, we introduce a lemma developed by Jiang et al. [14].

**Lemma 1** For any constant $\mu > 0$ and $G > 0$, the following inequality holds:

$$
2z^\top w \leq \frac{1}{\mu}z^\top Gz + \mu w^\top G^{-1}w, \quad \forall z, w \in \mathbb{R}^n.
$$
Proof of Theorem 1 In order to prove Theorem 1, we consider the following Lyapunov function candidate

$$V(k) = x(k)^T P x(k), \quad P > 0.$$  

(14)

Then, we can obtain that

$$V(k + 1 + i) - V(k + i)$$

$$= x(k + i)^T P (k + i) x(k + i) + 2 x(k)^T P G (k + i) x(k + i)$$

$$+ 2 x(k)^T P (k + i) P B (p(k + i)) \Delta f_2 (k + i)$$

$$+ \Delta f_2 (k + i)^T P (k + i) P (p(k + i)) \Delta f_2 (k + i)$$

$$- \delta x(k)^T P x(k + i).$$  

(15)

for \(i \geq 0\), where \(\Gamma (k + i) := A (p(k + i)) - B (p(k + i)) K\). Based on Lemma 1 and (9), it is easy to show that

$$2 x(k + i)^T \Gamma (k + i) x(k + i)$$

$$\leq \frac{1}{\mu} x(k + i)^T G x(k + i) + a^2 \mu \lambda_{max} (\Gamma (k + i)^T P B (p(k + i)))$$

$$\leq a^2 \lambda_{max} (B (p(k + i))^T P B (p(k + i))).$$  

(16)

Substituting (16) and (17) into (15), it can be rewritten as

$$V(k + 1 + i) - V(k + i)$$

$$\leq 2 x(k + i)^T P B (p(k + i)) \Delta f_2 (k + i)$$

$$\leq a^2 \lambda_{max} (B (p(k + i))^T P B (p(k + i))).$$  

(17)

with

$$\Theta (k + i) := \Gamma (k + i)^T P \Gamma (k + i) - P + \frac{1}{\mu} G,$$  

(19)

$$\delta (k + i) := a^2 (\mu \lambda_{max} (\xi_1 (k + i)) + \lambda_{max} (\xi_2 (k + i))).$$  

(20)

where

$$\xi_1 (k + i) := (\Gamma (k + i)^T P B (p(k + i))) G^{-1} (\Gamma (k + i)^T P B (p(k + i))),$$  

(21)

$$\xi_2 (k + i) := B (p(k + i))^T P B (p(k + i)).$$  

(22)

When \(\Theta (k + i) < 0\), it can be obtained that

$$V(k + 1 + i) - V(k + i) < -\delta (k + i) x(k + i)^2 + \delta (k + i).$$  

(23)

with

$$\varepsilon (k + i) := \lambda_{min} (-\Theta (k + i)).$$  

(24)

Since \([A (p(k + i)) B (p(k + i))]\) varies inside the prescribed polytope \(\Omega\), we define

$$\varepsilon := \min_{[A (p(k + i)) B (p(k + i))] \in \Omega} \lambda_{min} (-\Theta (k + i)),$$  

(25)

$$\delta := \max_{[A (p(k + i)) B (p(k + i))] \in \Omega} a^2 (\mu \lambda_{max} (\xi_1 (k + i)) + \lambda_{max} (\xi_2 (k + i))).$$  

(26)

Then, it follows that

$$V(k + 1 + i) - V(k + i) < 0$$

for \(\|x(k + i)\|^2 > \frac{\delta}{\varepsilon}, i \geq 0\).  

(27)

Based on the above inequality, we will analyze the region of stable attraction. Suppose, at step \(k + i\), \(\|x(k + i)\|^2 \leq \frac{\delta}{\varepsilon}\) holds. Then, from (18), we have

$$V(k + 1 + i) \leq V(k + i) + x(k + i)^T \Theta (k + i) x(k + i) + \delta$$

$$= x(k + i)^T P (\Theta (k + i)) x(k + i) + \delta$$

$$\leq \lambda_{max} (P + \Theta (k + i)) \|x(k + i)\|^2 + \delta \leq \frac{\delta}{\varepsilon} \lambda_{max} (P + \Theta (k + i)) + \delta.$$  

(28)

In addition, we know that

$$\lambda_{min} (P) \|x(k + 1 + i)\|^2 \leq V(k + 1 + i).$$  

(29)

From (28) and (29), we obtain

$$\lambda_{min} (P) \|x(k + 1 + i)\|^2 \leq \frac{\delta}{\varepsilon} \lambda_{max} (P + \Theta (k + i)) + \delta.$$  

(30)

Therefore, the region of stable attraction is represented as

$$D_0 \triangleq \left\{ x : \|x\|^2 \leq \frac{\delta}{\varepsilon} \frac{\lambda_{max} (P)}{\lambda_{min} (P)} + \frac{\delta}{\varepsilon} \right\}.$$  

(31)

with

$$\hat{\Theta} = \max_{[A (p(k + i)) B (p(k + i))] \in \Omega} \lambda_{max} (P + \Theta (k + i)).$$  

(32)

This implies that, by virtue of the fault compensator, there exists \(k_f\) such that \(\|x(k_f)\|^2 \leq \frac{\delta}{\varepsilon}\), and the state \(x(k)\) is bounded in the region (31) for \(k \geq k_f\).

Next, we consider the condition \(\Theta (k + i) < 0\). By using Schur complement, \(\Theta (k + i) < 0\) can be transformed to

$$P - \frac{1}{\mu} G^T P \frac{1}{P \Gamma (k + i) P} > 0.$$  

(33)

Then, since the inequality (33) is affine in \([A (p(k + i)) B (p(k + i))]\), it is easy to see that \(\Theta (k + i) < 0\) holds for any

$$[A (p(k + i)) B (p(k + i))] \in \Omega.$$  

(34)

if and only if there exist \(P > 0, G > 0, \) and \(\mu > 0\) satisfying (12). This proves the theorem.

By Theorem 1, it is guaranteed that the proposed compensator do not make the system unstable. The performance of the proposed compensator will be demonstrated in Section 5.

Remark 1: Though the input disturbances are not considered in our problem setting, we can treat the disturbance by including it in the term \(f_2 (k)\) as well as the actuator faults. As for the other types of disturbances, the control performance depends on the choice of the original controller \(K\). This is not the topic of this paper. The proposed fault compensator, however, actually reduces the effect of actuator faults as demonstrated in the numerical examples. We can expect that the proposed method is effective against more general disturbances provided the original controller (without actuator faults) works well against such disturbances. In order to quantify the effects of fault compensators, more rigorous analysis will be necessary, though the results of the paper can contribute the future works that take into account a more general setting of disturbances on the system.

Remark 2: When (6) holds for some \(P > 0\), it is straightforward to see that the condition of Theorem 1 is satisfied (note that we can take an arbitrarily large \(\mu\)). Therefore, \(x(k)\) is guaranteed to be bounded for any \(f_2 (k)\) satisfying (9) as long as \(K\) stabilizes the system quadratically.
Remark 3: If \( \Delta f_d(k) = 0 \), \( k > k_0 \), holds for some \( k_0 \), we can show that \( x(k) \) converges to 0 as \( k \) grows by a similar argument. Therefore, if the actuator fault \( f_d(k) \) becomes constant, the fault compensator achieves robust stability.

4. Output Feedback FTC Scheme

In this section, we propose an output feedback FTC scheme based on the fault compensator.

Here, since it is assumed that the state \( x(k) \) of the system, the manipulated control input \( u(k) \), and the actuator fault \( f_d(k) \) are unmeasurable, we adopt the following observer to estimate the state system and the actuator fault.

\[
\dot{\hat{x}}(k+1) = A(p(k))\hat{x}(k) + B(p(k))\hat{u}(k) + L_x(y(k) - C\hat{x}(k)),
\]
\[
\hat{f}_d(k+1) = \hat{f}_d(k) + L_f(y(k) - C\hat{x}(k))
\]
with
\[
\hat{u}(k) = u(k) + \hat{f}_d(k),
\]
where \( \hat{x}(k) \in \mathbb{R}^n \) is the estimated state of \( x(k) \) and \( \hat{f}_d(k) \in \mathbb{R}^n \) is the estimated actuator fault of \( f_d(k) \). \( L_x \) and \( L_f \) are the observer gains to be determined. Note that, since we know the current system matrices \([A(p(k))B(p(k))],\) the output \( y(k) \), the estimated state \( \hat{x}(k) \), and the estimated fault \( \hat{f}_d(k) \) at each step \( k \), \( \hat{x}(k) \) and \( \hat{f}_d(k) \) are obtained using (35) and (36), respectively.

4.1 Observer Design

We design the observer for LPV systems with actuator faults. From (1), (7), (35), and (36), the error dynamics can be derived as

\[
e_r(k+1) = e_r(k) + \Delta f_d(k+1),
\]
(37)

where \( e_r(k+1) := f_d(k+1) - \hat{f}_d(k+1) \). The above error dynamics can be rewritten as

\[
\dot{\bar{e}}(k) = \left( A(p(k)) - \bar{L}\bar{C} \right) \bar{e}(k) + \bar{D} \Delta f_d(k+1)
\]
(40)

with

\[
\bar{e}(k) = \begin{bmatrix} e_r(k) \\ e_f(k) \end{bmatrix}, \quad \bar{A}(p(k)) = \begin{bmatrix} A(p(k)) & B(p(k)) \\ 0 & I \end{bmatrix},
\]
\[
\bar{L} = \begin{bmatrix} L_x \\ L_f \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} C & 0 \end{bmatrix}, \quad \bar{D} = \begin{bmatrix} 0 & I \end{bmatrix}
\]

where \([A(p(k))B(p(k))] \in \Omega\). In the following, we determine the observer gains \( L_x \) and \( L_f \) in terms of LMI. First, we define

\[
V(k) = \bar{e}(k)^T \bar{P} \bar{e}(k), \quad P > 0,
\]
(41)

and consider the following condition

\[
V(k+1+i) - \rho^2 V(k+i) < -\bar{e}(k+i)^T L_e \bar{e}(k+i)
\]
(42)

for \( i \geq 0 \), the decay rate \( \rho (0 < \rho < 1) \), and the suitable weighting matrix \( L_e > 0 \), which are given by a designer. If the condition (42) is satisfied, then it is guaranteed that \( \bar{e}(k) \) is contained in a bounded region. The observer gains \( L_x \) and \( L_f \) are determined by the following theorem.

Theorem 2 If there exist \( P > 0, G > 0, \mu > 0, \) and \( Y \) satisfying

\[
\rho^2 P - \frac{1}{\mu} G - L_e \ast P \quad Y \ast P > 0, \quad j = 1, 2, \ldots, \ell,
\]
(43)

then the observer gains \( L_x \) and \( L_f \) are obtained by \( \bar{L} = P^{-1} Y \), and \( \bar{e}(k) \) is contained in a bounded region, where \( P \in \mathbb{R}^{(\ell n + n_0)(\ell n + n_0)}, \ G \in \mathbb{R}^{(\ell n + n_0)(\ell n + n_0)}, \mu \in \mathbb{R}, \) and \( Y \in \mathbb{R}^{(\ell n + n_0)(\ell n + n_0)} \).

Proof We consider the condition (42). Then, we can obtain that

\[
V(k+1+i) - \rho^2 V(k+i) = \bar{e}(k+i)^T A(k+i)^T P A(k+i) \bar{e}(k+i) + 2\bar{e}(k+i)^T A(k+i)^T P D \Delta f_d(k+1+i) + \Delta f_d(k+1+i)^T (\bar{D}^T \bar{P} \bar{D}) \Delta f_d(k+1+i) - \rho^2 \bar{e}(k+i)^T P \bar{e}(k+i)
\]
(44)

for \( i \geq 0 \), where \( A(k+i) := \bar{A}(p(k+i)) - \bar{L}\bar{C} \). Based on Lemma 1, it is easy to show that

\[
2\bar{e}(k+i)^T A(k+i)^T P D \Delta f_d(k+1+i) \leq \frac{1}{\mu} \bar{e}(k+i)^T G \bar{e}(k+i) + \mu \bar{e}(k+i)^T \lambda_{max}(Y(k+i))
\]
(45)

with

\[
Y(k+i) := (A(k+i)^T PD)^T G^{-1}(A(k+i)^T PD)
\]
(46)

for any \( \mu > 0 \) and any \( G > 0 \). In addition, we know that

\[
\Delta f_d(k+1+i)^T (\bar{D}^T \bar{P} \bar{D}) \Delta f_d(k+1+i) \leq \alpha^2 \lambda_{max}(\bar{D}^T \bar{P} \bar{D}).
\]
(47)

Substituting (45) and (47) into (44) and considering the condition (42), (44) can be rewritten as

\[
V(k+1+i) - \rho^2 V(k+i) + \bar{e}(k+i)^T L_e \bar{e}(k+i) \leq \bar{e}(k+i)^T \Xi(k+i) \bar{e}(k+i) + \Psi(k+i),
\]
(48)

where

\[
\Xi(k+i) := A(k+i)^T P A(k+i) - \rho^2 P + \frac{1}{\mu} G + L_e,
\]
(49)

\[
\Psi(k+i) := \mu \bar{e}(k+i)^T \lambda_{max}(Y(k+i)) + \lambda_{max}(\bar{D}^T \bar{P} \bar{D}).
\]
(50)

When \( \Xi(k+i) < 0 \), it can be obtained that

\[
V(k+1+i) - \rho^2 V(k+i) + \bar{e}(k+i)^T L_e \bar{e}(k+i) \leq -\zeta \| \bar{e}(k+i) \|^2 + \Psi(k+i)
\]
(51)

with

\[
\zeta := \lambda_{min}(-\Xi(k+i)).
\]
(52)

Since \([A(p(k+i))B(p(k+i))]\) varies inside the prescribed polytope \( \Omega \), it can be rewritten as

\[
\zeta = \min_{[A(p(k+i))B(p(k+i))] \in \Omega, \ z \geq 0} \lambda_{min}(-\Xi(k+i)),
\]
(53)

\[
\Psi = \max_{[A(p(k+i))B(p(k+i))] \in \Omega, \ z \geq 0} \alpha^2 (\mu \lambda_{max}(Y(k+i)) + \lambda_{max}(\bar{D}^T \bar{P} \bar{D})).
\]
(54)

Then, it follows that \( V(k+1+i) - \rho^2 V(k+i) + \bar{e}(k+i)^T L_e \bar{e}(k+i) < 0 \) for \( \zeta \| \bar{e}(k+i) \|^2 > \delta, \ i \geq 0 \). Similarly to the state feedback FTC
scheme, the region of stable attraction can be obtained. The region is represented as
\[ D_{\psi} = \left\{ \tilde{e} : \|\tilde{e}\|^2 \leq \frac{\gamma}{\zeta} \frac{\tilde{z}}{\lambda_{\text{min}}(P)} + \frac{\gamma}{\lambda_{\text{min}}(P)} \right\} \]  
with
\[ \tilde{z} = \max_{\{A(p(k+i))B(p(k+i))\in \Omega, i \geq 0\}} \lambda_{\text{max}}(\rho^2 P - L \epsilon + \Xi(k + i)). \]  
This implies that there exists \( k_T \) such that \( \|\tilde{e}(k_T)\|^2 \leq \frac{\gamma}{\zeta} \), and the state \( \tilde{e}(k) \) is bounded in the region (55) for \( k \geq k_T \). Similarly to the proof of Theorem 1, this is due to the use of the fault compensator.

Next, we consider the condition \( \Xi(k + i) < 0 \). Then, it can be rewritten as
\[ \rho^2 P - \frac{1}{\mu} G - L \epsilon - (P \Lambda(k + i))^{T} P^{-1}(P \Lambda(k + i)) > 0. \]  
By using Schur complement, it is possible to cast the inequality (57) in the following form:
\[ \begin{bmatrix} \rho^2 P - \frac{1}{\mu} G - L \epsilon \quad \ast \\ \ast \quad P \end{bmatrix} > 0. \]  
Substituting \( Y := PL \), we see that (58) is equivalent to
\[ \begin{bmatrix} \rho^2 P - \frac{1}{\mu} G - L \epsilon \quad \ast \\ \ast \quad P \Lambda(p(k + i)) - Y C \end{bmatrix} > 0. \]
Since the inequality (59) is affine in \( \{A(p(k+i))B(p(k+i))\} \), it is satisfied for all
\[ \{A(p(k+i))B(p(k+i))\} \in \Omega \] if and only if there exist \( P > 0, G > 0, \mu > 0 \), and \( Y \) satisfying (43). Then, the observer gains \( L_x \) and \( L_f \) are obtained by \( \tilde{L} = P^{-1}Y \).

4.2 Output Feedback FTC Based on Fault Compensator

We propose a new fault compensator based on the observer, but its structure is similar to that of (10). Figure 3 illustrates the output feedback FTC scheme based on the fault compensator represented as follows:
\[ z(k + 1) = \begin{bmatrix} \hat{u}(k) \\ u_s(k) \end{bmatrix}, \]
\[ v(k) = [1 - 1]z(k), \]  
where \( v(k) \in \mathbb{R}^n \) is the fault compensation input which is produced by the fault compensator. In order to compensate the effect of faults and guarantee bounded states of the system, we consider the control signal \( \hat{u}(k) \) represented as follows:
\[ u_s(k) = -K\hat{x}(k) - v(k). \]  
In the proposed output feedback FTC scheme, based on the estimated actuator fault at step \( k - 1 \), the fault compensation input \( v(k) \) continues to compensate the actuator faults. Then, as shown in Section 3, we can see that though the control input \( u(k) \) with unknown actuator faults \( f_a(k) \) is manipulated to the system (1), the state of the system is bounded in a region by the control signal \( u_s(k) \) with \( v(k) \) produced by (61). The analysis of the bounded region is similar to that of the state feedback FTC scheme and is omitted.

![Fig. 3 Output feedback FTC scheme based on fault compensator.](image)

Remark 4: Similarly to the state feedback FTC scheme, if \( \Delta f_a(k) = 0, k > k_s \) holds for some \( k_s \), then the state \( x(k) \) converges to zero as \( k \to \infty \).

Remark 5: Designing the feedback gain \( K \) using LMIs, we may construct a better FTC system. However, it is also important to design the fault compensator for a fixed \( K \) (instead of designing \( K \) as well). Because \( K \) should be designed from various viewpoints (not only from stabilization viewpoint) in many practical cases, and also engineers cannot often touch \( K \) in currently-operated control systems. Hence it would be convenient if we can achieve fault tolerance property by adding some compensator to the given feedback controller \( K \).

5. Numerical Example

In order to demonstrate the effectiveness of the proposed FTC technique, simulations are performed with the state feedback FTC scheme and the output feedback FTC scheme, respectively. In addition, the system disturbance and the sensor noise are considered for practical applications. The discrete-time LPV system is given by
\[ x(k + 1) = A(p(k))x(k) + Bu(k) + d(k), \]
\[ y(k) = Cx(k) + n_1(k) \]  
with
\[ A(p(k)) = 10^{-3} \times \begin{bmatrix} 999.6 & 0.2699 & 0.1646 & -4.558 \\ 0.4794 & 990 & -0.1761 & -40.01 \\ 0.9992 & 3.65 + p(k) & 993 & 14.07 \\ 0.005001 & 0.0183 & 9.965 & 1000 \end{bmatrix}, \]
\[ B = \begin{bmatrix} 0.004421 & 0.001754 \\ 0.03527 & -0.07554 \\ -0.05494 & 0.0446 \\ -0.0002751 & 0.0002235 \end{bmatrix}, \]
\[ C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \]
where \( d(k) \) and \( n_1(k) \) are the system disturbance and the sensor noise, respectively. Especially, in the state feedback FTC scheme, since there are sensors to measure the state \( x(k) \) and the manipulated control input \( u(k) \), the control signal (11) can be rewritten as
\[ u_s(k) = -K(x(k) + n_2(k)) - (v(k) + n_3(k)), \]  
where \( n_2(k) \) and \( n_3(k) \) are the sensor noises. Here, \( d(k) \), \( n_1(k) \), \( n_2(k) \), and \( n_3(k) \) are set as white noise, and the maximum amplitude is set as 0.05.

The initial states of the system (63) and the observer (35) are given as follows:
\[ x(0) = [-1.5 \, -0.2 \, 0.5 \, -1]^T, \]
\[ \dot{x}(0) = [-0.5 \, 1 \, 0.1 \, -0.3]^T. \]
We assume that the time-varying parameter \( p(k) \) belongs to the following region:
\[
p(k) \in [0, 5],
\]
and it is set as white noise. Under normal conditions, the control gain \( K \) is given as follows:
\[
K = \begin{bmatrix}
9.7672 & 0.0927 & -0.9981 & -4.1141 \\
6.1452 & -0.1364 & -0.5855 & -1.9789
\end{bmatrix}.
\]

5.1 Simulation with State Feedback FTC
The effectiveness of the proposed state feedback FTC scheme is demonstrated. A fault scenario is used to demonstrate the effectiveness of the proposed state feedback FTC scheme. The fault scenario is that the faults in the first and second actuators occur at \( k \geq 100 \) and \( k \geq 500 \), respectively. The actuator fault is shown in Fig. 4.

Under the above fault scenario, the fault compensation input (10) is updated on-line to compensate the effect of the actuator fault. Figure 5 shows the system state \( x_1(k) \) with and without the fault compensator. In the absence of the fault compensator, the deviation from the origin is considerably large and the performance is not satisfactory. On the other hand, the performance is much improved by using the fault compensator. From this simulation results, it is clear that the proposed state feedback FTC scheme works well.

5.2 Simulation with Output Feedback FTC
The effectiveness of the proposed output feedback FTC scheme is demonstrated. The decay rate \( \rho, \mu \), and the weighting matrix \( L_e \) are set as follows:
\[
\rho = \sqrt{0.6}, \quad \mu = 10^{10}, \quad L_e = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}.
\]

Then, from Theorem 2, the observer gains \( L_x \) and \( L_f \) are obtained as
\[
L_x = \begin{bmatrix}
1.0115 & -0.0320 & -0.0452 & -0.0048 \\
-0.0368 & 1.3606 & 0.0374 & -0.0400 \\
-0.0490 & -0.0135 & 1.3059 & 0.0157 \\
-0.0002 & -1.0000 & -0.9884 & 1.0000
\end{bmatrix},
\]
\[
L_f = \begin{bmatrix}
2.1561 & -5.7254 & -9.7064 & -0.0488 \\
1.5110 & -7.0901 & -4.5395 & -0.0228
\end{bmatrix}.
\]

The fault scenario considered here is that actuator faults occur at \( k \geq 300 \) in both the first and second actuators simultaneously, the fault values in the first and second actuators are 14.5 and 9.5, respectively. Under the above fault scenario, based on the estimated system states and the estimated faults, the fault compensation input (61) is updated on-line to compensate the effect of the actuator fault. Figure 6 shows the system state \( x_1(k) \) with and without the fault compensator. From this figure, it is obvious that although the actuator fault occurs, the state of the system is bounded by the proposed FTC scheme. Figure 7 shows the system state error \( x_1(k) - \hat{x}_1(k) \). The actuator fault \( f_a(k) \) and the estimated fault \( \hat{f}_a(k) \) are plotted in Fig. 8. Figures 7 and 8 show that the proposed observer works well. From this simulation results, it is obvious that the proposed output feedback FTC scheme works well.

6. Conclusion
A new FTC scheme for polytopic LPV systems against actuator faults has been presented. First of all, the authors have proposed a fault compensator. Its structure is simple to understand, but it is effective against actuator faults. Then, in order to show its basic idea, a state feedback FTC scheme based on the fault compensator has been given. In the proposed FTC scheme, it is not necessary to redesign or switch the control...
gain $K$, since the fault compensator is just added to the original controller designed under no fault conditions. Nevertheless, the state of the system can be bounded in a stable region since the fault compensator works well. Next, the algorithm has been extended to a output feedback FTC scheme. The FTC scheme for LPV systems involves the fault compensator based on the estimated faults and the observer based on LMI to estimate the system state and the actuator fault. The proposed FTC method is applicable to a variety of systems and guarantees bounded states of the system in the event of actuator faults. Simulation results have showed that the proposed FTC scheme works well.

As a future work, a new FTC method for systems subject to input saturation should be considered. In addition, although it is assumed that time-varying parameter is measurable on-line, it is an interesting topic to develop an output feedback FTC algorithm combined with the time-varying parameter estimation method.

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