Halftone Control of Distributed Generation Networks

Yuki Minami *, Shun-ichi Azuma **, and Toshiharu Sugie **

Abstract : The Smart Grid is a novel concept which integrates the power grid and the information-communication technology in order to increase the reliability of electricity and the efficiency of energy usage. Toward developing the smart grid, this paper focuses on the control of distributed generation networks composed of multiple generators such as gas-engines and gas-turbines. In this paper, we first formulate an operation mode control problem which is to determine ON/OFF modes of the generators subject to the balance of supply and demand. Then, as a solution to the problem, this paper proposes a simple and useful distributed control algorithm based on the idea of the halftoning which is one of image processing techniques. Finally, the effectiveness of the proposed method by several numerical simulations is evaluated.

Key Words : smart grid, balance of supply and demand, distributed generator, halftoning.

1. Introduction

The Smart Grid is a novel concept which refers to the restructuring of power grid systems, and it integrates the power grid and the information-communication technology. In recent years, the smart grid has been getting lots of attention for the reduction of carbon emission to combat global warming [1]–[8]. An image of smart grid is shown in Fig. 1 where distributed power generators such as small gas-turbines and wind generators are connected each other via communication channels, and they are controlled by softwares. By developing the smart grid technology, we reap most of the benefits, e.g., effective utilization of power equipment, large scale introduction of natural energy, and infrastructure construction of electric vehicle.

For the distributed generation networks which consist of multiple distributed generators and communication channels, one of important topics is to balance power demand with supply [9]–[11]. Thus, this paper focuses on the power demand and supply control problem for the networks. More precisely, we address the operation mode control problem formulated as follows: when a power demand distribution is given, determine the operation modes, i.e., the outputs of the distributed power generators such that the resulting supply distribution is similar to the demand distribution. For the problem, we assume that each generator has ON/OFF mode. The OFF mode means the rest of power generation; the ON mode means the operation with the highest generation efficiency.

As the solution to the problem, it should be expected that many generators have the ON mode in high power demand area while the OFF mode in low demand area. However, it is not easy to decide such operation mode in general: we have to appropriately choose either ON mode or OFF mode such that the amount and distribution of power supply are similar to those of demand. Typical approaches to the problem are, for example, (i) to solve a combinatorial optimization problem [12], (ii) to decide the ON/OFF mode of each generator by comparing a given demand value with an appropriate threshold value. For the method (i), we need not only communications to obtain total demand but also a centralized controller to control all power generators. Therefore, when the number of generators is large, the communications and the controller are complicated. In addition, it is difficult to solve the corresponding combinatorial optimization problem in a view point of computational cost. On the other hand, the method (ii) independently decides the operation modes without communications between generators, which could lead to an unbalance of supply and demand. Also, this method is separate from the mainstream of power supply that the generators are smartly controlled with network.

Motivated by the above background, this paper treats the distributed generation network as the dynamical system whose input and output are the information on demand and the supply distribution, respectively. Then, the authors propose the distributed control method for the system based on the idea of halftoning [13]–[15] which is one of the image processing techniques. Halftoning is to convert grayscale images to binary images which are perceptually similar to the original grayscale images. By using the analogy between the halftoning problem and the problem considered here, a simple and effective distributed control method is given in this paper. The effectiveness of the proposed method is evaluated by several numerical simulations.

Finally, it should be emphasized that the contributions of this
paper are (i) to formulate a novel problem for distributed generation networks, (ii) to reveal a weaker condition for networks such that the problem can be practically solved, and (iii) to present an algorithm which works well under the condition by using the idea of half-tone image processing. It is also noted that this paper is based on our previous version [16] published in a conference proceedings, and it includes full explanations and several simulation results.

**Notation:** Let $\mathbb{R}$ and $\mathbb{R}_+$ denote the real number field and the set of non-negative real numbers, respectively. For the set $\mathcal{S}$, let $\Pi(\mathcal{S})$ denote the power set of $\mathcal{S}$, e.g., $\Pi(\{1, 2, 3\}) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$. The symbol $\|\cdot\|_p$ expresses the $p$-norm. For $c \in \mathbb{R}^2$ and $s \in \mathbb{R}_+$, we define the set $\mathcal{B}_p(c, s) := \{z \in \mathbb{R}^2 | \|z - c\|_p \leq s\} \subset \mathbb{R}^2$. For example, $\mathcal{B}_2(c, s)$ is the 2-norm ball of center point $c$ and radius $s$. Finally, $|J|$ expresses the number of elements of the finite set $J$.

## 2. Problem Formulation

Consider a two-dimensional bounded region $\mathcal{P} \subset \mathbb{R}^2$ and suppose that the power demand distribution is given, e.g., as shown in Fig. 2 (left upper). Let the distribution be denoted by the function $\phi : \mathcal{P} \to [0, 1]$, and let the power demand value at the position $p \in \mathcal{P}$ be expressed as $\phi(p)$. In addition, we consider that $n$ power generators are deployed in the region $\mathcal{P}$ as shown in Fig. 2 (left down). The position of the $i$-th power generator is denoted by $p_i \in \mathcal{P}$, and let $\mathcal{P} := \{p_1, p_2, \ldots, p_n\}$ be the collection of $n$ positions.

The dispersed $n$ power generators are connected each other according to a network structure. The structure is represented by the directed graph $G = (\mathcal{V}, \mathcal{E})$ as shown in Fig. 3 where $\mathcal{V} := \{1, 2, \ldots, n\}$ is the vertex and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the edge. For the graph, we suppose that the direction of the arrow corresponds to the direction of information flow, namely, some information is transferred according to the direction. Furthermore, we define the set of the generators which send information to the $i$-th generator by $\mathcal{N}_i := \{j \in \mathcal{V} | (j, i) \in \mathcal{E}\}$. The set is called by the neighborhood of the generator $i$ (e.g., for the graph in Fig. 3, $\mathcal{N}_1 = \{4\}$, $\mathcal{N}_2 = \{1, 3\}$, $\mathcal{N}_3 = \{1, 2\}$, $\mathcal{N}_4 = \{1, 3, 5\}$, etc.).

In this paper, we assume that each power generator has two operation modes which are ON mode and OFF mode. The ON mode means the operation with the highest generation efficiency$^1$ and the OFF mode means the rest of power generation. Then, we determine each operation mode (ON/OFF) of the power generator by using two pieces of information: one is the local information on demand distribution and the other is the information transferred between power generators. Moreover, we assume that the energy supplied from the generator with the ON mode is spatially distributed in the area, which forms a power supply distribution as shown in Fig. 2 (upper right)$^2$.

In the following part of this section, the operation mode control problem is formulated after we introduce a distributed controller embedded in each generator and a diffusion equation which represents power distribution.

**Distributed controller:** Suppose that the distributed controller $K_i : (\mathcal{K}_{ui}, \mathcal{K}_{ni}, \mathcal{K}_{si})$, composed of the three sub-controllers $\mathcal{K}_{ui}$, $\mathcal{K}_{ni}$, and $\mathcal{K}_{si}$, is embedded in each generator as shown in Fig. 4.

In $\mathcal{K}_{ui}$, the operation mode and information which is sent to other generators are determined. In addition, $\mathcal{K}_{si}$ gets local information about demand distribution, and $\mathcal{K}_{si}$ deals with information transferred from nearby generators.

The distributed controller is explained in more detail.

The controller $\mathcal{K}_{ui}$ determines the operation mode of the $i$-th generator and the information transmitted between power generators, and it is given by

$$
\begin{align*}
\mathcal{K}_{ui} : \quad & x_i(k + 1) = ax_i(k) + b_1 r_1(k) + b_2 u_i(k), \\
& y_i(k) = c_1 x_i(k) + d_1 r_1(k), \\
& z_i(k) = Q(c_2 x_i(k) + d_2 r_1(k)),
\end{align*}
$$

where $x_i \in \mathbb{R}^m$ is the memory of $\mathcal{K}_{ui}$ (the initial value is $x_i(0) = 0$), $r_i \in \mathbb{R}$ and $u_i \in \mathbb{R}$ are the inputs, $y_i \in \mathbb{R}$ and $z_i \in \{0, 1\}$ are the outputs, $a \in \mathbb{R}^{m \times m}$, $b_1, b_2 \in \mathbb{R}^{m \times 1}$, $c_1, c_2 \in \mathbb{R}^{1 \times m}$, $d_1, d_2 \in \mathbb{R}$ are constant matrices. The function $Q : \mathbb{R} \to \{0, 1\}$

$^1$For distributed generators such as gas-turbines, there exists a certain rotational speed of turbine with the highest generation efficiency. This paper regards such operation of generators as ON mode.

$^2$This paper considers an energy distributed model. The model captures a qualitative nature that the high energy is supplied in a neighborhood of the generator with ON mode while the low energy is supplied in a distant place of the generator.
is the two-level static quantizer which rounds off to the nearest integer, and $k \in \mathbb{R}_+ \cup \{0\}$ is the step number. The output $y_i \in \mathbb{R}$ indicates the information transferred from the $i$-th generator to other generators, and the output $z_i \in \{0, 1\}$ is the operation mode: $z_i = 1$ and $z_i = 0$ correspond to ON and OFF modes, respectively. We denote the collection of $n$ operation modes at $k$-step by $\mathcal{Z}_k := \{z_i(k), z_2(k), \ldots, z_n(k)\}$.

Next, $K_{si}$ acquires local information on the power demand distribution $\phi$, and it is represented by

$$K_{si} : r_i(k) = \int_{\mathbb{B}_i(p, \gamma)} f(\phi(q))dq. \quad (2)$$

In (2), $\gamma \in \mathbb{R}$ characterizes the size of the circle area $\mathcal{B}_i(p, \gamma)$ where each power generator can get the local information on demand $\phi$. Note that $\gamma$ is given by sensor specification in advance. On the other hand, $\gamma : [0, 1] \to [0, 1]$ is the function which specifies how to use the local information about demand, and it can be designed by control designers.

Finally, $K_{si}$ handles the information $y_j (j \in N_i)$ received from neighbor power generators.

$$K_{si} : u_i(k) = g(y_1(k), y_2(k), \ldots, y_n(k), N_i). \quad (3)$$

We assume that the function $g : \mathbb{R}^n \times \Pi(\{1, 2, \ldots, n\}) \to \mathbb{R}$ depends only on the information $y_j (j \in N_i)$ transferred from the generators connected via communication channels. For example, if $N_i = \{1, 2\}$, then $g(y_1, y_2, \ldots, y_n, N_i) = g(y_1, y_2)$.

**Diffusion equation:** When all operation modes are determined by the above controllers $K_i : (K_{si}, K_{si}, K_{si}) (i = 1, 2, \ldots, n)$, the power generators with the ON mode supply the power energy. This paper supposes that each energy amount of the generator is same, and that the spatial distribution of the energy is given by the following solution to the diffusion equation.

$$D : \psi_{s-z_i}(p) = C \int_{\mathbb{B}_i(p, \gamma)} \exp \left(\frac{\|p - q\|^2}{2\sigma^2}\right) \xi(q; \mathcal{P}, \mathcal{Z}_i) dq \quad (4)$$

where $\psi_{s-z_i}(p)$ is the power supply at $p \in \mathbb{P}$, $C \in \mathbb{R}$ is the normalization constant, $\sigma \in \mathbb{R}$ is the diffusion weight coefficient, and $s \in \mathbb{R}$ gives the size of diffusion area. Although we use $\mathbb{B}_i(p, s)$ for numerical simulations shown in Section 4, we can consider the set $\mathbb{B}_i(p, s)$. Further, the function $\xi(q; \mathcal{P}, \mathcal{Z}_i)$ is given by

$$\xi(q; \mathcal{P}, \mathcal{Z}_i) := z_i(k) \delta(q - p). \quad (5)$$

where $\delta$ is the Dirac delta function. We use the function in order to set the supply energy zero at the area where the power generators are not disposed. For instance, when one generator is ON mode as shown in Fig. 5 (left), the supply distribution made by (4) is given by Fig. 5 (right).

**Operation mode control problem:** Before formulating the problem, some symbols and an assumption are prepared.

Consider the graph $G$ which omits the channels connecting from the generator of large index to that of small index from the graph $G^1$. For $G$, in a similar way to $N_i$, the neighborhood of the $i$-th generator is defined by $\tilde{N}_i$ (in Fig. 3, $\tilde{N}_1 = \emptyset$, $\tilde{N}_2 = \{1\}$, $\tilde{N}_3 = \{1, 2\}$, $\tilde{N}_4 = \{1, 3\}$, etc.). Moreover, we make the following assumption.

[Assumption] The graph $G$ is weakly connected.

This assumption means that the graph $G$ is not disconnected, i.e., no generator is isolated. The assumption is made as a minimum requirement for deriving a solution to the operation mode control problem considered here. In fact, the key idea of this paper is to use a halftoning algorithm which will be explained in Section 3. In order to apply the halftoning algorithm to the problem, we at least need the condition that no generator is isolated.

Then, the operation mode control problem is formulated as follows.

**Problem 1** The power demand distribution $\phi : \mathbb{P} \to [0, 1]$, the location of the generators $\mathcal{P} \subset \mathbb{P}^n$, the network structure $G$, the solution $\psi_{s-z_i}$ of diffusion equation are given, and consider the cost function

$$J(\mathcal{Z}_\infty) := \|\phi - \psi_{s-z_i}\|. \quad (6)$$

Then, find the distributed controllers $K_i : (K_{si}, K_{si}, K_{si}) (i = 1, 2, \ldots, n)$ (i.e., the constant matrices $a_i \in \mathbb{R}^{m_i \times m_i}, b_1, b_2 \in \mathbb{R}^{m_i}, c_1, c_2 \in \mathbb{R}^{m_i}, d_1, d_2 \in \mathbb{R}$, the function $f : [0, 1] \to [0, 1]$, and the function $g : \mathbb{R}^n \times \Pi(\{1, 2, \ldots, n\}) \to \mathbb{R}$), which minimize the function $J(\mathcal{Z}_\infty)$.

The value of the cost function (6) is the 2-norm of the scalar-valued function $\phi - \psi_{s-z_i}$ on the set $\mathbb{P}^n$. The cost function $J(\mathcal{Z}_\infty)$ evaluates the difference between the demand distribution $\phi$ and the supply distribution $\psi_{s-z_i}$ generated by power generators. Thus, if we determine the operation modes of generators by using the controller which is a solution to the problem, then it is expected that the supply distribution is similar to the demand one.

### 3 Halftone Control

This section presents a solution to Problem 1 based on the idea of halftone image processing.

#### 3.1 Halftoning

Halftoning is the process of transforming grayscale images into binary images composed of white and black pixels. The resulting binary images, which are called halftone images, are expected to be perceptually similar to their original grayscale versions. The key point of halftoning is that the density of white and black pixels depends on the intensity distribution of a grayscale image. Moreover, a halftone image is generated by considering human visual specification, i.e., lowpass characteristic. Therefore, a grayscale image converted from a halftone

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1 For example, in Fig. 3, the channels connecting from the third generator to second and from forth to first are omitted.
image by using a lowpass filter is perceptually similar to the original image.

One of the typical halftoning algorithms is the error diffusion method [13],[14]. The method consists of a feedback loop, and it works as follows. First, a grayscale pixel is converted to a binary pixel, and its binarization error is computed. Then, the error is distributed to neighbor pixels which have not yet been processed, namely, the error value is added to values of neighbor pixels. Figure 6 shows the original grayscale image and the binary one generated by the error diffusion method. From these images, we can see that the light part of the original image is converted to the cluster of white pixels, and that the dark part is converted to the cluster of black pixels.

3.2 Proposed Method

The operation mode control problem considered in this paper can be regarded as the halftoning problem by replacing power generators, operation modes (ON/OFF), and a power demand distribution with pixels of images, intensity values (white/black), and a concentration distribution, respectively. Therefore, if we apply the error diffusion algorithm to the problem, it is expected that the density of power generators with ON mode is determined depending on the magnitude of demand. Further, the solution to the diffusion equation is a type of low-pass filters. Thus, according to the feature of the halftoning, the generated power supply distribution could be similar to the demand one, namely, the value of $J(Z_{∞})$ could be small.

Based on the above idea, we propose the following distributed controller $K_i : (K_{ui}, K_{si}, K_{oi})$.

First, $K_{ui}$ is given by (1) with $a := 0$, $b_1 = b_2 := 1$, $c_1 = c_2 := 1$, and $d_1 = d_2 := 0$, that is,

$$K_{ui} : \begin{cases} x_i(k+1) = r_i(k) + u_i(k), \\
y_i(k) = x_i(k), \\
z_i(k) = Q(x_i(k)). \end{cases} \quad (7)$$

Two pieces of information about the demand distribution $r_i$ and the transmitted data $u_i$, which are determined by $K_{si}$ and $K_{oi}$, are saved at the memory $x_i$. Then, the controller determines the operation mode $z_i$ and the information $y_i$ transferred between power generators.

Next, for $K_{si}$, the function $f$ is given by

$$f(\phi(q)) := \begin{cases} \phi(p_i) & \text{if } q = p_i, \\
0 & \text{otherwise}, \end{cases} \quad (8)$$

i.e., $K_{si} : r_i(k) = \phi(p_i)$. Although it is necessary to decide $r_i$ considering the demand around each generator, we use the value of the demand information at the deployed position of the power generator for simplicity.

Finally, $K_{oi}$ is given by

$$K_{oi} : u_i(k) = - \sum_{j \in N_i} a_{ij} (Q(y_j(k)) - y_j(k)), \quad (9)$$

with

$$a_{ij} := \frac{2}{|I_i|} \frac{\|p_i - p_j\|}{\sum_{k \in I_i} \|p_i - p_k\|}, \quad (10)$$

where $I_i := \{ j \in \mathcal{V} | i \in \tilde{N}_j \}$. The element of the set $I_i$ is the power generator which receives the information from the $i$-th generator. In the proposed method, the difference between $y_j$ and its quantized value $Q(y_j)$ is used. This discrepancy $Q(y_j) - y_j = Q(x_j) - x_j$ implies the error which is generated during the determination of the operation mode. The error corresponds to a binarization error in halftoning. Therefore, we can find that the proposed method is based on the halftoning algorithm. From this fact, the total error $J(Z_{∞})$ is expected to be small. In addition, $a_{ij}$ in (10) is the weight for error propagation, and it is given so as to be large when the distance of two generators $i$ and $j$ is short.

4. Simulation

Consider 100 distributed power generators as shown in Fig. 7 (a), and suppose that the demand distribution $\phi$ in Fig. 7 (b) is given. Then, we use the proposed method to determine the operation modes $Z_{∞}$. The result is depicted in Fig. 7 (c) where the marks $\circ$ and $\times$ indicate ON and OFF modes, respectively. It follows that many generators are ON mode in the high demand area (white part) while the generators are OFF in the low one (black part). In addition, we obtain the power supply distribution based on the solution to the diffusion equation. In (4), we set $\sigma := 10$, $s := 25$, and $C := 0.19$, and then we obtain the result shown in Fig. 7 (d). This result illustrates that the supply distribution is similar to the demand one (Fig. 7 (b)).

Furthermore, when we consider the simple quantization of $r_i$, i.e., $z_i = Q(r_i)$, the operation mode $Z_{∞}$ is given as Fig. 7 (e).
Note that the simple method means an operation mode control without communication between generators. Then, the supply distribution Fig. 7 (f) is obtained by using (4). Compared Fig. 7 (d) with (f), we can see that the proposed method is superior to the simple method. In fact, the value of cost function is $J(Z_\infty) = 12.6345$ for the proposed method, while $J(Z_\infty) = 15.2956$ for the simple method.

In order to evaluate the effectiveness of the proposed method for any demand distribution, we consider four cases shown in the left column of Fig. 8. Then, by using the proposed method, we obtain the results in Fig. 8: the middle figures show the operation modes of generators and the right ones indicate the supply distributions. For each case, it turns out that the supply distribution is similar to the demand one. Moreover Table 1 shows the value of $J(Z_\infty)$ for the proposed method is less than that for the simple static quantization in each case. This means that the proposed method is effective and promising.

From these results, one can conclude that the proposed method is useful for determining the operation modes of the power generators.

Finally, we should notice the following two points.

One is the optimality of the proposed method. Although the distributed controller proposed in this paper minimizes the cost function in (6), it is still not clear that the controller is optimal for the optimization problem. Indeed, considering the combinatorial optimization problem for the above example, the number of candidate solutions to the problem is more than $10^{30}$, which means it is impossible to solve the problem. In this regard, however, we have tried to solve a minor optimization problem which has $10^6$ candidate solutions, and we have confirmed that the proposed method provides the smallest value of the cost function (6) within the $10^6$ candidate solutions.

The other is the supply and demand balance. The proposed method determines operation modes of generators such that the supply distribution is similar to the demand one. However, it is not theoretically guaranteed that the supply amount is greater than the demand one. To this problem, a practical solution is to use the modified demand information $\bar{\phi} = \beta \phi$ where $\beta > 1$.

### Table 1

<table>
<thead>
<tr>
<th>Case</th>
<th>Proposed method</th>
<th>Simple quantization</th>
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<tbody>
<tr>
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<tr>
<td>case 2</td>
<td>14.4871</td>
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<tr>
<td>case 3</td>
<td>9.8338</td>
<td>22.2550</td>
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<tr>
<td>case 4</td>
<td>12.8530</td>
<td>18.0589</td>
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</table>

### 5. Conclusion

In this paper, we have considered the problem of finding the operation modes of power generators in the distributed generation network. We first formulated the problem, and then proposed a distributed control method based on the idea of halftone image processing. Next, we have validated the effectiveness of the proposed method by several numerical simulations. We have concluded that the proposed method is simple and useful.

This paper assumes that each energy amount of generator is same. Thus, as a future work, it is expected to consider the case that each energy amount is different. In addition, this paper has not been considered a generator model and real-world problems such as energy transmission loss, frequency constraint, and so on. Therefore, it is one of the interesting future works to consider those problems. Furthermore, the proposed method is derived under unidirectional communication. Thus, the design of control methods for bidirectional communication should be taken into consideration, because such methods might provide better performance than the proposed method.

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Fig. 8 Simulation results.

References


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