Fictitious Reference Iterative Tuning to Modified IMC for Unstable Plants

Hien Thi Nguyen *, Osamu Kaneko **, and Shigeru Yamamoto **

Abstract: This paper proposes a data-driven controller parameter tuning of the modified internal model control (IMC), which was proposed by Yamada in 1999, for unstable plants. Here the authors apply fictitious reference iterative tuning (FRIT) to the parameterized modified IMC with only one-shot experimental data. The proposed approach enables us to simultaneously obtain the optimal controller for a desired performance and an appropriate model of the actual plant, and it is applicable for unstable plants in both of the minimum phase and the non-minimum phase cases.

Key Words: fictitious reference iterative tuning (FRIT), data-driven approach, internal model control (IMC), unstable plants.

1. Introduction

Internal Model Control (IMC) [1], which is shown in Fig. 1, has a great advantage to design a control system for a desired tracking property. In this figure, \( P \) is the controlled plant, \( P \) is a plant model, \( C_{IMC} \) is the controller, and \( r, u, y \) are the reference signal, the input and the output, respectively. One of the features of IMC is that it requires an explicit model of the plant to be used as a part of the controller. Thus, tuning the controller parameters, in general, requires an identification. However, in practice, there are many cases where it is difficult to apply a persistent signal to the actual plant for executing the identification from the viewpoints of the safe operation. Moreover, from the viewpoints of the management, there are also many cases where it is preferable to reduce time and expense. Thus, in cases where a plant model is unavailable, the direct use of the collected data to update the controller parameters (which we call data-driven approach) is an effective way since a model identification is omitted and we can avoid the difficulties of undermodeling. Moreover, since the IMC controller includes the plant model internally, it is expected that the data-driven approach to IMC enables us to obtain both a desired controller and an appropriate model of the actual plant.

From such points of view, there are a number of studies on data-driven approaches to IMC. In [2], De Buyne proposed IMC with iterative feedback tuning (IFT) [3]. However, the approach with IFT requires many experiments to update the parameters of the controller, thus it has some crucial problems with respect to practical points of view. In [4], another IMC approach with virtual reference feedback tuning (VRFT) [5] was proposed. This approach has a great advantage with only a single experiment to achieve a desired specification, and hence the time and expense for obtaining the optimal parameters are drastically reduced.

Nevertheless, the controller used in [4] is restricted as the class of linearly parameterized controllers. In [6]–[8], fictitious reference iterative tuning (FRIT) [9] was utilized for the controller parameter tuning of the IMC. Similarly to VRFT, FRIT is a controller parameter tuning method that achieves the desired specification with only a single experiment. However, FRIT considers the minimization of the error between the fictitious desired output and the actual one while VRFT focuses on the error between the virtual input and the actual one, and hence FRIT is intuitively understandable with respect to obtaining a desired output. Moreover, the approaches proposed in [6]–[8] treat the controller whose denominator and numerator are parameterized, which implies that more effective tuning of the IMC can be performed. In fact, by focusing on the feature that IMC controller includes the plant model internally, the approaches in [6]–[8] yield not only a desired controller but also a model of the plant. However, all of them are only applicable to the stable systems.

On another front, IMC, despite widely utilized in practical applications, cannot be applied for the unstable plants, which are with poles on the right half plane (RHP). In literature, there are many efforts to exploit the IMC for the unstable plants. In [10]–[12], a tuning method based on IMC in which the final controller is approximated as a PID structure was proposed. However, they are only applicable for the typical plants such as: the first-order, second-order unstable processes and they cannot retain the IMC architecture. In [13] and [14], a modified IMC and a modified Smith predictor have been proposed, respectively. In these researches, three controllers are designed for different objectives and they can achieve well the performance index and disturbance rejection. However, the architectures are quite complicated.

In [15], Yamada proposed a modified IMC structure based on inner-outer factorization. Compared to
above-mentioned approaches, this modification is natural and still keeps the advantage of the conventional IMC.

Based on the proposal in [15], this paper proposes a data-based tuning method with FRIT for unstable plants. Here, the authors clarify how to tune parameters of the modified IMC controller with only a one-shot experiment. The proposed approach is applicable for both minimum phase and non-minimum phase plants, and it enables us to simultaneously obtain a desired controller and an appropriate model of the actual plant.

The paper is organized as follows. Section 2 are the preliminaries with some notations. The problem statement, a brief review of FRIT, and the modified IMC in [15] for unstable plants are also addressed in this section. Section 3 expresses a mathematical plant model with the modified IMC. Here, we consider how to simultaneously obtain a desired controller and a mathematical model of the plant. In Section 4, FRIT is applied to obtain such a simultaneous attainment with a one-shot experiment. Section 5 shows the validity of the proposal with illustrative examples. Finally, some concluding remarks are given in Section 6.

2. Preliminaries

2.1 Notations

Let \( \mathbb{R} \) and \( \mathbb{R}^n \) denote the set of real numbers and that of real vectors of size \( n \), respectively. \( \mathbb{R}(s) \) is a space of all real, proper rational transfer functions. For a time signal \( w \), we denote a value of \( w \) at the time \( t \) as \( w(t) \). Let \( u \) and \( y \) denote the input and output data of a plant. By using these notations, the data obtained in the finite time with the sampling period \( \Delta \) are described as \( u(\Delta) \cdot w(\Delta) \cdot \ldots \cdot w(\text{N}\Delta) \) and \( y(\Delta) \cdot y(2\Delta) \cdot \ldots \cdot y(\text{N}\Delta) \), where \( \text{N} \) denotes the number of the sampled data.

For a transfer function described by \( G(s) = \frac{A(s)}{B(s)} \) such that \( A(s) \) and \( B(s) \) are coprime, the output \( y \) of \( G \) with respect to \( u \) is the solution of the differential equation \( B\left(\frac{d}{dt}\right)y = A\left(\frac{d}{dt}\right)u \). However, for the enhancement of the readability, we use the notation \( y = Gu \). For \( G(s) \in \mathbb{R}(s) \), we use \( G^{\theta}(s) \) to denote \( G(-s) \). Throughout of this paper, we omit the argument ‘\( s \)’ or ‘\( t \)’ from transfer functions or time series whenever there is no danger of confusion.

For a time series \( w = (w(\Delta) \cdot w(2\Delta) \cdot \ldots \cdot w(\text{N}\Delta)) \), we use the following notation \( \|w\|_2^2 = \frac{1}{N} \sum_{k=1}^{N} (w(k\Delta))^2 \).

2.2 Problem Statement

Let us consider a closed loop system with IMC in Fig. 1 where the controlled plant is linear, time-invariant, and unstable. Let \( P \) denote the transfer function of the actual plant. We assume that \( P \) is unknown except the degrees of the denominator and the numerator. The unstable plant to be controlled may be minimum phase or non-minimum phase, and it is with no zero or pole on the imaginary axis. In the non-minimum phase case, the number of the unstable zeros is assumed to be a priori known.

Assume that the plant model \( \hat{P} \) is parameterized with a tunable vector \( \rho_P := [\rho_{P,0} \cdot \rho_{P,1} \cdots \rho_{P,\nu_P+1}] \in \mathbb{R}^{\nu_P+1} \) as

\[
\hat{P}(\rho_P) = \frac{\rho_{P,0} s^{\nu_P} + \cdots + \rho_{P,1} s + \rho_{P,0}}{\rho_{P,\nu_P+1} s^{\nu_P} + \cdots + \rho_{P,0} + 1}
\]

(1)

Similarly, the controller \( C_{\text{IMC}} \) is parameterized with a tunable vector \( \rho_C := [\rho_{C,0} \cdot \rho_{C,1} \cdots \rho_{C,d+1}] \in \mathbb{R}^{d+1} \)

\[
C_{\text{IMC}}(\rho_C) = \frac{\rho_{C,0} \cdot s^{d} + \cdots + \rho_{C,1} s + \rho_{C,0}}{\rho_{C,d+1} s^{d+1} + \cdots + \rho_{C,0} + 1}
\]

(2)

Since the tunable parameter vector \( \rho := [\rho_P \rho_C] \) influences the input \( u \) and output \( y \), we explicitly denote them as \( u(\rho) \) and \( y(\rho) \), respectively (Fig. 2).

Let \( T_d \) denote a reference model of the closed loop system, then \( y_d := T_d r \) denotes the desired output. The problem here is to find a parameter vector \( \rho \) such that it minimizes the model-reference criterion

\[
J(\rho) := \|y(\rho) - T_d r\|_2^2
\]

(3)

with the direct use of experimental data.

On the other hand, since the controller includes a tunable mathematical model of the plant, it is expected that we can also simultaneously obtain an appropriate model of the actual plant. Moreover, it is preferable that the simultaneous attainment can be performed with as few data as possible in a practical sense. For this purpose, FRIT, which is briefly explained in the next subsection, is utilized.

2.3 Fictitious Reference Iterative Tuning-FRIT

Fictitious reference iterative tuning (FRIT) [9] is one of the effective data-based tuning methods for parameters of a controller with only a one-shot experiment. Consider Fig. 3 for a conventional closed loop system where the plant \( P \) is unknown. The feedback controller \( C \) is parameterized by a vector \( \rho \), then we denote it as \( C(\rho) \).

With the initial parameter vector \( \rho_0 \), we perform a one-shot experiment on the closed loop system to obtain the data: \( u(\rho_0) \) and \( y(\rho_0) \). The controller \( C(\rho_0) \) is assumed to be able to stabilize the closed loop system such as to yield a bounded output. By using the data \( u(\rho_0) \) and \( y(\rho_0) \), we compute the fictitious reference signal \( \hat{r}(\rho) \) [16] with a tunable parameter \( \rho \) as

\[
\hat{r}(\rho) = C^{-1}(\rho) u(\rho_0) + y(\rho_0)
\]

(4)

such that the output with respect to this signal always equals to the initial one \( y(\rho_0) \). Indeed, we can validate this by using the trivial relation \( P u(\rho_0) = y(\rho_0) \) as

\[
\frac{P(\rho)}{1 + P(\rho)} \hat{r}(\rho) = y(\rho_0)
\]

(5)

for any parameter vector \( \rho \). By using \( \hat{r}(\rho) \), the criterion to be minimized is introduced as

\[
J_F(\rho) = \|y(\rho_0) - T_d \hat{r}(\rho)\|_2^2
\]

(6)

The fictitious reference signal was proposed by [16] in the unsaturated control framework. In FRIT, we use it for the different purpose.
Grybilized by a local controller proposed by Yamada [15]. Here, the unstable plant (Fig. 4), which was motivated from Theorem 5.1-1 in [1], was

2.4 Modified IMC for Unstable Plants [15]

The conventional IMC in Fig. 1 cannot be implemented for unstable plants since it yields an unbounded output for any bounded input. To overcome this problem, a modified IMC (Fig. 4), which was motivated from Theorem 5.1-1 in [1], was proposed by Yamada [15]. Here, the unstable plant \( P(s) \) is stabilized by a local controller \( K \) and it is factorized as

\[
P(s) = P_s(s)P_{un}(s)
\]

where \( P_s(s) \) is a stable proper rational function and \( P_{un}(s) \) is an all-pass unstable and minimum phase function. Correspondingly, its model is expressed as

\[
P(s) = \tilde{P}_s(s)\tilde{P}_{un}(s)
\]

Internal stability of the closed loop system is guaranteed as follows.

**Theorem 1** Assume that \( \tilde{P}(s) = P(s) \). Then the closed-loop system in Fig. 4 is internally stable if and only if the following conditions hold:

1. \( C_{IMC} \) is stable.
2. \( \frac{\rho}{P_{un}} \) is stable.
3. \( \frac{\tilde{P}_s}{1+KP} \) is stable.
4. \( \tilde{P}_s \) is stable.

It is seen that, with a stable controller \( C_{IMC} \), the architecture in Fig. 4 guarantees the internal stability if \( \tilde{P} = P \). For the detailed proof and discussions, see [15].

3. Utilization of Modified IMC for the Simultaneous Attainment of a Controller and a Plant Model

Let us consider a model reference problem in which the desired specification is represented by a reference model \( T_d(s) \), the controlled plant is unstable as stated in subsection 2.2. Here, we use the modified IMC described in Fig. 4 for the simultaneous attainment of a controller and a plant model. Note that the factorization (9) of the controlled plant can be detailed as

\[
P(s) = \frac{N(s)}{D_s(s)P_{un}(s)}\frac{D_{un}(s)}{P_{un}(s)}
\]

where \( N(s) \) denotes the numerator of \( P(s) \), \( D_s(s) \) is the polynomial whose roots are only stable poles and \( D_{un}(s) \) is the polynomial whose roots are all unstable poles of \( P(s) \). Assume that \( P(s) \) is unknown except the degrees of \( N(s), D_s(s) \) and \( D_{un}(s) \).

### 3.1 The Minimum Phase Case

Along the factorization (11) where \( P(s) \) is minimum phase, i.e., \( N(s) \) contains only the roots in the left half plane (LHP), the transfer functions \( P_s \) and \( P_{un} \) in (10) are parameterized with tunable parameter vectors \( \rho_s = [a_0 \cdots a_n b_1 \cdots b_m]^T \in \mathbb{R}^{n+m+1} \) and \( \rho_{un} = [c_1 \cdots c_k]^T \in \mathbb{R}^k \) as

\[
P_s(\rho_s, \rho_{un}) = \frac{\sum_{i=1}^m a_is^i}{(\sum_{i=1}^m b_is^i + 1)(\sum_{i=1}^k c_is^i + 1)}
\]

and

\[
P_{un}(\rho_{un}) = \frac{\sum_{i=1}^k c_is^i + 1}{\sum_{i=1}^k c_is^i + 1}
\]

Hence, (10) can be rewritten as

\[
P(\rho) = \tilde{P}_s(\rho_s, \rho_{un})\tilde{P}_{un}(\rho_{un})
\]

where

\[
\rho := \begin{bmatrix} \rho_s \\ \rho_{un} \end{bmatrix}
\]

with constraints such that the numerator and denominator of \( \tilde{P}_s \) are Hurwitz polynomials.

Consider the closed loop system in Fig. 4, the transfer function \( G_{ry} \) from \( r \) to \( y \) is described as

\[
G_{ry} = \frac{C_{IMC}P(1 + K\tilde{P})}{\tilde{P}_{un}(1 + KP) + C_{IMC}(P - \tilde{P})}.
\]

If it is possible to put \( \tilde{P} = P \) and the controller described as

\[
C_{IMC} = T_d\tilde{P}_s^{-1},
\]
then we can see that
\[
G_{r_2} = \frac{C_{ IMC} P(1 + KP)}{P_{wm}(1 + KP) + C_{ IMC}(P - P)} = \frac{T_d \tilde{P}_2^{-1}P}{P_{wm}} = T_d.
\] (18)

Conversely, if \(G_{r_2} = T_d\), by using the controller (17), (16) can be rewritten
\[
T_d = \frac{T_d \tilde{P}_2^{-1}P(1 + KP)}{P_{wm}(1 + KP) + T_d \tilde{P}_2^{-1}(P - P)}
\] (19)
or
\[
T_d P_{wm}(1 + KP) + T_d^2 P_2^{-1}(P - P) = T_d \tilde{P}_2^{-1}P(1 + KP),
\]
which leads to
\[
T_d P_{wm}(1 - T_d) = T_d \tilde{P}_2^{-1}P(1 - T_d).
\] (20)

With \(T_d(1 - T_d)\) is not identically equal to zero, \(\tilde{P} = P\) holds.

From the above observation, it is seen that the structure (17) of the controller plays a crucial role in the simultaneous attainment of a plant model and a controller for the desired specification. Thus, if the controller \(C_{ IMC}\) is parameterized as
\[
C_{ IMC}(\rho) = T_d \tilde{P}_3(\rho - \rho_{un})^{-1},
\] (21)
the simultaneous attainment is concluded as follows.

**Theorem 2** Assume that the controller \(C_{ IMC}\) in Fig. (4) is parameterized as (21), then \(G_{r_2}(\rho) = T_d\) if and only if \(\tilde{P}(\rho) = P\).

Note that the plant model \(\tilde{P}(\rho)\) is parameterized based on the structure of the actual plant. This implies that the existence of \(\rho^*\) such that \(\tilde{P}(\rho^*) = P\), thus \(G_{r_2}(\rho^*) = T_d\) and then \(J_F(\rho^*) = 0\), can be guaranteed. Moreover, it follows from Theorem 2 that the controller \(C_{ IMC}\) parameterized as (21) yields the equivalence of the achievability of \(G_{r_2}(\rho^*) = T_d\) and that of \(\tilde{P}(\rho^*) = P\). On the other hand, the internal stability is guaranteed if four conditions hold under assumption \(P = \tilde{P}(\rho^*)\) as stated in Theorem 1. That is, if we can achieve \(G_{r_2}(\rho^*) = T_d\), then we can also guarantee the internal stability of the system described in Fig. 4 once four conditions in Theorem 1 can be satisfied.

Regarding the conditions for the internal stability, it is seen that, if above optimal parameter yields a minimum phase, stable \(\tilde{P}_3\), i.e., \(\tilde{P}(\rho^*)\) is minimum phase and stable, then the conditions guaranteeing the internal stability of the closed loop system are satisfied. Indeed, condition 1, 3, 4 in Theorem 1 are obviously guaranteed with such a \(\tilde{P}_3(\rho^*)\). By using [11] for the factorization of the model \(\tilde{P}(\rho^*)\), the transfer function in condition 2 can be rewritten as
\[
1 + KP = 1 + K \frac{\sum_{i=0}^{m} g_i s^i}{\sum_{j=0}^{l} s^j} + K \frac{\sum_{i=0}^{m} \sum_{j=0}^{l} d_{ij} s^j}{\sum_{i=0}^{l} s^j}.
\] (22)

It is seen that, the denominator of \(\frac{1 + KP}{P_{wm}}\) itself is the denominator of \(\tilde{P}_3\). Thus, if \(\tilde{P}(\rho^*)\) is guaranteed to be stable, then the condition 2 in Theorem 1 is also guaranteed.

### 3.2 The Non-Minimum Phase Case

Besides unstable poles, the plant may also include unstable zeros. For this case, the stable part \(P_s\) can be described as
\[
P_s = \frac{N_s(s)N_{s,1}(s)}{D_s(s)D_{s,1}(s)} = \frac{N_s(s)N_{s,2}(s)}{D_s(s)D_{s,2}(s)}\frac{N_s(s)}{P_s}\frac{D_{s,1}(s)}{P_{s,1}},
\] (23)
where \(N_s(s)\) denotes the polynomial whose roots are only the stable zeros and \(N_s(s)\) contains all the RHP zeros of the plant. In this case, besides \(\rho_{un}\) defined above, we use \(P_{s,2} = [g_0 \ldots g_1 b_1 \ldots b_m]^T \in \mathbb{R}^{m+1}\) and \(\rho_{s,2} = [d_0 \ldots d_l]^T \in \mathbb{R}^{l+1}\) to parameterize \(\tilde{P}_3\) as
\[
\tilde{P}_3(\rho_{s,2}, \rho_{s,1}) = \tilde{P}_{s,2}(\rho_{s,2}, \rho_{s,1})P_{s,2}(\rho_{s,1})\tilde{P}_{s,1}(\rho_{s,1})\tilde{P}_{s,1}(\rho_{s,1})
\] (24)
where
\[
P_{s,2}(\rho_{s,2}, \rho_{s,1}) = \frac{\sum_{i=0}^{m} g_i (-s)^i}{\sum_{i=0}^{l} s^i} + (\sum_{i=1}^{l} c_i (-s)^i + 1)
\] (25)
is minimum phase and invertible.

\[
P_{s,1}(\rho_{s,1}) = \frac{\sum_{i=0}^{l} d_{ij} (-s)^i}{\sum_{i=0}^{l} s^i},
\] (26)
is an all-pass function and contains all non-minimum elements of the plant.

Then, the internal plant model in this case is parameterized as
\[
P(\rho) = \tilde{P}_{s,2}(\rho_{s,2}, \rho_{s,1})\tilde{P}_{s,1}(\rho_{s,1})\tilde{P}_{s,1}(\rho_{s,1})P_{s,2}(\rho_{s,1})\tilde{P}_{s,1}(\rho_{s,1})
\] (27)
where
\[
\rho := \begin{bmatrix} \rho_{s,2} \\ \rho_{s,1} \\ \rho_{un} \end{bmatrix}
\] (28)

As discussed in [7],[8] and [17], the performance of the closed loop system is deeply related to the non-minimum phase behavior of the system, e.g., initial undershoot, zero crossing or overshoot on the step response. Thus, the reference model should include the same RHP zeros as the controlled plant. However, there is no information on the unstable zeros of the plant, the reference model is introduced as
\[
T_{un}(\rho_{s,1}) = T_d \tilde{P}_{s,1}(\rho_{s,1}).
\] (29)

Since \(\tilde{P}_{s,1}(\rho_{s,1})\) is an all-pass function, \(T_{un}(\rho_{s,1})\) still keeps the same gain characteristic as the given reference model \(T_d\) with any parameter \(\rho_{s,1}\). This technique was explained in the authors’ previous results for the non-minimum phase systems [7]. The controller in this case is parameterized as
\[
C_{ IMC} = T_d \tilde{P}_{s,1}(\rho_{s,1}, \rho_{s,2}, \rho_{un})^{-1}.
\] (30)

Similarly to the minimum phase case, it is seen that, with the controller (30), the closed loop system matches the desired specification as follows.

**Theorem 3** In cases of unstable, non-minimum phase plants, assume that the controller \(C_{ IMC}\) is defined as (30), then \(\tilde{G}_{r}(\rho) = T_{un}(\rho_{s,1})\) if and only if \(P = \tilde{P}(\rho)\).

Theorem 2 or Theorem 3 implies that we can simultaneously obtain a controller for the desired specification and a mathematical model of the actual plant.
4. FRIT to Modified IMC for Unstable Plants

4.1 One-Shot Parameter Tuning for Modified IMC

Assume that the unstable plant \( P(s) \) is stabilized by a local controller \( K \) and this controller is fixed during the controller parameter tuning. With the controller \( C_{IMC} \) determined by (21) or (30), the feedback controller in Fig. 3 is expressed as

\[
C(p) = \frac{C_{IMC}(p)1 + K P(p)}{1 - C_{IMC}(p) P_r(p) P_w(p)}. \tag{31}
\]

The fictitious reference signal can be rewritten

\[
\hat{r}(p) = \frac{1 - C_{IMC}(p) P_r(p)}{C(p)} P_w(p) u(p^0) + y(p^0). \tag{32}
\]

Implement (32) into (6), we obtain the cost function \( J_F(p) \) to be minimized. By using the Gauss-Newton method, we minimize the cost function and obtain

\[
ρ^* = \arg \min_{ρ} J_F(ρ). \tag{33}
\]

If \( J_F(ρ) = 0 \) can be achieved, \( G_{rγ}(ρ) = T_d \) (or \( G_r(ρ) = T_{dy}(ρ_{sc}) \) in the non-minimum phase case) generically holds. Moreover, from Theorem 2 or Theorem 3, we can also see that \( P(ρ) = P \) generically holds. The internal stability of the closed loop system therefore holds if the conditions in Theorem 1 are satisfied. From above analysis, it is said that the minimization of \( J_F(ρ) \) yields both a desired controller and an appropriate model of the plant. Particularly, this simultaneous attainment can be achieved with only a single experiment.

4.2 Remarks

**Remark 1** The system in Fig. 4 is internally stable under the assumption that \( \hat{P}(ρ) = P \). As the result of Theorem 2 (or Theorem 3), this condition is obtained in cases where the cost function \( J_F(ρ) = 0 \). In fact, \( J_F(ρ) \) is minimized as small as possible, implicitly \( P \neq \hat{P}(ρ) \) or even if we can achieve \( J_F(ρ) = 0 \), the fact that the system always exists a model uncertainty, e.g., un-modeled dynamics and/or disturbances, makes \( P \neq \hat{P}(ρ) \).

Assume that the actual plant \( P(s) \) belongs to a set described by the model with arbitrary multiplicative uncertainty \( Δ(s) \) as

\[
P = \hat{P}(ρ)(1 + Δ), \quad |Δ(jω)| < |\hat{Δ}(jω)|, \quad ∀ω ≥ 0, \tag{34}
\]

where \( \hat{Δ}(s) \) is \( \mathcal{H}(s) \) stable. \( Δ(s) \) is assumed that it does not affect the number of unstable poles of \( P(s) \).

Yamada [15] showed that the modified IMC in Fig. 4 is not only internally stable in the sense of \( Δ(s) = 0 \) but also robustly stable for arbitrary uncertainty \( Δ(s) \) satisfying \( |Δ| < |\hat{Δ}| \) if and only if

\[
\left\| \frac{(K P_w + C_{IMC})P_r}{1 + K P} Δ \right\|_∞ ≤ 1 \tag{35}
\]

holds.

By using \( C_{IMC}(ρ^*) \) and \( \hat{P}(ρ^*) \) resulted from the optimization problem, with a fixed stabilizing controller \( K \), (35) enables us to determine the upper bound of the plant uncertainty \( Δ(s) \) such that within this bound, the closed-loop system is still stable. In the other words, we can determine a set of the controlled plants that is stabilized by the optimal controller \( C_{IMC}(ρ^*) \).

**Remark 2** As shown in (6), the minimization of \( J_F(ρ^*) \) is done with a nonlinear optimization. This implies that the result depends on the initial input and output data. On the other hand, the choice of a stabilizing controller \( K \) may also affect to the result. More theoretical analysis on these issues should be addressed as the future researches.

**Remark 3** From the viewpoint of data-driven tuning, it is normal to put the coefficients of the numerator and denominator as tunable parameters. On the other hand, it is also usual in the area of robust control that an inner function can be represented by a rational function whose denominator is described by the product of the first order polynomials and the numerator is its conjugate. Thus, it is also possible to represent the parameterized polynomials (12) and (13) with \( ρ_s = [a_0 \cdots a_n b_0 \cdots b_m]^T \) and \( ρ_w = [c_1^* \cdots c_n^*]^T \) as

\[
P_s(ρ_s, ρ_w) = \sum_{i=0}^{n} a_i s^i \left( \sum_{i=1}^{m} b_i s_i + 1 \right) \prod_{i=1}^{n} (s + c_i^*) \tag{36}
\]

and

\[
P_w(ρ_w) = \prod_{i=1}^{m} \frac{s + c_i^*}{s - c_i^*} \tag{37}
\]

where \( c_i^* \) denotes the complex conjugates of \( c_i \).

Similar parametrization can also be available for the non-minimum phase part.

Moreover, there might be also a case where the property of the stable part can be known a priori. For example, if a system is with no-damping property then we know that the roots are real. In such cases, the stable denominator and numerator can be also parameterized with respect to real roots. That is, we have the flexibility on choosing what types of the parametrization to be used.

**Remark 4** In this paper, we assume that noise is small so as to be neglected. However, from the practical points of view, if the noise cannot be neglected, it should be eliminated. In such a case, similarly to Remark 2 in [7] by the authors, the technique used in the IFT [3] and [5] can be employed here. We repeat initial experiment with the same controller \( C(ρ^0) \) twice under the assumption that noises in the different experiments are uncorrelated each other and also uncorrelated to \( u(ρ^0) \) and \( y(ρ^0) \). With two sets of experimental data, we construct the cost function which can eliminate the effect of the noise. For more details, one is referred to [3] and [5].

4.3 Algorithm

The proposed approach can be summarized as follows.

1. Parameterize the unstable plant model as (14) for the minimum phase case or (27) for the non-minimum phase case.
2. The parameter vector \( ρ \) is correspondingly determined by (15) or (28) for minimum phase or non-minimum phase cases, respectively.
3. Prepare the initial parameter vector \( ρ^0 \) and perform a one-shot experiment to obtain the data \( u(ρ^0) \) and \( y(ρ^0) \).
4. Compute \( \hat{r}(ρ) \) by using (32), construct the cost function \( J_F(ρ) \) as (6) and minimize it by an off-line nonlinear optimization.
5. Obtain \( \rho^* = \arg \min_{\rho} J_F(\rho) \) which yields both a desired controller and an appropriate model of the plant.

Note that, at each step of the optimization process, \( \rho \) is checked with the constraints such that the numerator and denominator of \( \tilde{P} \) (or those of \( \tilde{P}_s \) in the non-minimum phase case) are Hurwitz polynomials. The process will continue if the constraints are satisfied. Otherwise, it stops, we either take the parameter vector of the previous step or adjust the initial parameters and then repeat the procedure.

5. Numerical Example

5.1 Example 1

Assume the controlled plant is unstable and minimum phase as

\[
P = \frac{s + 1}{s^2 + s - 6} = \frac{s + 1}{(s - 2)(s + 3)}.
\]

(38)

The reference model is given by

\[
T_d = \frac{1}{2s + 1}.
\]

(39)

We take the internal model described as

\[
\tilde{P}(\rho) = \frac{\rho_1 s + \rho_0}{(\rho_2 s + 1)(s + \rho_3)} \frac{s + \rho_3}{s - \rho_3}.
\]

(40)

where \( \rho = [\rho_0 \rho_1 \rho_2]^T \) and \( \rho_{in} = \rho_3 \).

We use a stabilizing proportional controller \( K = 7 \) which is fixed during tuning procedure. With initial parameter vectors \( \rho_0^0 = [1 1 1]^T \) and \( \rho_{in}^0 = 1 \), we perform a one-shot experiment to obtain the input \( u(\rho_0^0) \) and the output \( y(\rho_0^0) \). Figure 5 shows the reference signal \( r \), the initial output \( y(\rho)^0 \), and the desired output \( T_d r \) as the dot-and-dash line, the solid line and the dotted line, respectively. By applying the proposed algorithm, we obtain the optimal parameter vectors \( \rho_0^* = [0.3273 0.3311 0.3515]^T \) and \( \rho_{in}^* = 1.9638 \) with the cost function \( J_F(\rho^*) = 2.8594 \times 10^{-4} \). Perform experiment with \( \rho^* = [\rho_0^* \rho_1^* \rho_2^*]^T \), we obtain the outputs that are illustrated as Fig. 6. It is seen that, the optimal output \( y(\rho^*) \) (the solid line) and the desired output \( T_d r \) (the dotted line) are well matched. From this result, it is concluded that we can obtain the desired output by using the optimal vector \( \rho^* \).

Simultaneously, this optimal vector yields a plant model \( \tilde{P}(\rho^*) \). Figure 7 and Fig. 8 show the frequency characteristics of the actual plant and its model. In these two figures, the characteristics of the actual plant, the model, and the reference model are drawn as the solid lines, the dotted lines, and the dot-and-dash lines, respectively. It is said that the dynamics of the model reflects well that of the actual plant, or on the other word, the actual plant is well identified by using the optimal parameter vector \( \rho^* \).

5.2 Example 2

Let us consider an unstable non-minimum phase plant to be controlled as

\[
P = \frac{-s + 2}{0.3 s^2 + 0.75 s - 1} = \frac{-s + 2}{(0.3 s + 1)(s - 1)}.
\]

(41)

We use the same reference model (39) and the internal model described by

\[
\tilde{P}(\rho) = \frac{\rho_0 (s + \rho_2)}{(\rho_1 s + 1)(s + \rho_3)} \frac{-s + \rho_2}{s + \rho_2} \frac{s + \rho_3}{s - \rho_3}.
\]

(42)

where \( \rho_{in} = [\rho_0 \rho_1]^T; \rho_{in} = \rho_2 \) and \( \rho_{in} = \rho_3 \).
The PD controller with $K_p = 0.6$ and $K_d = 0.16$ was used to stabilize the controlled plant in this case. We set initial parameters as $\rho_{\text{in}}^0 = [0.5, 0.5]^T$; $\rho_{\text{ef}}^0 = 1$ and $\rho_{\text{st}}^0 = 0.2$. The initial output is illustrated in Fig. 9. By applying the proposed method, we obtain the optimal parameter vectors $\rho_{\text{in}}^* = [1.0054, 0.2984]^T$; $\rho_{\text{ef}}^* = 1.9668$ and $\rho_{\text{st}}^* = 0.9882$ with $J_F(\rho^*) = 6.8989 \times 10^{-3}$. Perform the experiment with $\rho^*$, we see that the desired output can be achieved as illustrated in Fig. 10.

As for the identification of the plant model, the gain and phase characteristics of $P$ and those of $\tilde{P}$ are drawn in Fig. 11 and Fig. 12, respectively. It is seen that the controlled plant is also well-identified with $\rho^*$.

### 6. Conclusion

Based on the results in [15], this paper has proposed a data-driven approach with the fictitious reference iterative tuning (FRIT) for the unstable plants with only partial information on the actual plant. The approach can be applicable for unstable plant in both of minimum phase and non-minimum phase cases. And as same as the previous results, the proposal sounds attractive since it yields both a desired controller and an appropriate model of the actual plant with only one-shot experiment. However, the approach is solved under assumption that we know the number of unstable poles/zeros and the degree of the numerator/denominator. In general cases, without any information on the actual plants, the approach should be more theoretically analyzed. This issue will be clarified in the future researches.

This work was partially supported by the JSPS Grant in Aid for Scientific Research (B) No. 23360183 and (C) No. 22560442.

---

**Fig. 9** The reference signal $r$ (the dot-and-dash line), the initial output $y(\rho^0)$ (the solid line), and the desired output $T_d\tilde{P}_{\text{st}}(\rho_{\text{st}}^0)r$ (the dotted line) in Example 2.

**Fig. 10** The reference signal $r$ (the dot-and-dash line), the optimal output $y(\rho^*)$ (the solid line), and the desired output $T_d\tilde{P}_{\text{st}}(\rho_{\text{st}}^*)r$ (the dotted line) in Example 2.

**Fig. 11** Gain characteristics: The actual plant $P$ (the solid line), the plant model $\tilde{P}(\rho^*)$ (the dotted line), and the reference model $T_d\tilde{P}_{\text{st}}(\rho_{\text{st}}^*)$ (the dot-and-dash line) in Example 2.

**Fig. 12** Phase characteristics: The actual plant $P$ (the solid line), the plant model $\tilde{P}(\rho^*)$ (the dotted line), and the reference model $T_d\tilde{P}_{\text{st}}(\rho_{\text{st}}^*)$ (the dot-and-dash line) in Example 2.

**References**

[10] G.E. Rotstein and D.R. Lewin: Simple PI and PID tuning for...


---

**Hien Thi NGUYEN**

She received her B.E. and M.E. degrees from the Hanoi University of Agriculture, Vietnam in 2000 and 2002, respectively. In 2001, she joined Faculty of Engineering, Hanoi University of Agriculture, as a Lecturer. She is currently a doctoral student at Kanazawa University, Japan. Her research interests include control systems and its applications.

**Osamu KANEKO (Member)**

He received his B.E. and M.E. degrees from the Nagaoka University of Technology, Japan, and Ph.D. degree from Osaka University, Japan, in 1992, 1994, and 2005, respectively. In 1999, he joined Osaka University as an Assistant Professor. In 2009, he moved to Kanazawa University as an Associate Professor. His research interests include control system design and system theory.

**Shigeru YAMAMOTO (Member)**

He received his B.E., M.E., and Ph.D. degrees from Osaka University, Japan in 1987, 1989, and 1996, respectively. In 1989, he joined Osaka University as an Assistant Professor of the Faculty of Engineering. In 1994, he moved to the Faculty of Engineering Science, Osaka University, and served there as a Lecturer and an Associate Professor. In 2007, he moved to Kanazawa University as a Professor. His research interests include control system design and system theory.