A Design Method of Delta-Sigma Data Conversion System with Pre-Filter

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Abstract: AD/DA conversion has become a core technology in digital signal processing. Signal compression is one of the important components for AD/DA conversion systems. Delta-sigma modulator (DSM) is well known as an effective method for encoding analog signals into digital signals. Traditional data conversion systems are composed of the post-filter and the DSM. It is required to satisfy small quantization noise and small signal distortion characteristics by appropriate design of the filter and DSM. In this study, the authors propose a new design method of data conversion system that includes pre-filter in addition to the traditional data conversion system. To design easier, an evaluation framework of the data conversion system is proposed. Then, a design algorithm based on the evaluation framework is proposed by using particle swarm optimization algorithm. Post and pre-filters are designed so as to minimize the effect of the noise and the signal distortion due to quantization. In this paper, the authors verify the AD/DA systems with post and pre-filters by using the voice signal compression system. The effectiveness of the proposed design method is illustrated by the numerical simulations.

Key Words: AD/DA converter, delta-sigma modulator, data compression, noise reduction.

1. Introduction

An analog-to-digital converter is a device that converts an analog signal to a digital signal. AD/DA conversion has become a core technology in digital signal processing [1]. Signal compression is one of the most important functions to make an appropriate AD/DA conversion system.

Signal compression is used in many fields such as the sound processing and the image processing. It is also important if we use communication channel in the networked control system. Delta-sigma analog-to-digital converter is known to achieve high resolution conversion [1]–[6]. Delta-sigma modulator (DSM) is a kind of noise-shaping type converter and its one characteristic is that it can remove the effect of low-frequency noise.

There exist many researches about the DSM. Not only the delta-sigma modulator but also the entire system design is required to achieve good noise reduction characteristics based on evaluation of the entire AD/DA conversion system. The DSM output includes noise in higher frequency part compared to the input signal’s frequency range. Quantization noise is removed by using low-pass post-filter located after the demodulator. The difference between the output signal and the input signal should be minimized to achieve good noise reduction performance. This performance depends on design of the post-filter and the DSM. If we choose cut-off frequency as lower, the signal distortion becomes large. On the other hand, if we choose cut-off frequency as higher to avoid signal distortion, the post-filter cannot remove the quantization noise sufficiently. Usually, it is required to design the post-filter in the demodulator part based on the trade-off between the signal distortion and the quantization noise.

To overcome this problem, the authors propose a delta-sigma data conversion system with not only the post-filter but also a pre-filter for signal shaping. By using the pre-filter, we can divide the effect of the quantization noise and the signal distortion. Design problem is formulated based on the proposed evaluation system. The authors design post-filter and pre-filter by using a particle swarm optimization (PSO) algorithm which is a kind of the optimization method based on the swarm behavior [7]. The PSO algorithm is used in many control systems design [8],[9]. To confirm the effectiveness of our proposed system, two types of music, which have large frequency range, are used as the input signals.

This paper is organized as follows: At first, the outline of AD/DA conversion system with delta-sigma modulator is presented. Then, the proposed structure and its evaluation system is explained. The parameter design problem is formulated and the problem is solved by using the PSO algorithm with constraints. The effectiveness is shown by simulations. This paper is the extended version of [10]. In particular, the detailed analysis results and discussions are added in Section 4.

2. Preliminary

2.1 Outline of Traditional Delta-Sigma Data Conversion System

Figure 1 shows the traditional delta-sigma data conversion system. Figure 1 is composed of a DSM and a post-filter F. An output of the DSM is a coarse signal when we use one bit quantization. The magnitude of the quantization noise depends on its quantization width and the number of the quantization...
level.

In this paper, the DSM is assumed to be given as follow:

\[ Q_{\Delta \xi} : \begin{cases} \xi(k+1) = \xi(k) - (u(k) - v(k)), \\ v(k) = Q_{\Delta \xi} [-\xi(k) + u(k)]. \end{cases} \]

(1)

\( \xi \in R \) is the state of the DSM, \( u(k) \) is the input and \( v(k) \) is the output. Initial state is given as \( \xi(0) = 0 \). Number of the quantization levels is denoted as \( M \). Input-output level is given as \( d \) and dynamic range for \( Q_{\Delta \xi} \) is given as follows:

\[ u(k) \in [-(M - 1)d/2, (M - 1)d/2]. \]

(2)

A discrete valued set \( V \) is determined as \( V := \{(M - 1)d/2, -(M - 3)d/2, \cdots, (M - 1)d/2 \} \). Then, \( Q_{\Delta \xi} : R \rightarrow V \) is the standard uniform quantizer with a saturation. In this paper, \( M = 3 \) and \( d = 1 \) is assumed and is called as a 3-level quantizer. The permissible input range for the 3-level DSM is given as follow:

\[ u(k) \in [-1, 1]. \]

(3)

Block diagram of (1) is shown in Fig. 2. The quantization noise, which is generated by \( Q_{\Delta \xi} \), is denoted as \( n_q \). \( n_q(k) \) is a random disturbance which satisfy \( n_q(k) \in [-d/2, d/2], \forall k \). The relationship between \( u \) and \( v \) are described by using \( n_q \) as follow:

\[ v = u + \left(1 - \frac{1}{z}\right)n_q, \]

(4)

where \( z \) is the delay operator. The term \( (1 - 1/z) \) indicates the derivation transfer function. Figure 3 shows the frequency characteristics of the quantization noise. In delta-sigma conversion systems, the quantization noise is further reduced at low frequency part, which is the band where the input signal. The quantization noise is increasing at the higher frequency part. Therefore, the high frequency noise is removed by using low-pass filter. Filter design is important because the signal distortion occurs when we set the lower cut-off frequency than the primary part of the input signal frequency. Design objective is to minimize the difference between \( u_{out} \) and \( u_{min} \) in Fig. 1 under the trade-off between the quantization noise and the signal distortion.

Fig. 2 Block diagram of DSM.

2.2 Particle Swarm Optimization Algorithm [7]

In this section, we describe the conventional particle swarm optimization algorithm which is a kind of the optimization method based on the swarm behavior [7]. The following statements are the standard form of the minimization problem.

\[
\begin{align*}
\min_{x \in \mathbb{R}^n} J(x) \\
\text{subject to } h(x) < 0
\end{align*}
\]

(5)

(6)

\( J : \mathbb{R}^n \rightarrow \mathbb{R} \) is the objective function and \( x = (\xi_1, \cdots, \xi_n)^T \) is the design variable vector. \( h(x) < 0 \) denotes the constraint for \( x \). The optimal solution \( x^* \) for (5), (6) is required to obtain from an optimization algorithm.

Particle swarm optimization (PSO) is a computation method for optimizing a problem by iteratively trying to improve a solution. Multiple particles \( x_1, \cdots, x_m \) are used in the PSO algorithm where \( m \) denotes the number of particles. To solve (5) by the PSO algorithm, the following objective function \( J^p(x) \) is assumed to be given.

\[
J^p = \begin{cases} J(x) & (h(x) < 0) \\
J^b & \text{(otherwise)} \end{cases}
\]

(7)

\( J^b \) is the larger value compared to a value of \( J(x) \) which is an acceptable solution. Position and velocity of \( i \)-th particle is denoted as \( x_i = (\xi_{i1}, \cdots, \xi_{in})^T \) and \( v_i = (\varphi_{i1}, \cdots, \varphi_{im})^T \), respectively. \( x_i \) is updated based on the following update laws.

\[
x_{i1}^{k+1} = x_{i1}^k + v_{i1}^{k+1} \\
v_{i1}^{k+1} = c_1 x_{i1}^k + c_2 r_2(k)(x_{best}^k - x_{i1}^k) + c_3 r_3(k)(x_{gbest}^k - x_{i1}^k)
\]

(8)

\( k \) denote the iteration number and its initial value is \( k = 0 \). \( c_1, c_2 \) and \( c_3 \) are the weighting coefficients which is given by the designer. The random numbers \( r_1(k) \) and \( r_2(k) \) are selected in \([0, 1]\). In (8), \( x_{best}^k \) means the personal best solution which is determined by the following statements.

\[
x_{best}^k := \arg \min_{x \in \mathbb{R}^n} J^p(x)
\]

(9)

\( x_{best}^k \) means the global best solution which is determined by the following statements.

\[
x_{gbest}^k := \arg \min_{x \in \mathbb{R}^n} J^p(x)
\]

(10)

The PSO algorithm is given as following steps:

STEP 0: Set \( k = 0 \). Initial position \( x^0 \) and velocity \( v^0 \) are selected randomly and evaluate the corresponding objective function at each position.

STEP 1: Update \( x_{best}^k \) and \( x_{gbest}^k \) by (9) and (10), respectively. Then, apply update laws (8) for all particles, and go to STEP 2.

STEP 2: Evaluate all position \( x^k \) by (7). Set \( k = k + 1 \) and go to STEP 1 if \( k < k_{max} \). Else, update \( x_{best}^{k+1} \) and \( x_{gbest}^{k+1} \). \( x_{best}^{k+1} \) and \( x_{gbest}^{k+1} \) is the designed parameters.

The PSO algorithm is simple and a few or no assumption is required to solve the problem. Many classes of the control design problems can be applied easily.
3. Proposed Data Conversion System

3.1 Delta-Sigma Data Conversion System with Pre-Filter

We design delta-sigma data conversion system with a pre-filter as shown in Fig. 4. A discrete time pre-filter $F_1$ and a post-filter $F_2$ are included in the system and it can be considered as a two degree of freedom. It is required to design the pre-filter $F_1$ and the post-filter $F_2$ in this paper. To design simplicity, we give the form of $F_1$ and $F_2$ as follows:

\[ F_1(z) = D(z), \]
\[ F_2(z) = F(z)D(z)^{-1}, \]

where $D(z)$ is assumed to be stable and the relative degree is zero. $D(z)$ is a minimum phase transfer function. $F(z)$ is also a filter. The traditional data conversion system can be represented by $D(z) = 1$. Therefore, Fig. 4 has a potential to improve the data conversion performance in the meaning of the quantization noise and the signal distortion.

\[ u_{M} = F(z)F_{piano}(z), \]
\[ e_2(z) = F(z)D(z)^{-1}(1 - \frac{1}{z})n_{q}(z) \]

Therefore, it is possible to design SN ratio by $u_{in}$ with pre-filter $D(z)$. The effect of quantization noise for $u_{out}$ is minimized. The input signal for DSM is defined as $u_{in}(z) = D(z)u_{in}(z)$. $u_{D}(k)$ should satisfy signal range constraint given in (3).

Noise reduction characteristics is given by the post-filter $F(z)D(z)^{-1}$. If we choose $D^{-1}(z)$ as low-pass filter, it is good for minimize the effect of quantization noise as the following equation. By using (4), $z$-transformation of $e_2(z)$ is written as follow.

\[ e_2(z) = F(z)D(z)^{-1}(1 - \frac{1}{z})n_{q}(z) \]

We obtain good performance filter $D(z)^{-1}$ to minimize $e_2(k)$ under the constraint in (3).

To obtain small error signal, the following evaluation function is used to design $F$ and $D$:

\[ J = f_1|\sigma(e_1)| + f_2|\sigma(e_2)|, f_1, f_2 > 0, \]

where $\sigma^2$ denote the variance. $f_1$ and $f_2$ are the weight of each error signal. We can handle trade-off between the signal distortion $e_1$ and the quantization noise $e_2$ by tuning $f_1$ and $f_2$. When $u_{in}, F$ and $D$ are given, we can calculate the value $J$.

4. Numerical Simulations of Filter Design

In this section, we design the filters $F$ and $D$ based on the evaluation system in Section 3.2. The parameters in $F$ and $D$ are derived by using the PSO algorithm in Section 2.2. To confirm the effectiveness of the pre-filter $D$, we design $D$ after designing $F$.

4.1 Input Signal for Design Filters

Characteristics of the input signal $u_{in}$ is explained in this section. Figure 6 shows a source signal cutting from mp4 data of a classical piano sound. We can see from Fig. 6 that the signal range is normalized and satisfies the range constraint (3) in case $D = 1$. The sampling frequency of $u_{in}$ is 44,100 Hz. Figure 7 shows its frequency analysis result using FFT. In Fig. 7, main frequency range is (0 to 2) kHz.

4.2 Design of Filter $F_{piano}$

Post-filter $F_{piano}$ is assumed to be given as $D_{piano} = 1$ which represents the traditional DSM data converter written in Fig. 1. At first, we design continuous-time filter $F_c$ as a Butterworth filter which is designed to have as flat a frequency response as possible in the input signal passband. The following two parameters are assumed as the design parameters for obtaining the Butterworth filter $F_c$.  

\[ e_1(z) = (z^{-m} - F(z))u_{in}(z), \]

where $z^{-m}$ is the delay component and we set $m = 0$ in this paper. To obtain small $e_1$, the gain of $(z^{-m} - F(z))$ should be smaller in the dominant frequency range of $u_{in}$.

Then, we consider about pre-filter $D(z)$ design. The quantization noise characteristics do not depend on the input signal.
By using $W_p$ and $n$, gain of $F_c$ can be written by the following equation.

$$|F_c(j \omega)|^2 = \frac{1}{1 + \left(\frac{\omega}{W_p}\right)^{2n}} \quad (16)$$

To design the Butterworth low-pass filter, continuous time Butterworth low-pass filter is derived by using MATLAB function "butter.m". The obtained filter $F_c$ is discretized by bilinear $z$-transform and we obtain $F_{\text{piano}}$ which is a discrete time transfer function.

Evaluation function and constraints for the PSO algorithm is determined as follows. To evaluate $J_{\text{piano}}$, (15) with $f_1 = f_2 = 1$ is used as the evaluation function $J_{\text{piano}}$. Following conditions are used to achieve appropriate filter.

$$W_p > 0 \quad (17)$$
$$n > 0 \quad (18)$$

Penalty cost $J_{\text{piano}}^b = 5000$ is applied if these conditions are not satisfied.

Iteration number of the PSO algorithm is set as $k_{\text{max}} = 100$ and the particle number is 50. Design variable for $F_c$ is determined as $x = [W_p, n]^T$. Initial positions and velocities of the design parameters are as follow. $W_p$ is selected randomly from the range $[0, 5000]$. Corresponding velocity is selected randomly from the range $[-500, 500]$. $n$ is an integer and is selected from the range $[0, 30]$. Corresponding velocity is selected randomly from the range $[-2, 2]$. $c_1 = 0.9, c_2 = 1, c_3 = 1$ are selected in the velocity update law (8).

As a result of solving the PSO algorithm in Section 2.2, the evaluation function value is $J_{\text{piano}} = 0.0315$ which is shown in Fig. 8. The optimal filter $F_c$ is obtained as shown in Fig. 9. The characteristics of the obtained filter $F_c$ are as follows.
• cut-off frequency $W_p = 2163$ Hz
• order of the filter $n = 4$

In case of the classical piano, $F_c$ is designed as a 4th-order Butterworth filter. Figure 10 shows the output spectrum of the DSM. We can find that the magnitude of the signal is larger in high frequency part compared to Fig. 7. Figure 11 shows the error between $u_{in}$ and $u_{out}$ by the traditional method. We can find that the maximum absolute error is larger than 0.1. Figure 12 shows the spectrum of $u_{out}$. By comparing Fig. 10 and Fig. 12, quantization noise is removed by $F_{piano}$ of our proposed method in high frequency part. In case of $u_{in}$ in Fig. 7, magnitude is almost zero for the frequency higher than 4 kHz. On the other hand, magnitude of Fig. 12 in high frequency is smaller than that of Fig. 7 because of the low-pass filter $F_{piano}$.

4.3 Design of Filter $D_{piano}$

Pre-filter $D_{piano}$ is designed by the PSO algorithm. $F_{piano}$ obtained by Section 4.2 is used in $D_{piano}$ design. Design parameters for the PSO algorithm is $a$, $b$, and $c$ as follow.

$$D_{piano}(z) = \frac{z^2}{z^2 - a z - b}$$ (19)

The design variable for $D_{piano}$ is determined as $x = [a, b, c]^T$. It is required to satisfy the signal range constraint which is written in (3).

We set the evaluation function as same for the case of $F$ design. As the conditions, we have to check the signal range of $u_{in}(k)$ for each calculation. If $u_{in}(k)$ does not satisfy signal range constraint, $J_{piano}^b = 5000$ is updated to the evaluation function. $D_{piano}(k)$ should be stable and its conditions are given as follows:

$$-1 < a < 1, -1 < b < 1$$ (20)

The iteration number of the PSO algorithm is set as $k_{max} = 100$ and the particle number is 100. The particle number is large because the number of the design variables is larger than the case of $F$ design. Evaluation function $J_{piano}$ is given by the same function in Section 4.2. Initial positions of $a$ and $b$ are selected randomly from the range $[-1, 1]$. Corresponding velocities are selected randomly from the ranges $[-100, 100]$ and $[-10, 10]$, respectively. Coefficients in velocity update law is the same for the case of $F$ design.

As a result of solving the PSO algorithm, the best value of the evaluation function $J_{piano}$ is shown in Fig. 13. The value of the evaluation function becomes smaller by increasing the iteration number. The designed filter $D_{piano}$ is shown as follow.

$$D_{piano} = 10.71 \frac{z - 0.893}{z + 0.625}$$ (21)

We can find that $D_{piano}$ has high pass characteristics as shown in Fig. 14. $J_{piano} = 0.0262$ is obtained by using (21). Figure 15 shows the spectrum of the output of DSM. By using $D_{piano}$, $u_{in}$ becomes larger around 2 kHz and its SN ratio becomes larger than that of Fig. 10. Figure 16 shows the error between $u_{in}$ and $u_{out}$ by the proposed method. We can find that the maximum absolute error is smaller than the case of traditional method in Fig. 11. Figure 17 shows the spectrum of $u_{out}$. The magnitude of the quantization noise in Fig. 17 is smaller than that in Fig. 12 for the frequency around 2 kHz.
We can find that our proposed system in Fig. 4 is useful because error in Fig. 17 and cost value $J_{piano}$ are smaller than the case with $D = 1$ which represents the traditional system.

4.4 Design of $F$ and $D$ for J-POP Music

In this section, J-POP music is used as $u_{in}$ to analyze the effectiveness of our design method for various inputs. Figure 18 shows a part of the J-POP music. Figure 19 shows its FFT analysis result. Frequency range of Fig. 19 is more larger than the case of the classical piano source signal in Section 4.1. $F_{jp}(z)$ and $D_{jp}(z)$ are designed for input signal shown in Fig. 18 by same sequence in Sections 4.2 and 4.3.

Optimal filter $F_{jp}(z)$ has the following characteristics and its gain diagram is shown in Fig. 20.

- cut-off frequency $W_p = 3099$ Hz
- order of the filter $n = 2$

The following filter is obtained by the PSO algorithm.

$$D_{jp}(z) = 1.79 \frac{z - 0.335}{z + 0.229}$$  \hspace{1cm} (22)

The obtained parameters are drastically different from the case of the classical piano. In particular the effect of the designed pre-filter is smaller than the case of the classical piano.

We can find that $D_{jp}$ has high pass characteristics in Fig. 21. The post-filter $F_{jp}(z)D_{jp}(z)^{-1}$ is obtained as a low-pass filter by combining Fig. 20 and Fig. 21. In particular, cut-off frequency is different from the result for the classical piano because the frequency range of J-POP music is larger than that of the classical piano. Cut-off frequency for J-POP music is also larger than that of the classical piano. Therefore, the error between $u_{in}$ and $u_{out}$, which is shown in Fig. 22, is larger than that of the classical piano because of the quantization noise. We can find that the performance of the data converter depends on the characteristics of the input signal $u_{in}$. $J_{jp} = 0.0727$ is obtained by the PSO algorithm. Figure 23 shows the frequency analysis result of the output signal. We can find that Fig. 23 is similar to Fig. 19 in the low frequency part.

Finally, when we apply the obtained filters $F_{jp}$ and $D_{jp}$ to the classical piano input in Fig. 6. Its evaluation value is given as $J_{piano} = 0.0374$ and we obtain the simulation result in Figs. 24 and 25. The evaluation value $J_{piano}$ using $F_{jp}$ and $D_{jp}$ is larger than that of $F_{piano}$ and $D_{piano}$. Error is also larger in Fig. 24. We can find that the quantization noise does not removed in high frequency part in Fig. 25 compared to the case of the piano sound. We can find that individual design for each input signal
is significant to minimize the effect of the quantization noise.

5. Conclusions

In this paper, the delta-sigma data conversion system with pre-filter and its evaluation framework are proposed to achieve good approximation of the input signal under the data quantization. The pre-filter and the post-filter are designed by our evaluation system with the PSO algorithm. The effectiveness of our proposed method is verified by the numerical simulations.

This paper considers the two-step design of $F$ and $D$. When number of the design parameters is increasing, the difficulty of the problem becomes exponentially increasing in any meta-heuristic design method like PSO algorithm. Advantage of the proposed design method is that number of the design parameters for each PSO problem is smaller in the two-step methods compared to the simultaneous design. Detailed comparison to the simultaneous design of $F$ and $D$ is one of our future works.

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References


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