Residual Vibration Suppression Using Simple Motion Trajectory for Mechanical Systems

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Abstract: This paper proposes motion-trajectory generation for mechanical systems that can be widely used in industrial applications. Many industrial systems, such as feed drive tables, gantry cranes, and material handling systems, are operated using S-curve acceleration and deceleration trajectory, and simple controllers that allow only such a simple motion trajectory are employed from the viewpoints of controller hardware and implementation cost efficiency. This study deals with simple mechanical dynamics including vibration properties and derives a necessary and sufficient condition for the motion trajectory to suppress residual vibration. Simulation and experimental results demonstrate the validity of the proposed conditions.

Key Words: mechanical motion trajectory, residual vibration suppression, natural frequency, S-curve trajectory.

1. Introduction

To achieve a shorter motion cycle time in industrial systems, residual vibration must be suppressed by using appropriate motion trajectories for drive systems. Cranes and material handling systems are typical examples of systems that require residual vibration suppression, and their controller design have been widely studied, such as optimal control [1]–[5], gain scheduling control [6]–[8], sliding mode control [9]–[11], adaptive control [12], back-stepping control [13], input-shaping control [14], disturbance observer based control [15], nonlinear control based on the Lyapunov stability theorem [16],[17], and rotary crane control using only boom-horizontal motion [18]–[22]. Feed drive tables also require control systems that can suppress undesirable vibration [23].

Although it is assumed that any motion trajectory can be implemented in most existing vibration control schemes, many industrial systems allow the implementation of only a simple motion trajectory, such as a trapezoidal velocity curve or S-curve acceleration and deceleration trajectory (hereafter referred as S-curve trajectory), from the viewpoints of controller hardware and implementation cost efficiency. This study investigates the use of S-curve trajectory for residual vibration suppression for typical mechanical systems with a single vibration natural frequency. Because the proposed approach is based on the feedforward control with respect to the vibration, a sensor for vibration measurement is not required, and the noise problem on the vibration measurement may be avoided.

The main original contribution is derivation of necessary and sufficient conditions for completely suppressing residual vibration for S-curve trajectory motion based on a cycloid function, which is widely used in many industrial machines, because it provides smooth connection between the acceleration/deceleration periods and the constant velocity period [24].

Although B-spline and Bezier curves are widely used for complex trajectory motion, the S-curve is mostly used for the fundamental point to point motion, and the presented trajectory in this paper may be extensively implemented in inexpensive industrial controllers. Although previous studies have similar objectives [25],[26], the authors derive necessary and sufficient conditions. In [27], a condition for suppressing the residual vibration in a trajectory with a constant acceleration/deceleration period is presented, although there is a disadvantage in the constant acceleration trajectory that the acceleration is discontinuously changed from an acceleration period to a constant velocity period. The motion trajectory considered in this paper is based on a cycloid function that provides continuous connection of acceleration from the acceleration period to the constant velocity period. Hence, the contribution of this paper has a certain advantage. In addition, the motion trajectory considered in this paper includes a trigonometric function to achieve the continuous connection of acceleration, and thus the approach for deriving the necessary and sufficient condition for residual vibration suppression is completely different from that in [27]. The obtained condition is different as well. References [28] and [29] take approaches of changing the trajectory profile from the original S-curve to other profiles (by including some trigonometric functions in [28] and power functions in [29], respectively), they cannot be applied to an well-used industrial controllers that allow only the S-curve trajectory for implementation. Simulation and experimental results demonstrate the validity and effectiveness of the proposed conditions.

2. Mechanical System Dynamics and Motion Trajectory

Here a typical mass-spring dynamics shown in Fig. 1 is considered. This dynamics can be widely used for representing the properties of a mechanical system with vibration. For example, the longitudinal vibration property of a ball screw of a feed drive system can be represented by the mass-spring dynamics [30]. The equation of motion for the system in Fig. 1 is given as follows:

\[ m \ddot{x} + c \dot{x} + kx = F(t) \]
\[ m_p \ddot{y} + k_p (y - x) = 0 \]  
where \( x \) is the input position, \( y \) the output position (controlled variable), \( m_p \) is the mass and \( k_p \) is the spring constant. The load sway dynamics of cranes can also be represented by Eq. (1) by regarding the gravity effect as the spring term. Equation (1) also models the vibration effect caused by the compliance of transmission gears such as the harmonic drives used in industrial robots.

Equation (1) can be rewritten as follows:

\[ \ddot{y} + \omega^2 y = \omega^2 x, \quad \omega = \sqrt{\frac{k_p}{m_p}} \]  
where \( \omega \) is the angular natural frequency. This study assumes that velocity \( \dot{x} \) follows the S-curve trajectory, as shown in Fig. 2. This trajectory has the following form:

\[ \dot{x} = \begin{cases} \frac{v}{t} (1 - \cos \omega t t) & t \in [0, t_1) \\ \frac{v}{t} (1 + \cos \omega t (t - t_1 - t_2)) & t \in [t_1, t_1 + t_2] \\ \frac{v}{t} t & t \in [t_1 + t_2, t_1 + t_2 + t_3] \end{cases} \]  
where \( t_i (i = 1, 2, 3) \) is the time for acceleration, constant velocity, and deceleration periods, respectively, and \( \omega t_i = \pi / t_i \).

Integrating Eq. (3) with respect to time, the following equation for position \( x \) is derived:

\[ x = \begin{cases} \frac{v}{t} \left( \frac{1}{\omega^2} \sin \omega t t \right) & t \in [0, t_1) \\ vt - \frac{v}{2} & t \in [t_1, t_1 + t_2] \\ \frac{1}{\omega^2} \sin \omega (t - t_1 - t_2) t_2 & t \in [t_1 + t_2, t_1 + t_2 + t_3] \end{cases} \]  

\[ y_1 = C_1 \cos \omega t + C_2 \sin \omega t - \frac{v \omega}{2(\omega^2 - \omega_r^2)} \]  
\[ \dot{y}_1 = -\omega C_1 \sin \omega t + \omega C_2 \cos \omega t - \frac{v \omega^2 \cos \omega t}{2(\omega^2 - \omega_r^2)} \]  
where \( C_1 \) and \( C_2 \) are the constants determined from initial conditions \( y_1(0) = 0 \) and \( \dot{y}_1(0) = 0 \), respectively.

\[ C_1 = \frac{v \omega}{2(\omega^2 - \omega_r^2)} - \frac{v}{2 \omega} \]  
\[ C_2 = \frac{v \omega^2 \cos \omega t}{2(\omega^2 - \omega_r^2)} + \frac{v}{2 \omega} \]  

Displacement \( y \) and its derivative \( \dot{y} \) for \( t_1 \leq t < t_1 + t_2, y_2 \) and \( \dot{y}_2 \), are derived from Eqs. (2) and (4) as follows:

\[ y_2 = C_3 \cos \omega t + C_4 \sin \omega t + vt - \frac{v t_1}{2} \]  
\[ \dot{y}_2 = -\omega C_3 \sin \omega t + \omega C_4 \cos \omega t + v \]  

where constants \( C_3 \) and \( C_4 \) can be determined as follows from boundary conditions \( y_2(t_1) = y_1(t_1) \) and \( \dot{y}_2(t_1) = \dot{y}_1(t_1) \) from Eqs. (5)–(10) as follows:

\[ \begin{bmatrix} C_3 \\ C_4 \end{bmatrix} = -M(t_1) \begin{bmatrix} \frac{v \omega}{2(\omega^2 - \omega_r^2)} - \frac{v}{2 \omega} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{v \omega}{2(\omega^2 - \omega_r^2)} - \frac{v}{2 \omega} \end{bmatrix} \]  

where \( M(t) \) is the rotation matrix as follows:

\[ M(t) = \begin{bmatrix} \cos \omega t & -\sin \omega t \\ \sin \omega t & \cos \omega t \end{bmatrix} \]  

Displacement \( y \) and its derivative for \( t_1 + t_2 \leq t \leq t_1 + t_2 + t_3, y_3 \) and \( \dot{y}_3 \), are derived as follows:

\[ y_3 = C_5 \cos \omega t + C_6 \sin \omega t + \frac{v \omega^2 \sin \omega (t - t_1 - t_2)}{2(\omega^2 - \omega_r^2)} + \frac{vt(t + t_2)}{2} \]  
\[ \dot{y}_3 = -\omega C_5 \sin \omega t + \omega C_6 \cos \omega t + \frac{v \omega^2 \cos \omega (t - t_1 - t_2)}{2(\omega^2 - \omega_r^2)} \]  

where constants \( C_5 \) and \( C_6 \) can be determined from boundary conditions \( y_3(t_1 + t_2) = y_2(t_1 + t_2) \) and \( \dot{y}_3(t_1 + t_2) = \dot{y}_2(t_1 + t_2) \) from Eqs. (11)–(14) as follows:

\[ \begin{bmatrix} C_5 \\ C_6 \end{bmatrix} = -M(t_1 + t_2) \begin{bmatrix} 0 \\ \frac{v \omega}{2(\omega^2 - \omega_r^2)} - \frac{v}{2 \omega} \end{bmatrix} + \begin{bmatrix} C_3 \\ C_4 \end{bmatrix} \]  

To suppress the residual vibration of the system in Eq. (2), \( x = y \) and \( \dot{y} = 0 \) must be achieved at time \( t = t_1 + t_2 + t_3 \). Then, the acceleration \( \ddot{y} = 0 \) at \( t = t_1 + t_2 + t_3 \) is satisfied from Eq. (2). These conditions can be represented as follows:

\[ y_3 = \frac{v t_1}{2} + t_2 + \frac{t_3}{2} \]  
\[ \dot{y}_3 = 0 \]  

at \( t = t_1 + t_2 + t_3 \). By considering Eqs. (13)–(17), the following condition is derived to achieve residual vibration suppression by the S-curve trajectory:

\[ M^{-1}(t_1 + t_2 + t_3) \times [-M(t_1 + t_2) V_2 + M(t_1) V_1 + V_3] - V_3 = 0, \]  

\[ V_i = \begin{bmatrix} 0 \\ \frac{v \omega}{2(\omega^2 - \omega_r^2)} - \frac{v}{2 \omega} \end{bmatrix}, \quad i = 1, 3 \]
Because it is common that the acceleration and deceleration periods are the same, the following relation is assumed:

$$t_1 = t_2 \quad \text{(i.e., } \omega_1 = \omega_3)$$

(20)

Then, the following equation is derived:

$$M^{-1}(2t_1 + t_2) \times \begin{bmatrix} -M(t_1 + t_2) + M(t_1) + I - M(2t_1 + t_2) \end{bmatrix} V_1 = 0$$

(21)

where $I$ is a $2 \times 2$ identity matrix. Because the first row of $V_1$ is zero, Eq. (21) is satisfied for any $V_1$ if and only if the following two equations are satisfied:

$$\sin \omega(t_1 + t_2) - \sin \omega t_1 + \sin \omega(2t_1 + t_2) = 0$$

(22)

$$- \cos \omega(t_1 + t_2) + \cos \omega t_1 + 1 - \cos \omega(2t_1 + t_2) = 0$$

(23)

Equations (22) and (23) can be rewritten as follows:

$$\begin{bmatrix} 1 + \cos \omega t_1 & - \sin \omega t_1 \\ \sin \omega t_1 & 1 + \cos \omega t_1 \end{bmatrix} \begin{bmatrix} \sin \omega(t_1 + t_2) \\ 1 - \cos \omega(t_1 + t_2) \end{bmatrix} = 0$$

(24)

When the $2 \times 2$ matrix in Eq. (24) is nonsingular, the necessary and sufficient condition to satisfy Eq. (24) is

$$t_1 + t_2 = \frac{2\pi k}{\omega}, \quad k = 1, 2, \ldots$$

(25)

by which a $2 \times 1$ vector in Eq. (24) becomes a zero vector. The $2 \times 2$ matrix in Eq. (24) is singular if and only if

$$t_1 = \frac{(2n - 1)\pi}{\omega}, \quad n = 2, 3, \ldots$$

(26)

by which the $2 \times 2$ matrix becomes a zero matrix, and Eq. (24) is satisfied. Therefore, either of Eq. (25) or (26) must be satisfied to suppress the residual vibration.

Note that, from Eq. (19), $n = 1$ (i.e., $\omega_1 = \omega_3 = \omega$) is not included in Eq. (26). Next, by considering $\omega_1 = \omega_3 = \omega$, the following equations are obtained instead of Eqs. (5)–(14) under the same boundary conditions:

$$y_1 = C_1 \cos \omega t + C_2 \sin \omega t + \frac{v t}{2} + \frac{v}{4} \cos \omega t$$

(27)

$$\dot{y}_1 = -\omega C_1 \sin \omega t + \omega C_2 \cos \omega t + \frac{v}{4} \cos \omega t - \frac{v t}{4} \sin \omega t$$

(28)

$$y_2 = C_3 \cos \omega t + C_4 \sin \omega t + \frac{v t}{2}$$

(29)

$$\dot{y}_2 = -\omega C_3 \sin \omega t + \omega C_4 \cos \omega t + v$$

(30)

$$y_3 = C_5 \cos \omega t + C_6 \sin \omega t - \frac{v t}{4} \cos \omega(t - t_1 - t_2) + \frac{v f(t + t_2)}{2}$$

(31)

$$\dot{y}_3 = -\omega C_5 \sin \omega t + \omega C_6 \cos \omega t + \frac{v t}{4} \sin \omega(t - t_1 - t_2) - \frac{v}{8} \cos \omega(t - t_1 - t_2) + \frac{v}{8}$$

(32)

$$C_1 = 0, \quad C_2 = -\frac{3v}{4\omega}, \quad C_3 = \frac{v t_1}{4}, \quad C_4 = 0,$$

$$C_5 = \frac{C_3}{C_6} - M(t_2) \begin{bmatrix} \omega(t_1 + t_2) \\ 4 M(t_2) \end{bmatrix}$$

(33)

To suppress residual vibration, Eqs. (16) and (17) must be satisfied at $t = t_1 + t_2 + t_3 = 2t_1 + t_2$. This condition is represented by the following condition derived from Eqs. (31) and (32):

$$\begin{bmatrix} v(t_1 + t_2) \\ 0 \end{bmatrix} = M(2t_1 + t_2)^{-1} \begin{bmatrix} C_5 \\ C_6 \end{bmatrix} + \frac{v(t_1 + t_2)}{4}$$

(34)

By considering $M(2t_1 + t_2) = M(t_2)$ when $\omega_1 = \omega_3$ and Eqs. (33) and (34), the following conditions are derived:

$$\begin{bmatrix} v(t_1 + t_2) \\ 0 \end{bmatrix} = M(t_2)^{-1} \begin{bmatrix} \omega(t_1 + t_2) \\ 0 \end{bmatrix} + \frac{v(t_1 + t_2)}{4}$$

(35)

The above condition is satisfied if and only if

$$t_2 = \frac{(2m - 1)\pi}{\omega}, \quad m = 1, 2, \ldots$$

(36)

Note that $\omega_1 = \omega_3 = \omega$ and Eq. (36) satisfy Eq. (25).

In conclusion, to achieve residual vibration suppression by the S-curve trajectory considered in [27] is different from that in this paper.

4. Simulation

To verify the validity of the proposed conditions given in Eqs. (37) and (38), simulations are conducted under two conditions assuming that the acceleration and deceleration are the same (i.e., $t_1 = t_3$). The fourth-order Runge-Kutta method with 1 ms step size is employed for the simulation. The first simulation is conducted under $f = \omega/2\pi = 10$ Hz, $t_2 = 0.03$ s and six different acceleration times. The simulation results are shown in Fig. 3, in which residual vibration ($y - x$) is suppressed only when $n$ is an integer and is greater than one as given in Eq. (37). Although Figs. 3 (b), (d) and (f) correspond to the cases with the condition presented in [27], residual vibration occurs, because the S-curve trajectory considered in [27] is different from that in this paper.

Figure 4 demonstrates the validity of Eq. (38) for $f = 10$ Hz and $t_1 = 0.05$ s, in which residual vibration is suppressed only when $k$ is an integer, as given in Eq. (38). Because the proposed condition in Eqs. (37) and (38) is the necessary and sufficient condition, the residual vibration always occurs when the condition is not satisfied. In order to verify the practical usability of the proposed condition, the variation of the residual vibration to the parameter (natural frequency) variance is verified by simulation as shown in Fig. 5, where $f$ and $f_2$ are the actual natural frequency and the nominal one used for the trajectory design, respectively. It is shown in Fig. 5 that the proposed condition still provides small magnitude of vibration under the practical range of parameter variance.
Fig. 3 Simulation results for verifying condition 1 ($T = 1/f = 0.1 \text{ s}, t_2 = 0.03 \text{ s}$).

Fig. 4 Simulation results for verifying condition 2 ($T = 1/f = 0.1 \text{ s}, t_1 = 0.05 \text{ s}$).

Fig. 5 Robustness verification to natural frequency variance.

5. Experiment

The proposed condition in Eqs. (37) and (38) does not have any robustness to plant modeling errors and disturbance because it is the necessary and sufficient condition for residual vibration suppression. The robustness is required for practical applications, and hence the practical usability of the proposed condition was verified experimentally by using a crane system in Fig. 6, because the actual plant certainly has some plant modeling errors and disturbance such as friction. In another viewpoint, there is always a possibility that the residual vibration can be suppressed by adjusting the acceleration and constant velocity periods so that the condition (37) or (38) is satisfied even when the model uncertainty exists. Therefore, the proposed condition is practically effective for the model uncer-
tainty in that the residual vibration can be suppressed by slight adjustment of the acceleration and constant velocity periods.

The linear dynamics of the crane system can be represented in the left equation in Eq. (2), in which \( x \) and \( y \) correspond to the moving stage position and the end position of the pendulum in Fig. 6. Two pendulums with different natural periods \( T_n = \frac{2\pi}{\omega_n} = 1.01 \) s and 0.55 s were considered. Figure 7 shows the experimental results in which the necessary and sufficient conditions proposed in this paper are not satisfied. In both pendulums, significant residual vibration occurs. Figure 8 shows experimental results that satisfy Eq. (38) and \( k = 1 \). In all cases, residual vibration is well suppressed although required motion time is shorter than the case in Fig. 7 (b) as well. From the results in Figs. 8 and 9, it can be confirmed that the proposed condition is effective for actual plants under practical disturbance and parameter variation.

6. Conclusion

This paper investigates simple motion trajectory generation for suppressing residual vibration of mechanical systems. Because some controllers for industrial systems allow the implementation of only an S-curve acceleration and deceleration trajectory from the viewpoint of cost efficiency, this study proposed a method to generate a simple motion trajectory that can effectively suppress the residual vibration of machines. Although there have been some studies with the similar objective, the authors derived the necessary and sufficient condition. The residual vibration can be suppressed if and only if the acceleration and deceleration periods are \((2n - 1)T_n/2\), \(n = 2, 3, \ldots\) or the sum of acceleration period and the constant velocity period is \(kT_n\), \(k = 1, 2, \ldots\) where \(T_n\) is the vibration natural period. Simulation and experimental results demonstrate the validity of the proposed condition.

The authors considered the similar simple mass-spring dynamics in Fig. 1 for an actual industrial robot in the previous paper [31], and showed the effectiveness of the simple model for the industrial robot (although controller design is different from that in this paper). Application of the presented condition to the industrial robot is left for future work.

Acknowledgments

This work was partially supported by JSPS KAKENHI Grant Number 24560131.

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