1. Introduction

The global community is currently looking for fuel-efficient and environmentally viable alternatives for many traditional energy conversion approaches. One of the efficient ways is to use "dual-fuel vehicles" (vehicles with multifuel engines), which are used in certain regions. For improving life cycle value of the vehicles, Yanmar has developed a reliable "dual-fuel engine" [1] that can use diesel and gas for the marine engine sector.

Evaluating air/fuel mixture and calculating the equivalence ratio in these dual-fuel engines requires certain new techniques that can facilitate the complicated treatment of two or more mixed fuels. In this study, a universal relationship is established between a modified refractive index of a premixed gas of fuel and air and its equivalence ratio. Through this relationship, we can calculate the equivalence ratio of any premixed gas (except those of light fuels) from its refractive index using a unique coefficient. The uniqueness of the relation coefficient enables us to obtain the equivalence ratio of even a multi-fuel premixed gas from its refractive index. In this study, the novel relationship is applied to a non-scanning 3D-CT (computerized tomography) technique using a multi(20)-directional quantitative schlieren optical system with a flashlight source. Instantaneous equivalence ratio distributions of a dual-fuel premixed gas flow with two jets of ethanol/air and propane/air mixtures are measured by CT-reconstruction. Good agreement was observed between the CT-reconstructed data and the proposed comprehensive relation.

Key Words: Multi-directional quantitative schlieren optical technique, Premixed gas field, Flow visualization, Computerized tomography (CT), 3D measurement, Equivalence ratio distribution, Refractive index, Bi-fuel, Dual-fuel, Multi-fuel premixing fuel-air technology. For calculating the equivalence ratio, it is necessary to know the type of fuel and its concentration ratio. It is complicated to measure the equivalence ratio in a flow field with multiple fuel types using conventional investigation techniques. Hence, the objective of this work is to study the equivalence ratio, which is important for combustion. Further, we propose a measurement method for the equivalence ratio of premixed gas with multiple types of fuels of unknown components, based on the modified refractive index of the premixed gas field. Furthermore, this is applied to the three-dimensional computerized tomography (3D-CT) technique using a multi-directional quantitative schlieren optical system to try to measure the instantaneous 3D equivalence ratio distributions of the flow field where the premixed gas exists. First, the "3D modified refractive index" distributions data of the target flow are obtained using 3D-CT reconstruction of the projection’s images. Further, the corresponding "equivalence ratio" is calculated using the proposed "universal relationship" between a modified refractive index and its equivalence ratio.

Kawahara et al. [2] developed a heterodyne interferometry system with a fiber-optic sensor. This system provides a
refractive index (consequently, temperature) measurement technique for unburned gas in a commercially produced spark-ignition (SI) engine. Sharma et al. [3] experimentally determined the refractive index and density for binary liquid mixtures of eucalyptol with hydrocarbons, and from measured values, refractive index deviations at different temperatures were computed. Nita et al. [4] present experimental data of the refractive index for pseudo-binary mixtures of reformate gasoline with ethanol, isopropanol, and n-butanol over the entire composition range and for temperature ranging from 293.15 K to 313.15 K. The refractive index mixture rules are discussed by Heller [5].

The schlieren imaging technique has proven to be particularly well suited in combustion and fluids and has been widely implemented [6]. This technique provides useful qualitative and quantitative schlieren system information on the variations in fluid density, temperature, static pressure, and flows can be depicted in detail.

In previous studies [7–17], by employing and developing a non-scanning three-dimensional computerized tomography (3D-CT) technique using a delicate multi-directional quantitative schlieren optical system with a flashlight source, the following have been successfully measured and certain new techniques introduced:

(a) local burning velocity of a turbulent premixed flame [7],
(b) light-emission distributions of premixed flames [8],
(c) instantaneous 3D density distributions of high-speed premixed turbulent flames [9–11],
(d) 3D density distributions of a steady non-axisymmetric premixed flame [10],
(e) instantaneous 3D density distributions of spark-ignited premixed spherical flame kernels in laminar and turbulent flows [12, 13],
(f) instantaneous 3D density distributions of spark-ignited flame kernels in propane/air-fuel jets from direct injection nozzle [14],
(g) instantaneous 3D density distributions of supersonic microjets issuing from circular and square micro nozzles [15],
(h) a novel "multi-path integration" technique for image-noise reduction, and
(i) a new "inverse process" technique to obtain vertical schlieren brightness gradient from horizontal experimental data [16, 17].

2. Methods and Experimental Setup

2.1. Overview of the Proposed Method

In this study, we obtained an instantaneous 3D modified refractive index distributions using the 3D-CT reconstruction method. Using the 20 modified optical path length $O_{\text{path}}$ images derived from the 20 quantitative schlieren images as projection data, the CT-reconstruction procedure has been successfully performed.

We propose a new method for obtaining the equivalence ratio distributions from the modified refractive index distributions. Using this method, it is possible to obtain instantaneous 3D distributions of equivalence ratios in the presence of multiple types of fuels with unknown components for hydrocarbon fuels with three or more carbon atoms.

2.2. Derivation of Equivalence Ratio $\varphi$ based on Modified Refractive Index $n_m$

The method for obtaining the equivalence ratio from the measured modified refractive index is discussed as follows.

The gas refractive index $n$ is expressed by equation (1) using the modified refractive index $n_m$ as,

$$n = 1 + n_m.$$  (1)

It can be expressed by equation (2) using the Gladstone-Dale relation ($\rho$: density, $K$: Gladstone-Dale constant) as,

$$n = 1 + \rho K.$$  (2)

Therefore, the modified refractive index $n_m$, density, and Gladstone-Dale constant of a gas have the following relationship,

$$n_m = \rho K.$$  (3)

In the combustion of hydrocarbon-type fuels $C_xH_{y}O_z$, the air fuel
ratio (AFR) of the premixed gas based on the equivalence ratio $\phi$ and hydrocarbon components is,

$$AFR = \left( a + 0.25b - 0.5c \right) / \left( 0.21\phi \right).$$

(4)

The density of the premixed gas $\rho_{mix}$ can be obtained by the following equation using the mass fraction $X_i$ known from the mass ratio obtained by equation (4).

$$\rho_{mix} = \Sigma X_i \rho_i = \rho_{\text{Air}} X_{\text{Air}} + \rho_{\text{Fuel}} X_{\text{Fuel}} + \ldots,$$

(5)

where,

$$X_{\text{Air}} = AFR / ( 1 + AFR ),$$

(6)

$$X_{\text{Fuel}} = 1 / ( 1 + AFR ),$$

(7)

Therefore, the product of the density of the premixed gas and Gladstone-Dale constant $K$ is obtained by the following equation (8).

$$\rho_{mix} K_{mix} = \Sigma \rho_i K_i X_i.$$  

(8)

Further, based on equation (3) $n_m = pK$ (i.e. $n_m = \rho_{mix} K_{mix}$), therefore, equation (8) shows the relationship between the equivalence ratio $\phi$ and modified refractive index $n_m$ (= $n - 1$, $n$: refractive index) of the premixed gas. The results of this relationship are illustrated in Fig. 1 for certain hydrocarbon-type fuels. The Gladstone-Dale constant $K$ [18] in Table 1 is used to calculate the modified refractive index.

In Fig. 1, except for methane and ethane, it can be observed that the relationship between the equivalence ratio and modified refractive index can be represented by a single straight line regardless of the fuel type because they are present in a close range and their differences are very small and negligible. We propose the following correlation (equation (9)), which reflects the variation of the results shown in Fig. 1.

$$n_m = 0.0000326 \phi + 0.0002726.$$  

(9)

Although, the Gladstone-Dale constants in Table 1 differ by more than 20%. The uncertainty is for the upper limit (Octane C8H18 +8.9%, the lower limit (Propane C3H8) -11%, and Ethanol +4.6%. Hence, the equivalence ratio can be obtained using a universal relationship that does not depend on the fuel type, and the equivalence ratio can be measured even when multiple fuels with unknown components are mixed together. This method can be applied to gas alarms, which utilize refractive index sensors [19–22].

2.3. Multi-Directional Quantitative Schlieren System

Figure 2(a)–(c) illustrates the concept of the 20-directional schlieren camera. The target "two premixed jets/non-uniform density field" is observed from a 180° angle using numerous schlieren optical systems simultaneously. Twenty systems capture the multi-directional view in the 20 different angle positions from $\theta = -85.5^\circ$ to $\theta = +85.5^\circ$ at an interval of $9^\circ$. Here angle $\theta$ is defined as the horizontal angle from the $x$-axis. The center between two-nozzle exits has been selected as the origin of the $xyz$-coordinate system.

A single instance of the multi-directional quantitative schlieren camera system is illustrated in Fig. 2(d). This system is composed of two convex achromatic lenses with 50 mm diameter and 300 mm focal length, a flashlight source unit, a vertical knife-edge (for obtaining horizontal schlieren brightness gradient images), a digital camera (mirrorless single-lens camera, Nikon, J1) and a camera lens (focal length 80 mm, lens diameter 21 mm). The digital cameras present 8-bit color depth JPEG images this equates to $(2^8)^3=16,777,216$ total potential RGB color values. In the 3D-CT quantitative schlieren measurement technique, all JPEG images convert to the portable gray map (PGM) format. It contains 256 monochrome grayscale values that represent different shades of gray from black (0) to white (256). The light unit is a xenon flashtube, full-spectrum white with a
Fig. 2  Multi-direction schlieren optical and coordinate systems.
uniform luminance rectangular area of 0.25 mm × 2 mm and 35 μs duration. The light intensity is adjusted by using a neutral density (ND) filter (Fuji 1, exposure adjustment multiple 10). The stepped ND filter is used for calibrating the cameras by obtaining images of grayscale in 11 steps without the knife-edge.

From a technological point of view, this experimental method (CT non-scanning, Fig. 2(e)) has a similarity with a CT-scanner that used in medical engineering for medical diagnostics (Fig. 2(f)). A CT-scan combines a series of X-ray images obtained from different angles to produce cross-sectional (tomographic) images of specific areas of a scanned object (human body). In the schlieren 3D-CT measurement technique employs schlieren photography (instead of X-ray images) from 20 directions to reconstruct 3D-density distributions data (cross-sectional density distributions) of a target flow.

2.4. Image Processing

To obtain the modified refractive index distributions, the 3D-CT measurement requires the distribution of the "modified optical path length \( O_{\text{plm}} \)" integrated on the ray as projection data. The modified optical path length distribution \( O_{\text{plm}} \) is obtained based on the diagram in Fig. 3. Figures 3(b)-(e) show the schlieren observation process from an angle \( \theta \) for the refractive index profile \( n^*(x, y) \) in Fig. 3(a). Quantities with an asterisk sign (*) express the actual values and without asterisk represent derived values from the CT-reconstruction process. Fig. 3(b)-(h) show observations in the direction of \( \theta \) from x-axis in the inclined coordinates which are denoted by \( X(\theta) \) and \( Y(\theta) \). The primary goal is to obtain the modified path length \( O_{\text{plm}}* \) of deviation modified refractive index \( (\Delta n_{m}* \) ), which is shown in Fig. 3(d) with the unit of length (m). Here, the modified refractive index deviation \( \Delta n_{m}* \) is defined as the modified refractive index deviation \( \Delta n_{m} = n_{m}* - n_{m} \) from the modified refractive index \( n_{m}* \) of the surrounding gas (air) region of constant modified refractive index \( n_{m}* (= 2.726 \times 10^{-3}) \) on the periphery of the observed range of radius \( R \), as \( n_{m}* > n_{m} \). \( \Delta n_{m}* \) is non-negative and CT-reconstruction is possible. The modified optical path length of deviation modified refractive index \( (O_{\text{plm}}*(X(\theta))) \) is called modified optical path length \( O_{\text{plm}}*. \) It is automatically obtained from the schlieren observation by spatially-integration of deviation modified refractive index \( \Delta n_{m}* (X(\theta), Y) \) along the line of sight. The modified refractive index distributions of the target flow field are reconstructed using 2D distributions (image) of modified optical path length \( O_{\text{plm}} \) as the projection in the CT-reconstruction process.

In practice, for obtaining modified optical path length \( O_{\text{plm}} \) (Fig. 3(h)), the image processing activity starts from Fig. 3(e) by obtaining two sets of images, "with target" and "without target" (without any disturbance in the test section) with a horizontal brightness gradient in the x-direction. Two sets of images present by the schlieren observation as \( B(X) \) and \( B_{nj}(X) \) (brightness of schlieren image in no-jet/no-disturbance condition). \( B_{nj}(X) \) is a schlieren brightness distribution when there is no refractive index distribution \( (n^*(x, y) = n_{m}*) \). To obtain the modified optical path length \( O_{\text{plm}}*(X(\theta)) \) from \( B(X) \) and \( B_{nj}(X) \), both are processed as depicted in Fig. 3(f)-(h). As indicated in Figs. 3(f) and (g), deviation brightness in schlieren image \( \Delta B(X) \)

\[
\Delta B(X) = B(X) - B_{nj}(X),
\]

(10)

is scaled to \( d(O_{\text{plm}}*(X(\theta)))/dX(\theta) \) by the next expression.

\[
d(O_{\text{plm}}*(X(\theta)))/dX(\theta) = \alpha \times (B(X) - B_{nj}(X)),
\]

(11)

where \( \alpha \) depends on transparent width of the light source image on schlieren stop location (\( \Delta s = 0.25 \text{ mm} \times 2 \text{ mm} \)) and focal length of the convergent lens \( (f = 300 \text{ mm}) \). Therefore, modified optical path length \( O_{\text{plm}} \) is reproduced by transverse-integration of \( d(O_{\text{plm}}*(X(\theta)))/dX(\theta) \) from schlieren images, as shown in Fig. 3(h). Furthermore, \( O_{\text{plm}}*(X(\theta) = -R) = O_{\text{plm}}*(X(\theta) = +R) = 0 \).

2.5. CT-Reconstruction

The images of modified optical path length \( O_{\text{plm}} \) are used as projections for CT-reconstruction by maximum likelihood-expectation maximization (ML-EM) [23] an appropriate CT algorithm to obtain the 3D reconstruction of modified refractive index distributions. ML-EM (distributed back projection) method [7–17] is employed for CT-reconstruction. The CT-reconstruction procedure is performed in each horizontal plane of z-axis for the reconstruction of deviation modified refractive index distributions \( \Delta n_{m}(x, y) \) from linear data set of modified optical path length \( O_{\text{plm}}(X(\theta)) \) (Fig. 3(c)). Further, 2D modified refractive index distribution (Fig. 3(b)) is obtained as follows.

\[
n_{m}(x, y) = \Delta n_{m}(x, y) + n_{m}*.
\]

(12)

The reconstruction is performed for each cross-section, and the cross-sections are stacked to form the 3D modified refractive index distribution. Therefore, the 2D distributions \( n_{m}(x, y) \) are accumulated in layers to form 3D-CT distribution \( n_{m}(x, y, z) \). In this study, the modified optical path length \( O_{\text{plm}}(X(\theta)) \) projections images of 900(\( H \)) × 5(\( V \)) pixel (27 mm × 0.15 mm) are used for CT-reconstruction to produce 3D data 900(\( x \)) × 900(\( y \)) × 5(\( z \)) pixel (27 mm × 27 mm × 0.15 mm). The voxel size is 0.03 mm in each direction.

2.6. Experimental Injection Nozzle

Figures 4 shows an overview of the premixed gas injection device used in this experiment and an outline of its nozzles shape
and size. This injection device can inject two different fuels simultaneously by filling the parts shown in red in Fig. 4 with premixed gas and rotating the lower motor. A stepping motor (manufactured by Plexmotion, SSA-PR-42D2, built-in driver) was used in this experimental setup. As depicted in Fig. 4(top), the convergent nozzle shape is provided with a constricted flow part with an inlet diameter of 12 mm and outlet diameter of 6 mm to inject the premixed gas in a laminar flow. The flow rate is adjusted for the airflow 50 l/min and for the propane flow 3 l/min to maintain the laminar flow. In this experiment, propane-air and ethanol-air premixed gases with an equivalence ratio $\phi = 1$ are injected simultaneously, and 20-directional simultaneous quantitative schlieren imaging is performed. The center between two-nozzle exits has been selected as the origin of the $xyz$-coordinate system. The ethanol-air premixed gas nozzle outlet center coordinate is $(x, y, z) = (0, -8, 0)$ and propane-air premixed gas nozzle outlet center coordinate is $(x, y, z) = (0, +8, 0)$.

3. Results and Discussion

3.1. Quantitative Schlieren Images

Figure 5 shows quantitative schlieren images of the two premixed gas jets obtained simultaneously by the multi(20)-directional schlieren camera system. The insets in each image are

![Fig. 3 Process of formation of schlieren brightness and conversion to projections $O_{plm}$ of CT.](image-url)
the values of the shooting angles $\theta$. The images of Fig. 5 are processed by the aforementioned conversion procedure to projection images of modified optical path length $O_{opl}$. Using the modified optical path length $O_{opl}$ images as projections for CT-reconstruction, 3D instantaneous modified refractive index $n_m$ distributions of flow targets of two premixed gas jets were successfully obtained.

3.2. CT-Reconstruction Results

Figure 6(a) shows a sample of horizontal distributions of modified refractive index $n_m$ for $z = 11.1$ mm position for stoichiometric ($\varphi = 1$) premixed jets.

Figure 6(b) shows the corresponding contour diagram of Fig. 6(a). The modified refractive index is indicated by the brightness level as shown by the brightness bar charts in the inset. Figure 7 shows the radial distribution diagram of the modified refractive index $n_m$ and equivalence ratio $\varphi$ for $x = 0$ and $z = 11.1$ mm. In Fig. 7, the modified refractive index $n_m$ is obtained from CT-reconstructed data and corresponding equivalence ratio $\varphi$ is calculated from the universal relationship (equation (9)) between a modified refractive index and its equivalence ratio. It is observed that the peak value of the equivalence ratio in the potential core of the premixed gas jets is approaching to $\varphi = 1$ on the left and right sides, despite the different fuel types (ethanol, propane). The uncertainty value of potential core for ethanol and propane is $+2.4\%$ and $-18\%$, respectively. A good agreement is observed between the CT-reconstructed data and proposed universal relationship between a modified refractive index of a premixed gas of fuel and air, and its equivalence ratio.

4. Conclusions

An experimental study was performed to obtain the detailed information required for understanding the relationship between the modified refractive index of a premixed gas of fuel and air and its equivalence ratio. Here, a "universal relationship" between the "modified refractive index $n_m$" of a premixed gas of fuel and air and its "equivalence ratio $\varphi$" is proposed. The results
confirm good agreement between the CT-reconstructed data and the proposed universal relationship. Therefore the equivalence ratio can be obtained by a universal coefficient that does not depend on the fuel type and can be measured when multiple fuels with unknown components are combined. This method can be applied to gas alarms technology. Its application in other fields should be investigated.

Acknowledgments

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References


Fig. 6 3D-CT reconstructed horizontal distribution result of $n_m$ for stoichiometric ($\phi = 1$) premixed jets.

(a) 3D-CT reconstructed horizontal distributions data of $n_m$ for propane (top) - ethanol (bottom), $z = 11.1$ mm

(b) Corresponding contour diagram of Fig. 6(a)

Fig. 7 Radial diagram of $n_m$ (from CT-reconstructed result) and equivalence ratio $\phi$ (using equation (9)) for ethanol (left) - propane (right), $x = 0$ and $z = 11.1$ mm.


**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>Brightness of schlieren image [-]</td>
</tr>
<tr>
<td>Bnj</td>
<td>Brightness of schlieren image in no-jet (no disturbance) condition in the test section [-]</td>
</tr>
<tr>
<td>f</td>
<td>Focal length of convergent lens in schlieren system [m]</td>
</tr>
<tr>
<td>K</td>
<td>Gladstone-Dale constant [m$^3$/kg]</td>
</tr>
<tr>
<td>n</td>
<td>Refractive index [-]; derived value</td>
</tr>
<tr>
<td>n$^*$</td>
<td>Refractive index [-]; actual value</td>
</tr>
<tr>
<td>nma$^*$</td>
<td>Refractive index of ambient gas of air [-]; actual value</td>
</tr>
<tr>
<td>nm$^*$</td>
<td>Modified refractive index [-]; derived value, ($n_m = \Delta n_m + nma$*)</td>
</tr>
<tr>
<td>nm$^*$</td>
<td>Modified refractive index [-]; actual value</td>
</tr>
<tr>
<td>nma$^*$</td>
<td>Modified refractive index of ambient gas of air [-]; actual value</td>
</tr>
<tr>
<td>O_plm</td>
<td>Modified optical path length [m]; derived value</td>
</tr>
<tr>
<td>O_plm$^*$</td>
<td>Modified optical path length [m]; actual value</td>
</tr>
<tr>
<td>R</td>
<td>Radius of reconstruction area [m]</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Angle of observation [º (degree)]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density [kg/m$^3$]</td>
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<tr>
<td>$\phi$</td>
<td>Equivalence ratio [-]</td>
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