Survey on Secondary School Teachers’ Statistical Knowledge for Teaching: The Need of Developing Venezuelan Teachers’ Competence to Teach Statistics

Orlando GONZÁLEZ

Despite the inclusion of several variability-related contents into the secondary school mathematics curriculum by recent reforms in many countries including Venezuela, studies on the professional competence to teach such contents held by mathematics teachers are lacking in the literature. In order to help close this research gap, a novel framework for statistical knowledge for teaching (SKT) is introduced, as well as a survey instrument developed on its basis, designed to assess the eight dimensions of professional competencies for teaching variability-related contents identified by this study: the six aspects of teachers’ professional knowledge comprising SKT, teachers’ conceptions of variability, and teachers’ beliefs about statistics teaching and learning. In this article, an analysis of the answers collected from 53 in-service secondary school mathematics teachers working at the metropolitan area of Caracas, Venezuela, is carried out, focusing on teachers’ common content knowledge, knowledge of content and curriculum, and conceptions of variability. Finally, some interesting findings, trends and implications yielded from the data analysis are discussed.

Key words: statistical knowledge for teaching, statistical literacy, teachers’ conceptions of variability, knowledge of content and curriculum.

1. Introduction

Aiming towards statistical literacy—which is a tremendously valuable asset in today’s knowledge-based society—, curricular reforms carried out in recent years at many countries have brought into the secondary school mathematics curriculum topics related to statistics (e.g., NCTM, 2000; MEXT, 2008a, 2008b, 2009). In the particular case of Venezuela, topics on statistics and probability were introduced into lower secondary education (i.e., in Grades 7–9) in 1985, while their introduction into upper secondary education (i.e., in Grades 10–11) occurred in 1972, year until which the study of such topics was left exclusively to university students (ME, 1985, 1997; CENAMEC, 1990).

It is noticeable that variability—a property which accounts for the propensity of an statistical object to vary or change, which is considered by several researchers not only as a fundamental concept in statistics, but also as its raison d’être(e.g., Shaughnessy, 2007)—may arise naturally in many different ways in all the statistical topics included in the aforementioned mathematics curricula. Consequently, nowadays secondary school mathematics teachers in Venezuela, as well as in other countries worldwide, must teach
several variability-related ideas—such as the ones of graphical representations of data, measures of variation, distribution and sampling—, and such work demands from them specific professional competencies—which basically depend on professional knowledge and teacher beliefs (Döhrmann, Kaiser & Blömeke, 2012, p. 327)—, without which the aims of the mathematics curriculum regarding statistics education cannot be achieved.

With all this in mind, it seems very clear that teachers’ professional knowledge about variability-related concepts is of paramount importance in the teaching and learning of the statistical contents within the mathematics curriculum, as well as teachers’ conceptions of variability and statistics-related beliefs. However, despite the apparent importance of these traits in statistics education, practically no research on them was reported prior the turn of the millennium. This paucity of research in the aforementioned issues is particularly true in the case of Venezuela, country in which the few reported researches on statistics education to date have been centered on the statistical contents in the school curriculum or on students’ knowledge about statistics and probability (cf. Salcedo, 2008), with no studies reported, to the knowledge of the present author, on teachers’ professional competencies to teach statistics at any school level. Hence, it is by no means surprising the urgent call for increasing research on these areas made by a number of concerned researchers, particularly for studies on teachers’ professional knowledge and practices while teaching variability (e.g., Sánchez, da Silva & Coutinho, 2011, p. 219), as well as for teachers’ beliefs on statistics itself and on what aspects of statistics should be taught in schools and how (e.g., Pierce & Chick, 2011, p. 160), and the conceptions of variability held by school mathematics teachers (Makar & Canada, 2005). Accordingly, the purpose of this paper is to respond to such calls by proposing a conceptual framework for mathematics teachers’ professional competencies to teach variability-related contents, which integrates statistical knowledge for teaching—henceforth SKT, the professional knowledge entailed by the work of effectively teaching statistics—, conceptions of variability, and statistics-related beliefs. Based on such framework, an array of indicators is presented, in order to examine their implementation for data collection, and then gather information on the aforementioned traits in secondary school mathematics teachers. In that way, it would be possible to get a clearer picture about their level of competence to teach variability-related contents.

This article reports and discusses particular results obtained from the practical implementation of the aforementioned framework on a group of 53 Venezuelan secondary school mathematics teachers. Moreover, this article is the final installment of a continuing series, which started exploring in-service Japanese elementary, middle and high school mathematics teachers’ statistical literacy from the viewpoint of variability (Isoda & González, 2012). Then, an article expanding the scope from statistical literacy and knowledge of stochastical phenomena to professional competencies to teach variability-related ideas included in the mathematics school curricula followed (González, 2012), proposing the framework depicted in Fig. 2 and the assessment tool shown in Fig. 3. Finally, a series of articles reporting results regarding a practical implementation of this framework, in relation to a case study of four Japanese senior high school mathematics teachers, followed (González, 2013, 2014).
2. Conceptualizing teachers’ professional competencies for effective teaching of variability-related ideas

2.1. The MKT model

Ball, Thames and Phelps (2008) developed the notion of mathematical knowledge for teaching—henceforth MKT—, a practice-based framework focused on what knowledge and skills teachers need in order to be able to teach mathematics effectively. This model describes MKT as being made up of two domains—namely subject matter knowledge (SMK) and pedagogical content knowledge (PCK)—, each of them structured in a tripartite form (see Fig. 1). According to Ball et al. (2008), SMK can be divided into common content knowledge (CCK), specialized content knowledge (SCK), and horizon content knowledge (HCK). Furthermore, Ball and her colleagues presented a refined division of PCK, comprised of knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum (KCC) (the interested reader should refer to the original article for a detailed discussion of these constructs).

![Fig. 1. Domains of Mathematical Knowledge for Teaching (MKT), according to Ball et al. (2008)](image)

Through this model, Ball and her colleagues not only clarified the distinction between SMK and PCK and refined previous conceptualizations of such constructs found in the literature, but also made significant progress in identifying the relationship between teacher knowledge and student achievement in mathematics. However, as it has been highlighted by some researchers (e.g., Petrou & Goulding, 2011, p. 16), by not considering the role of either beliefs or conceptions about the subject matter in teachers’ practices, the MKT model is less well appreciated. In the case of beliefs—defined by Philipp (2007, p. 259) as “psychologically held understandings, premises, or prepositions about the world that are thought to be true”—, previous studies have suggested their influence on the way teachers transform the subject matter into teaching practice to make it comprehensible to others (e.g., Philipp, 2007; Petrou & Goulding, 2011; Eichler, 2011). For example, if teachers believe that mathematics is mainly a subject of rules and procedures that should be memorized, then pedagogical aspects such as how they implement the curriculum and plan their lessons will be constrained (Petrou & Goulding, 2011). In the case of conceptions—the set of internal representations and the corresponding associations that a concept evokes in an individual, often explained in the literature as “conscious beliefs”—, they have been reported to be related to teachers’ level of statistical literacy and understanding of specific statistical objects (Makar & Canada, 2005; Isoda & González, 2012). For example,
Orlando GONZÁLEZ

if teachers conceptualize distribution of data as an aggregate—which is fundamental in statistics—, their approach to data handling will be oriented toward the entire data as a focus of attention—a statistical habit of mind sought to be developed in students—, rather than focus on individual cases (Shaughnessy, 2007; Isoda & González, 2012). Therefore, it seems that not acknowledging beliefs and conceptions as factors affecting teachers' professional knowledge could be a drawback, since the significant influence of these traits on the work of teaching that has been reported in the literature (cf. Philipp, 2007; Batanero & Díaz, 2010; Eichler, 2011; Pierce & Chick, 2011).

2.2. A New Conceptualization of Statistical Knowledge for Teaching

To date, few conceptualizations of SKT have been reported in the literature, with almost all of them employing as a basis the aforementioned framework for MKT—cf. Groth (2007); Noll (2011); Burgess (2011). For example, Groth (2007) proposed a theoretical SKT framework for teaching statistics at the high school level, focusing on the constructs CCK and SCK. Noll (2011) provided a framework for characterizing graduate teaching assistants' SKT required for the teaching sampling in undergraduate statistics, focusing also on the constructs CCK and SCK. Burgess (2011) developed a SKT framework for a classroom-based investigation of the professional knowledge held by upper primary level teachers, focusing on the constructs CCK, SCK, KCS and KCT. From this review of previous studies, it is evident that only limited understanding of the professional knowledge entailed by the work of effectively teaching statistics has been achieved so far, since these previous efforts to conceptualize SKT has neither considered all the six categories of professional knowledge identified by Ball et al. (2008), nor reported empirical results on the SKT held by secondary school mathematics teachers. Furthermore, the aforementioned SKT frameworks consider the role of neither beliefs nor conceptions in teachers’ professional practice and statistical literacy. Consequently, it seems that these gaps in the literature result in an inaccurate picture of how prepared are secondary school mathematics teachers to teach statistical contents related to variability.

In an effort to fill the aforementioned gaps, a conceptual model for secondary mathematics teachers' professional competencies to teach variability-related contents was proposed by the author (see Fig. 2) (cf. González, 2012). This conceptualization was based on the following four conjectures, which were the result of an extensive literature review intended to identify influential factors pertinent to the professional knowledge entailed to teach statistics at school mathematics (cf. González (2012) for a detailed discussion of these conjectures):

1. Due to the common grounds shared by mathematics and statistics, using a model for MKT could be useful to explain the SMK and PCK required for teaching statistics at school: there is considerable overlap and cooperation between the two disciplines, not only content-wise and skill-wise, but also curriculum-wise, since statistics is typically taught within the school mathematics curricula. Thus, this relationship between mathematics and statistics suggests that current research on the structure of MKT could provide a viable starting point for examining SKT (Groth, 2007).

2. Due to the differences between mathematics and statistics, the model for MKT used must be adapted, in order to acknowledge such differences and meet the requirements of teaching statistics: Although mathematics and statistics share some common grounds, the two disciplines are different in several ways, especially in the use of numbers: mathematics has a deterministic nature, while statistics has a stochastic...
one. Ball et al. (2008) seem to disregard this fundamental difference in their MKT framework. This is quite evident in their definition of CCK, described as "simply calculating an answer or, more generally, correctly solving mathematics problems" (Ball et al., 2008, p. 399), which does not quite work for the case of statistics—except for the deterministic view of teaching. The statistical skills expected from any individual after completing compulsory schooling go beyond just correctly solving problems. They include, among others, the ability of acknowledging and measuring variability, as well as the ability of describing graphs and distributions, which are related to statistical literacy (cf. Gal, 2004; Pfannkuch & Ben-Zvi, 2011). Moreover, such skills are also common to a wide variety of settings, and hence not unique to teaching, similarly to the skills related to CCK. Therefore, in order to acknowledge the differences between mathematics and statistics and meet the case of statistics education, in the present study CCK will be seen as statistical literacy. The rest of knowledge components in this framework are defined in the same way as in the model of MKT by Ball et al. (2008), but rephrased in some cases to meet the requirements of teaching statistics.

(3) As beliefs are seen by many researchers to be playing a crucial role in shaping teachers’ practice, consideration of teachers’ beliefs about statistics teaching and learning is fundamental for the present study.

(4) Since conceptions are "pictures" held by an individual about a concept from the world of objective knowledge which may affect teachers’ problem-solving and teaching approaches, and conceptions of variability have been reported to be strongly related to statistical literacy, consideration of teachers’ conceptions of variability is fundamental for the present study.

Reflection on these conjectures resulted in a framework depicted in Fig. 2, comprised of the two facets: one cognitive and one affective. The cognitive facet is a six-fold conceptualization of SKT, comprised of all the six knowledge categories identified by Ball et al. (2008) in their MKT model, with the construct CCK being understood in the present study as statistical literacy, in order to meet the case of statistics education. The affective facet of the model proposed in this article is comprised of two components: statistics-related beliefs and conceptions of variability held by teachers, since, as explained before, the former appear to influence how they teach—and subsequently may influence how their students learn (cf. Philipp, 2007)—while the latter have been reported to be strongly related to the level of statistical literacy, and are important for researchers to recognize teachers’ potentially limiting views of variability in particular settings (cf. Makar & Canada, 2005; Batanero & Díaz, 2010; Isoda & González, 2012).

Following a literature review and consultation with specialists, the six elements comprising SKT in Fig. 2 were paired with twelve qualitative indicators, as shown in Table 1. Each of these indicators was built from the definition of each cognitive category identified by the present study, in order to provide a comprehensive framework for SKT (e.g., Gal, 2004; Ball et al., 2008). For example, according to Ball et al. (2008, p. 391), one characteristic of the so-called knowledge of content and curriculum (KCC) is knowledge about “topics and issues that have been and will be taught in the same subject area in the preceding and later years”, since teachers must know at which grade levels particular topics are typically taught. This cognitive feature is acknowledged by Indicator F-1 (see Table 1). The rest of indicators identified in this study were developed in a similar way.
Orlando GONZÁLEZ

3. Assessing teachers’ professional competencies for effective teaching of variability-related topics

3.1. The Survey Instrument: Development and Analysis

Based on the aforementioned 12 cognitive indicators and the Venezuelan secondary school mathematics curriculum, a pen-and-paper instrument was developed. For such instrument, designed to be completed in one hour, a task dealing with data-handling in histograms, comparing distributions and decision-making—originally developed by Garfield, delMas & Chance (1999)—was chosen, mainly for two reasons: (1) tasks comparing distributions not only demand knowledge about many variability-related ideas in the field of descriptive statistics, but also have been reported in the literature as an effective means of exploring teachers’ understanding of distribution, conceptions of variability and statistical literacy (Makar & Confrey, 2005; Isoda & González, 2012); and (2) most of the statistical contents in the Venezuelan secondary school mathematics curriculum are ideas related to descriptive statistics. For the present study, the chosen task was modified to facilitate the calculations that could be made by the respondents, as well as enriched with seven questions aiming to elicit empirical evidence on all 12 indicators of SKT identified in the present study, which resulted in the 7-question item—henceforth Item
1—shown in Fig. 3. Such questions were developed from studies with similar aims reported in the literature (e.g., Meletiou & Lee, 2003; Ball et al., 2008; González & Isoda, 2011; Isoda & González, 2012).

A mapping between the components of SKT that would be elicited by each question in Item 1, as well as the indicators associated to each cognitive aspect considered by this framework, is shown in Table 2.

---

**ITEM 1**
Please, read carefully the following task and answer the questions below:

Choosing the distribution with more variability. Look at the histograms of the following two distributions:

![Histograms of Distributions A and B](image)

Which distribution (A or B) do you think has more variability? Briefly describe why you think this.

(a) Answer this task in as many different ways as you can. Please, be sure to show every step of your solution process.
(b) What are the important ideas that might be used to answer this task?
(c) Suppose that, after posing this task to your students, three of them come up with the following answers:

**STUDENT 1:** "Distribution A has more variability because it’s not symmetrical."

**STUDENT 2:** "Distribution A ranges from 3 to 14, while Distribution B ranges from 1 to 14. Then, Distribution B has more variability."

**STUDENT 3:** "The bars in Distribution A are clumped closer to the central bar than they are in Distribution B. Then, Distribution B has more variability."

Dealing with each student separately, please comment briefly on each of these answers, focusing on whether the answer is correct or not, why you think so, and what reasoning might have led students to produce each answer.

(d) Suppose you pose this task to your students. What are the most likely responses (correct and incorrect), and difficulties you would expect from them? Briefly explain why you think so. (Regarding to the most likely answers that you might get from the students, please do not include those mentioned in part (c))

(e) Mathematically/statistically speaking, is any of the answers given by the students interesting or significant? If yes, briefly explain why and on what aspects. (Please, focus your response on whether there is a significant mathematical/statistical insight in the student’s answer, and whether there are forthcoming contents in future classroom subjects connected to the notions being said or implied in such answer.)

(f) Briefly describe how the important ideas involved in the solving process of the given task are addressed in official curriculum documents across the different grade levels of schooling.

(g) Suppose you want to plan a lesson (or a series of lessons) to introduce the meaning of variability in the setting of the given problem to your students. Briefly describe as many instructional strategies, activities and/or tasks as you can think of that would be appropriate to use for this purpose, and sequence them accordingly, explaining why you chose to put them in such a particular order.

---

**Fig. 3. Item 1: “Choosing the distribution with more variability” task**

**Table 2. Knowledge components of SKT elicited by each of the questions posed in Item 1**

<table>
<thead>
<tr>
<th>Elicited knowledge component of SKT</th>
<th>Related indicator of SKT</th>
<th>Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>Common Content Knowledge (CCK, as Statistical Literacy)</td>
<td>A-1</td>
<td>(a)</td>
</tr>
<tr>
<td></td>
<td>A-2</td>
<td>(a)</td>
</tr>
<tr>
<td>Specialized Content Knowledge (SCK)</td>
<td>B-1</td>
<td>(c)</td>
</tr>
<tr>
<td></td>
<td>B-2</td>
<td>(c)</td>
</tr>
<tr>
<td>Horizon Content Knowledge (HCK)</td>
<td>C-1</td>
<td>(e)</td>
</tr>
<tr>
<td></td>
<td>C-2</td>
<td>(b)</td>
</tr>
<tr>
<td>Knowledge of Content and Students (KCS)</td>
<td>D-1</td>
<td>(d)</td>
</tr>
<tr>
<td></td>
<td>D-2</td>
<td>(d)</td>
</tr>
<tr>
<td>Knowledge of Content and Teaching (KCT)</td>
<td>E-1</td>
<td>(g)</td>
</tr>
<tr>
<td></td>
<td>E-2</td>
<td>(g)</td>
</tr>
<tr>
<td>Knowledge of Content and Curriculum (KCC)</td>
<td>F-1</td>
<td>(f)</td>
</tr>
<tr>
<td></td>
<td>F-2</td>
<td>(g)</td>
</tr>
</tbody>
</table>
In regard to the cognitive facet of the conceptual model for SKT proposed here, collected answers will be qualitatively analyzed, particularly by using a “bottom up” approach to coding (Coffey & Atkinson, 1996). This grounded form of analysis ensures that the themes or categories extracted are, in fact, grounded in the data and hence reflect the participants’ own knowledge base of each of the cognitive components of SKT examined by the designed instrument. An exception to this approach will be the analysis to Question (a), which will be undertaken based on and refining the categorizations employed by previous studies dealing with the chosen task (cf. Meletiou & Lee, 2003; Isoda & González, 2012). Thus, in the present study, teachers’ answers to Question (a) will be organized into the following categories:

a0: No response.
a1: Distribution A, giving no reason, just guessing, by arguing intuitive ideas, or based on a mistaken calculation.
a2: Distribution A, based on a misinterpretation related to symmetry and/or a poor fit to a normal distribution.
a3: Distribution A, based on arguments related to differences in the heights of the bars.
a4: Distribution B, giving no reason, just guessing, by arguing intuitive ideas, or by misinterpretation.
a5: Distribution B, based on arguments related to simple recognition of variability.
a6: Distribution B, based on arguments related to sophisticated recognition of variability.

In regard to the components in the affective facet of the conceptual model for SKT proposed here, it is anticipated that teachers’ answers to Question (a) will provide enough information about how the respondents conceptualize variability, based on the fact that teachers’ conceptions of variability can be made explicit by answering tasks in which knowledge and understanding of variability-related ideas, as well as the ability to connect and represent them, are required (González & Isoda 2011; Isoda & González, 2012). These conceptions of variability are going to be classified using as main reference the categorization proposed by Shaughnessy (2007, pp. 984–985). Thus, from the results obtained in previous researches (e.g., Noll, 2011; Isoda & González, 2012; González, 2013, 2014), the following four conceptions of variability identified by Shaughnessy (2007) are expected to be found in teachers’ answers:

Variability in particular values, including extremes or outliers: people holding this conception focus their attention on particular data values in a graph or a data set.

Variability as distance or difference from some fixed point: people holding this conception think of variability as an actual or a visual measurement of the distance of each or some elements of a dataset either from an endpoint value or from some measure of central tendency.

Variability as the sum of residuals: people holding this conception think of variability as the measure of the total variation of an entire distribution of data via the calculation of deviation-based metrics such as the mean absolute deviation, sum of residuals or averages of the absolute value differences from a measure of center.

Variation as distribution: people holding this conception are able to get involved in extensive transnumeration, in order to consider many theoretical features of a distribution, particularly when variability between or among a set of distributions is compared.

The first conception described in the list above is considered as a low level one, since people who hold it do not consider measures of central tendency, and hence do not discuss the connections between middles in data and the variability of data dispersed around a middle (Shaughnessy, 2007, 2008; Isoda & González, 2012). The latter three
conceptions described above are at an even higher level compared to the first one, since they discuss the connections between measures of central tendency and the variability of data dispersed around a center. In addition, a fifth conception not included in the categorization described by Shaughnessy (2007) is also anticipated, as previous studies have reported (Noll, 2011; Isoda & González, 2012; González, 2013, 2014). In the present study, this conception will be referred to as “Variability as visual cues in the graph”. People holding this conception regard variability as unevenness in the bars of a histogram, as symmetry, or as closeness (or lack thereof) of fit to a normal distribution.

In the case of teachers’ beliefs about statistics teaching and learning, the limited research on this topic (e.g., Pierce & Chick, 2011, p. 159) suggests that they could be identified through examining the features of the lesson plans that teachers produce, such as the tasks chosen to consider a particular statistical idea, and the types of instructional strategies teachers planned to use during the lesson. What teachers planned to do—which is related to the construct KCT, and hence with answers to Question (g)—will be analyzed using the four categories reflecting on teachers’ beliefs developed by Eichler (2011)—i.e., traditionalists, application preparers, everyday life preparers, and structuralists—, which will provide valuable information on teachers’ beliefs about the nature of statistics, as well as about the teaching and learning of statistics.

3. 2. Participants and Data Collection Methodology

Fifty-three in-service secondary school mathematics teachers, working in the metropolitan area of Caracas, Venezuela, voluntarily and anonymously participated in this study. Among these teachers, 19 were working at lower high school level, 15 at upper high school level, and 19 at both lower and upper high school levels. The age of the participants ranged from 21 to 71 years-old, with an average of 42.6 years-old. The classroom experience of the participants ranged from 0 to 45 years, with an average of 16.4 years. The teachers were contacted previous agreement with their respective institutions via the principal, and a person responsible for the distribution and gathering of the survey instruments was present at the moment that teachers filled in the questionnaires. The period of data collection was from July to September 2012.

Participants were given a question booklet with one question per page, and were required to answer each question in the order of appearance before proceeding to subsequent questions.

4. Results and Discussion

The results presented in this section were obtained by undertaking a qualitative analysis of the collected answers given by the teachers in this study, focused on verifying whether the indicators depicted in Table 1 were observed in such answers. In general, this analysis provided a comprehensive picture of the current state of the surveyed Venezuelan secondary school mathematics teachers’ knowledge base on SKT, conceptions of variability, and beliefs about statistics teaching and learning. While the present study is aimed at examining the eight components of professional competencies for teaching variability-related contents identified here, due to space limitations, the discussion in this article will be limited to the results observed in Questions (a)—related to CCK and conceptions of variability—, and Question (f)—related to KCC. While the present study is aimed at examining the eight components of professional competencies for teaching variability-related contents identified here, the discussion in this article will be limited to
the results observed in Questions (a)—related to statistical literacy and conceptions of variability—, and Question (f)—related to KCC. The reason for choosing Question (a) for discussion was that mathematics teachers must be statistically literate themselves—which includes the ability to perform sophisticated acknowledgement of variability—in order to meaningfully teach statistics and then promote statistical literacy among their students at any school level. Question (f) was chosen because that was the question on which participants showed the poorest performance in this survey.

4.1. Results and Discussion Regarding Question (a)

1. Results

Table 3. Results obtained from participants’ answers to Question (a) - Frequency and percentage

<table>
<thead>
<tr>
<th>Category</th>
<th>Frequency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower High School (19 teachers)</td>
<td>Upper High School (15 teachers)</td>
</tr>
<tr>
<td>a0: No response.</td>
<td>2 (10.5)</td>
</tr>
<tr>
<td>a1: Distribution A, giving no reason, just guessing, by arguing intuitive ideas, or based on a mistaken calculation.</td>
<td>1 (5.3)</td>
</tr>
<tr>
<td>a2: Distribution A, based on a misinterpretation related to symmetry and/or a poor fit to a normal distribution.</td>
<td>1 (5.3)</td>
</tr>
<tr>
<td>a3: Distribution A, based on arguments related to differences in the heights of the bars (e.g., “Distribution A is bumpier”; “the number of different heights in A is higher than in B”).</td>
<td>3 (15.8)</td>
</tr>
<tr>
<td>a4: Distribution B, giving no reason, just guessing, by arguing intuitive ideas, or by misinterpretation (e.g., “B has a larger span in frequency than A”).</td>
<td>4 (21.1)</td>
</tr>
<tr>
<td>a5: Distribution B, based on arguments related to simple recognition of variability (i.e., answers concerned only with extremes or the ranges of each distribution; e.g., “because it’s more spread out”).</td>
<td>2 (10.5)</td>
</tr>
<tr>
<td>a6: Distribution B, based on arguments related to sophisticated recognition of variability (i.e., answers connecting both middles and extremes e.g. “because the scores differ more from the center”).</td>
<td>6 (31.5)</td>
</tr>
</tbody>
</table>

The answers given by the surveyed teachers to Question (a) were qualitatively analyzed, and then organized into the six categories previously introduced in Section 3.1. These categories were developed considering both the distribution chosen as answer, and the reasons supporting the choice. To illustrate this point, let us consider the following example. Let us suppose that a teacher answered Question (a) as follows: “Distribution B has more variability, because in B more scores fell far from the mean (0.1 or 9.10)”. Note that the chosen distribution is the correct one—Distribution B—, and the reason given to support such choice is appropriate. Thus, Indicator A-1 seems to be met by this teacher. Note also that this teacher gives an answer mentioning both middles and extremes in data, while considering the distance between them. Thus, this teacher appears not only to acknowledge variability and correctly interpret its meaning in the given setting—meeting Indicator A-2 in this way—but also to be holding the conception of variability known as “Variability as distance or difference from some fixed point” (Shaughnessy, 2007, p. 985), which is regarded in the literature as an answer evidencing sophisticated recognition of variability, and hence will be labelled as belonging to category a6 in Table 3. After carrying out this analysis over the collected data, the results summarized in Table 3 were obtained.

2. Discussion

The setting of the task posed in Item 1—comparing distributions—requires from teachers the mastery of several variability-related concepts, such as the ones of frequency distribution table, distribution, measures of variation, and histogram. In Venezuela, these
concepts should be taught throughout the entire secondary school mathematics, from Grade 7 to Grade 11. Moreover, the Venezuelan mathematics curriculum for each grade at secondary level explicitly states that teachers must foster students' statistical literacy skills by asking them to interpret and analyze the data represented in histograms and frequency distribution tables (ME, 1985; CENAMEC, 1990), and particularly in Grade 11, this analysis and interpretation of data should be done through the use of measures of variation, such as range, quartiles, variance and standard deviation. Therefore, Question (a) was posed in order to see, among others, (i) whether by looking at the histograms of two distributions of scores, teachers could figure out which one has more variability, and then use data-based arguments to defend their answer, in the same way that their students are expected to do; and (ii) how the respondents conceptualize variability in the setting of the given task.

Table 4. Comparison of some measures of variation related to Distributions A and B

<table>
<thead>
<tr>
<th>Measure of Variation</th>
<th>Distribution A</th>
<th>Distribution B</th>
<th>Variability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>6 (discrete)</td>
<td>10 (discrete)</td>
<td>A &lt; B</td>
</tr>
<tr>
<td></td>
<td>7 (continuous)</td>
<td>11 (continuous)</td>
<td></td>
</tr>
<tr>
<td>Variance</td>
<td>3.2</td>
<td>4.5</td>
<td>A &lt; B</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>1.8</td>
<td>2.1</td>
<td>A &lt; B</td>
</tr>
<tr>
<td>Interquartile range</td>
<td>2.7</td>
<td>3</td>
<td>A &lt; B</td>
</tr>
<tr>
<td>Mean deviation</td>
<td>1.4</td>
<td>1.7</td>
<td>A &lt; B</td>
</tr>
</tbody>
</table>

From the information presented in Table 3, it is clear that, despite all the numerical evidence that can be deduced from the given histograms (see Table 4), only 41.5% of the surveyed teachers (22/53) gave a correct response to the posed task; that is, only 41.5% of the participants chose Distribution B and supported their selection on arguments based on simple—those in category a5—or sophisticated—those in category a6—recognition of variability. Among these 22 teachers, 5 of them supported their answer with only the actual or a visual calculation of the range and its interpretation, whereas the remaining 17 teachers engaged in more sophisticated ways to acknowledge variability, such as calculation of the mean deviation (5), standard deviation (4), variance (3), and residuals (1), or providing an answer based on a naked eye description of the degree of data clustering around the mean (7). Among the 17 teachers whose answers fell into category a6, only 6 of them (11.3%) engaged in answering Question (a) using more than one approach—5 teachers used 2 approaches, while 1 teacher used 4. It is noticeable that none of the surveyed teachers used a graphical approach (such as a cumulative frequency polygon) or the interquartile range to answer Question (a), even though these topics must be covered in the mathematics curriculum at both lower (Grade 9) and upper (Grade 11) high school levels, respectively.

Those teachers whose answers were exclusively based on the range—namely those in category a5—do not exhibit an aggregate view of data and distribution, since they are rather concerned with the variability of just the endpoints of the data set without considering a measure of central tendency, which could be interpreted as a very simple acknowledgment of variability (Shaughnessy, 2007, 2008). On the contrary, those teachers whose answers mention both middles and extremes in data, discuss the connections between middles in data and the variability of data dispersed around a measure of central tendency, or even point out deviations of data from the mean or median—i.e., those answers in category a6—, could be placed at an even higher level, since they provide evidence of holding an aggregate view of data and distribution.

It is also noticeable that the group of teachers working only at upper high school level was the one with the highest proportion of correct answers in Question (a), with a 60% (9/15), whereas 42% (8/19) of the surveyed teachers working exclusively on lower
high school, and 26.3% (5/19) of the teachers working at both levels of secondary education, managed to provide a correct answer to the posed task.

Misconceptions commonly exhibited by secondary and tertiary mathematics students were found among the incorrect answers given by the group of surveyed teachers. As it can be appreciated in Table 3, those misconceptions were grouped into categories a2, a3 and a4. Answers falling into category a2 were those in which teachers showed evidence of thinking of variability in terms of symmetry or degree of fit to a normal distribution. In the present study, 9.4% (5/53) of the surveyed teachers showed evidence of holding such misconception.

Into category a3 fall all those answers which selected Distribution A by giving arguments based on differences in the heights of the bars. Some examples of answers belonging to this category are the following: “Distribution A has more variability because it’s bumpier”; “Distribution A has more variability because it has more different heights than Distribution B”; “Distribution A has more variability because the heights of its bars fluctuate a lot”. 9 of the surveyed teachers (17.0%) fell into category a3. It is noticeable that the group of teachers working only at both lower and upper high school levels was the one with the highest proportion of answers in this category, with a 26.3% (5/19).

These findings are similar to those reported in the statistics education literature, which point out that both students and mathematics teachers tend to compare values on the vertical axis, and to conclude that the variable which has “more varied values on Y”, “less pattern on Y” or “is more random on Y” has a larger variability, which is a common misconception in this kind of problems (Garfield et al., 1999; Meletiou & Lee, 2003; Cooper & Shore, 2007; González & Isoda, 2011; Isoda & González, 2012).

Answers falling into category a4 are those that, despite having correctly chosen Distribution B as the one with more variability, provided a wrong justification to support this choice. Among those justifications, there is the common misconception of thinking of variability in terms of the largest span in the vertical axis—i.e., judging the variability of the data displayed in a histogram by the largest difference in height of its bars—, instead of looking at the horizontal spread of data around a measure of central tendency, which is a common mistake made by many students. In the present study, from the 9 teachers belonging to category a4, 5 of them—2 working at lower high school, and 3 working at both levels—gave arguments supporting the view that Distribution B has more variability because the difference in frequency between the highest and lowest bars of the histogram is bigger in Distribution B than in Distribution A. Since these teachers regard variability as the largest span in frequency, they might think that a histogram with narrow tails and a high peak has greater variability than one with bars of more similar heights, which is incorrect.

It is worthwhile to highlight that 4 out of the 17 teachers in category a6 committed errors in the calculations made to support their answers. Nevertheless, the arguments made by these teachers were consistent with the numerical results obtained by them. A similar phenomenon was appreciated in a case study research with similar aims to this study, which was carried out by the author on Japanese secondary school mathematics teachers (cf. González, 2013, 2014).

(3) Regarding the fulfillment of Indicators A-1 and A-2

From the analysis of the collected answers to Question (a), it seems fair to conclude that only 18 of the surveyed teachers were able to meet Indicator A-1, since from the 22 teachers whose answers fell into categories a5 and a6, only 18—those who did not commit committed calculation errors—were able to give a correct and appropriate response to the given task. In relation to Indicator A-2, it seems fair to say that it was fully met by those teachers in categories a5 and a6—i.e., those who showed evidence of simple or
sophisticated recognition of variability—; therefore, Indicator A-2 is fully satisfied only by 22 teachers in the present study.

(4) Regarding teachers’ conceptions of variability:

The answers given by the surveyed teachers provided evidence of the following five conceptions of variability:

Variability in particular values, including extremes or outliers: Teachers whose answers fell into category a5, as well as those teachers in category a4 who regarded variability as the largest span in frequency—10 teachers (18.8% overall)—, are examples of people holding this conception.

Variability as distance or difference from some fixed point: In this study, this conception is exemplified by those teachers in category a6 who provided answers describing the degree of data clustering around the mean—9 teachers (17.0% overall).

Variability as the sum of residuals: In this study, the six teachers in category a6 (11.3% overall) who calculated deviation-based metrics exemplifies this conception.

Variation as distribution: Examples of teachers holding this conception are the 10 teachers (18.8% overall) in category a6 who attempted to explain the data variability by translating the histograms into frequency distribution tables in order to calculate measures of variation such as the variance and standard deviation, and then compare both datasets on the basis of the interpretation of such measures.

Variability as visual cues in the graph: In the present study, the 17 teachers (32.1% overall) whose responses revealed misconceptions such as thinking of variability in terms of symmetry or degree of fit—or lack thereof—to a normal distribution—answers accounted for into categories a2 and a4—; or thinking of symmetrical or quasi-normal distributions as having less—answers accounted for into category a3—or more—answers accounted for into category a4—variability than its asymmetrical counterparts, showed evidence of holding this conception.

4.2. Results and Discussion Regarding Question (f)

(1) Results

The answers provided by the surveyed teachers were analyzed using a “bottom up” approach to coding, in order to identify categories grounded in the data from the identification of frequently occurring answers (Coffey & Atkinson, 1996). After a process of reduction and clustering, finally five categories emerged from the coding process. Table 5 shows a full description of the identified categories, as well as a numerical summary the answers given by the surveyed teachers to this question:

Table 5. Results obtained from participants’ answers to Question (f) - Frequency and percentage

<table>
<thead>
<tr>
<th>Category</th>
<th>Frequency (%)</th>
<th>Lower High School (19 teachers)</th>
<th>Upper High School (15 teachers)</th>
<th>Both Levels (38 teachers)</th>
<th>Total (53 teachers)</th>
</tr>
</thead>
<tbody>
<tr>
<td>f0: No response.</td>
<td>2 (10.5)</td>
<td>2 (13.3)</td>
<td>4 (21.0)</td>
<td>8 (15.1)</td>
<td></td>
</tr>
<tr>
<td>f1: I don’t know / I’m not familiar with the content/ I don’t teach statistics.</td>
<td>8 (42.1)</td>
<td>6 (40.0)</td>
<td>6 (31.6)</td>
<td>20 (37.7)</td>
<td></td>
</tr>
<tr>
<td>f2: General answer, without specification of grade or grade level.</td>
<td>4 (21.0)</td>
<td>0 (0.0)</td>
<td>3 (15.8)</td>
<td>7 (13.2)</td>
<td></td>
</tr>
<tr>
<td>f3: General answer, with specification of grade or grade level.</td>
<td>1 (5.3)</td>
<td>2 (13.3)</td>
<td>5 (26.3)</td>
<td>8 (15.1)</td>
<td></td>
</tr>
<tr>
<td>f4: Mention of specific topics, without specifying the grade or grade level.</td>
<td>1 (5.3)</td>
<td>2 (13.3)</td>
<td>1 (5.3)</td>
<td>4 (7.6)</td>
<td></td>
</tr>
<tr>
<td>f5: Mention of specific topics, specifying the grade or grade level.</td>
<td>1 (5.3)</td>
<td>2 (13.3)</td>
<td>0 (0.0)</td>
<td>6 (11.3)</td>
<td></td>
</tr>
</tbody>
</table>
(2) Discussion

In the present study, 45 of the participants answered this question. The collected answers were categorized into the groups shown in Table 5. Teachers whose answers fell into category f1 are those who admitted either unfamiliarity with how statistical contents are addressed in official curriculum documents across the different grade levels of schooling, or not teaching any statistical content. Unfortunately, this group represents the majority of surveyed teachers, with 37.7% (20/53) of them in this category.

Category f2 collects those answers in which teachers gave general answers, without mentioning a specific grade or grade level. For example, teachers who made mention of “methods to identify trends in data” or “analysis of graphs”, without pointing out which specific methods or graphs they were referring to, or at what grade level such contents are supposed to be taught. 13.2% (7/53) of the surveyed teachers’ answers fell into this category. Similarly to teachers in category f2, those in category f3 gave general answers, but they made mention of a specific grade or grade level. 15.1% (8/53) of the surveyed teachers’ answers fell into this category.

Category f4 consists of all those answers in which teachers made mention of specific statistical topics listed in the Venezuelan mathematics curriculum, without mentioning a specific grade or grade level. For example, teachers who specified “to determine the absolute and relative frequencies of a data set”, “to create a frequency distribution table from unsorted data”, or “to define and calculate the mean, mean deviation and variance”, but did not mention at what grade level such contents are supposed to be taught. Just 7.6% (4/53) of the surveyed teachers’ answers fell into this category. Teachers in category f5 also mentioned specific statistical topics, but contrarily to those in category f4, they pointed out at what grade or grade level such topics are supposed to be taught. Only 11.3% (6/53) of the participants in the present study fell into this category.

It was noticeable that only 3 (5.7%) of the surveyed teachers made explicit mention of embedding the study of particular statistical ideas in a daily-life context, which is vital to internalize in the students that statistics helps solve everyday problems and tasks (cf. Gattuso & Ottaviani, 2011, pp. 122–123, 129), and also recurrently stated in the Venezuelan mathematics curricula at all school levels (ME, 1985, 1997; CENAMEC, 1990). This fact might be a hint of the majority of the participants in this study being “traditionalists” (Eichler, 2011)—i.e., teachers who believe that statistics teaching should emphasize theory and students’ acquisition of algorithmic skills, and are less concerned about applications.

(3) Regarding the fulfillment of Indicator F-1

In order to fully meet this indicator, teachers must show evidence of knowledge about both the specific statistical concepts in the mathematics curriculum, and the grade levels and/or content areas at which students are typically taught such concepts. In the present study, it seems evident that the surveyed teachers whose answers fell into the category f5 are the only ones who may satisfy these conditions. Therefore, based on the gathered data, it would be fair to say that only 11.3% (6/53) of the participants in this study fully met Indicator F-1. However, those teachers in categories f3 and f4 showed evidence of partial fulfillment of this indicator, since they evidenced knowledge of either specific statistical topics present in the Venezuelan mathematics curriculum or the grade level in which such contents are supposed to be taught.
5. Conclusions

In this study, the author contributes to the area of statistics education literature by proposing an eight-fold framework for statistical knowledge for teaching (SKT), comprised by two facets: one cognitive—which takes into account all the six components of mathematics teachers’ professional knowledge identified by Ball et al. (2008)—and one affective-motivational—made up of teachers’ conceptions of variability and statistics-related beliefs. This consideration of eight components to explain SKT addresses specific gaps in statistics education research literature, since none of the previous MKT-based frameworks of SKT reported to date—cf. Groth, 2007; Noll, 2011; Burgess, 2011—takes into account either all the six components identified by Ball et al. (2008), or the role of beliefs and conceptions of variability held by the teachers, which could result in an inaccurate picture of mathematics teachers’ preparedness to teach statistical contents related to variability at any school level. In addition, a survey instrument was developed based on the framework for SKT proposed by the current study. Through this assessment tool, it was possible to gather qualitative evidence about each of the eight hypothesized components comprising SKT, which not only supported the content validity of the questionnaire, but also made possible to thoroughly examine the professional competence to teach variability-related contents held by mathematics teachers. Finally, the framework proposed here draws meaningful connections between the eight components comprising SKT, which could be the focus of further research work.

The survey was carried out on a sample of 53 in-service secondary school mathematics teachers working at the metropolitan area of Caracas, Venezuela, and results from the data collected from such implementation, focusing on teachers’ common content knowledge, knowledge of content and curriculum, and conceptions of variability, are analyzed and discussed in this article.

Based on teachers’ performance in Question (a) of Item 1, some answer tendencies were identified, such as considering the variable in the histograms as a discrete one; making errors in the computation of measures of variation; linking variability as visual attributes of a distribution, such as symmetry and closeness (or lack thereof) of fit to a normal distribution; and a disregard of graphical approaches or the use of quartiles to solve the posed task. Moreover, only the teachers categorized in this case study as a6 exhibited an aggregate view of data and distribution, and appeared to be holding rich conceptions of variability such as “Variation as distribution”—i.e., teachers who combined both centers, spreads and theoretical properties of the distributions in their reasoning to solve the given task. This means that only about a third of the surveyed teachers might have an aggregate-based reasoning, rather than thinking of data as individuals. Through considering data as an aggregate, it is possible to characterize group propensities that can include attention to variability, middles, spread, outliers, clusters, intervals, or residuals, view which is unavailable when focusing on specific data points. Thus, since the majority of the surveyed teachers might be using in their classrooms a data-handling approach focused on individual cases, rather than oriented toward the entire data as a focus of attention, Venezuelan secondary school students might be acquiring bad data-handling habits, which is likely to affect their development of statistical literacy skills (Shaughnessy, 2007; Isoda & González, 2012). Also, with only teachers in categories a5 and a6 answering correctly the posed task (41.5% overall), it seems that a big proportion of the surveyed teachers might lack the statistical literacy skills they must foster in their students.
On the other hand, it seems that the participants in this study have a very limited statistical horizon, since only 6 of 53 teachers (11.3% overall) engaged in answering Question (a) using more than one approach.

In the present case study, it was spotted that almost a third of the surveyed teachers exhibited particular statistical misconceptions which are found often in students at any school level—i.e., those teachers holding the conception “Variability as visual cues in the graph”. These misconceptions not only seem to have affected teachers’ performance in Question (a), but also appear to indicate a poor statistical literacy level of the teachers who hold them. Moreover, conceptions of variability have been reported in the literature to have influence on teachers’ problem-solving and teaching approaches, so these teachers might be passing on their misconceptions to their students (Philipp, 2007).

In relation to KCC, it seems that only 11.3% (6/53) of the surveyed teachers have knowledge of both the specific statistical concepts in the mathematics curriculum, and the grade levels at which students are typically taught such concepts. This finding might hinder the development of statistical literacy in Venezuelan students, since KCC is not only one of the main poles on which the professional knowledge required for teaching statistics stands, but also a fundamental aspect related to lesson planning. (Ponte, 2011, pp. 300–301). Also, due to the disregard by 50 of 53 participants of connecting the teaching of statistical concepts with a daily-life context when answering Question (f), the majority of the surveyed teachers might be considered as “traditionalists”—i.e., teachers who believe that statistics teaching should emphasize theory, memorization and students’ acquisition of algorithmic skills, and are less concerned about applications.

The fact that the surveyed teachers in this study showed, among others, statistical misconceptions, poor level of statistical literacy, a weak knowledge base of KCC, and evidence of being “traditional teachers”, several cognitive and pedagogical skills related to SKT should be strengthen, such as the abilities to relate the given task to different data representations, making appropriate interpretations of variability and knowing about the contents and goals of the curriculum related to the teaching and learning of statistics within school mathematics. Therefore, given the evidence gathered so far by the present study, Venezuelan lower and upper secondary mathematics teachers surely need to follow—during pre- and in-service training—not only courses in statistics to acquire statistical subject matter knowledge and develop their statistical literacy, but also they need training in the didactic of statistics to develop statistical pedagogical content knowledge and, then, be able to carry out their demanding role in both achieving the goals of mathematics curriculum related to statistics education and promoting statistical literacy in students.

References


Centro Nacional para el Mejoramiento de la Enseñanza de la Ciencia y la Matemática (CENAMEC) (1990). Programa de Articulación: Contenidos de Matemática para la
Survey on Secondary School Teachers’ Statistical Knowledge for Teaching: The Need of Developing Venezuelan Teachers’ Competence to Teach Statistics

Educación Media, Diversificada y Profesional. Primer y Segundo Año (Ciencias) (Caracas: CENAMEC).


**Author**

Orlando GONZÁLEZ

Graduate School of International Development and Cooperation, Hiroshima University

TEL: 082-422-6944

E-mail: ogonzalez@hiroshima-u.ac.jp